Spectral Enrichment in Brass Instruments Due to Nonlinear Sound Propagation: a Comparison of Measurements and Predictions

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Since the discovery nearly twenty years ago that nonlinear propagation of the internal sound wave in a trombone was primarily responsible for the "brassy" timbre in fortissimo playing, it has become increasingly clear that nonlinear distortion even at moderate sound levels can contribute significantly to the tonal character of a brass instrument. Previous work has explored the effects of bore profile and viscous damping on the rate at which spectral enrichment due to nonlinear distortion develops in brass instruments. This paper reviews evidence from experimental measurements and numerical simulations of nonlinear propagation in brass instruments with different bore shapes and sizes, and discusses the possibility of deriving a quantitative prediction of the relative importance of nonlinear spectral enrichment in a brass instrument from measurements of its bore.

1 Introduction

One of the most characteristic features of the sound of a brass instrument is the way in which the timbre changes during a crescendo. The frequency spectrum of a note played quietly on a trumpet, trombone or horn is usually dominated by the first few harmonics. As the dynamic level is increased the spectral centroid also increases, and at the fortissimo level the sound has an extremely bright, "brassy" timbre. The enrichment of the spectrum during a crescendo is slower in predominantly conical instruments like the tuba, and the brassy timbre may not develop even at the fortissimo level. While the nonlinear nature of the relationship between the air flow through the lips and the pressure difference across them is partly responsible for the spectral enrichment during a crescendo, the dominant factor at high dynamic levels is the nonlinear character of sound propagation in the air column inside the instrument [1]. The most spectacular consequence of nonlinear propagation is the brassy sound of a trumpet played fortissimo, but it is also a significant factor in the timbral character of instruments played at the mezzoforte dynamic level [2].

In a section of expanding bore in a brass instrument, the pressure amplitude of the forward travelling wave diminishes, and this in turn reduces the contribution of the widened section to the cumulative nonlinear distortion. This effect of expansion of a flaring tube of diameter $D(x)$ with distance x from the entrance can be modelled using the "stretched length"

$$z(x) = \int_0^x \frac{D(0)}{D(x)} dx \tag{1}$$

The relative importance of spectral enrichment due to nonlinear propagation in brass instruments with different bore profiles can be estimated using the brassiness potential parameter B, defined as

$$B = \frac{1}{L_{ecl}} \int_{x=0}^{x=L} \frac{D_{min}}{D(x)} dx, \tag{2}$$

where L is the total length of the instrument air column from mouthpiece to bell, and L_{ecl} is the equivalent cone length of the instrument [3]. D_{min} is the minimum diameter of the bore; since the instrument usually begins with a short inward tapering mouthpiece receiver, the minimum diameter is typically a few centimetres from the tube entrance.

The degree of spectral enrichment at a given dynamic level depends not only on the relative bore profile $D(x)/D_{min}$ but also on the absolute value of D_{min}. The major reason for this dependence on absolute radial scale is that the musically relevant dynamic level of a note is determined by the pressure amplitude in the radiated sound, while the extent of nonlinear distortion of the propagating wave depends on the pressure amplitude in the internal air column of the instrument. The pressure transfer function from the input of a wide bored instrument to a point in the external radiation field is larger than that for a narrow bored instrument, so a given amplitude of radiated sound will be generated by a smaller input pressure amplitude in the wider instrument. The wider instrument is therefore expected to have less spectral enrichment at a given output level than a narrow instrument with the same relative bore profile.

The absolute radial scale of the bore profile also has an influence on the spectral centroid of the propagating wave inside a brass instrument because of the dependence of viscothermal losses on the tube diameter. Since the losses increase with frequency they reduce the spectral centroid of the propagating wave; this effect is greater in tubes of narrower diameter, and therefore affects the timbre of the radiated sound in the opposite sense to the transfer function effect described in the preceding paragraph [4].

To aid in the understanding of the physical processes that determine the musically significant differences between brass instruments of different types, and to provide a convenient tool for brass instrument taxonomy, it is desirable to find a single parameter which provides an estimate of the likely spectral enrichment at the forte level in a given instrument. Such a parameter should be calculable from geometrical data about the instrument bore, and should incorporate the effects of both relative and absolute bore profiles. This paper describes a tentative approach to this goal.

Four brass instruments were selected from the extensive collection of brass instruments in the Edinburgh University Collection of Historic Musical Instruments. Relevant bore profile data for the four instruments are presented in Table 1

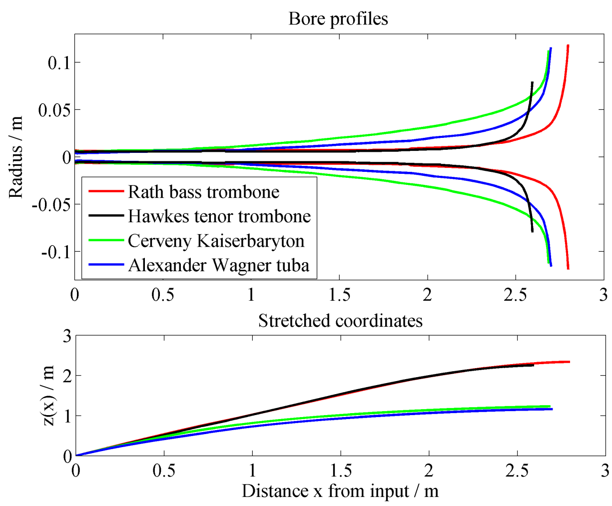

Figure 1: Above: Bore profiles for the four instruments studied. Below: Stretched coordinate $z(x)$ for each instrument.

Table 1: Data for the four brass instruments studied.

Maker	Instrument	$D(0)$/mm	D_{min}/mm	B
Rath	bass trombone eu5877	14.0	12.4	0.70
Hawkes	tenor trombone eu5717	11.6	10.6	0.70
Cerveny	Kaiserbaryton eu3412	12.3	10.9	0.37
Alexander	Wagner tuba eu2854	8.3	7.7	0.36

The bore profiles and stretched length curves for the four instruments are illustrated in Figure 1. The two trombones are predominantly cylindrical instruments with almost identical relative profiles, and each has a value of $B = 0.7$; however, the Rath bass trombone is significantly wider than the Hawkes tenor. The Cerveny Kaiserbaryton (a type of tuba) and the Alexander Wagner tuba are predominantly conical instruments with very similar values of B (0.37 and 0.36 respectively), but the Wagner tuba has a much smaller radial scale than the Kaiserbaryton. These bore characteristics were chosen to aid in separating the influences of B and $D(0)$ on spectral enrichment. The four instruments are shown in Figure 2.

Figure 2: Photograph of the four instruments used in the study. From left to right: Cerveny Kaiserbaryton, Alexander Wagner tuba, Rath bass trombone, Hawkes tenor trombone.

2 Experimental study

The experimental setup was similar to that used in previous studies of nonlinear propagation in brass instruments [3]. To minimise complications arising from the nonlinear behaviour of the lip valve and the existence of

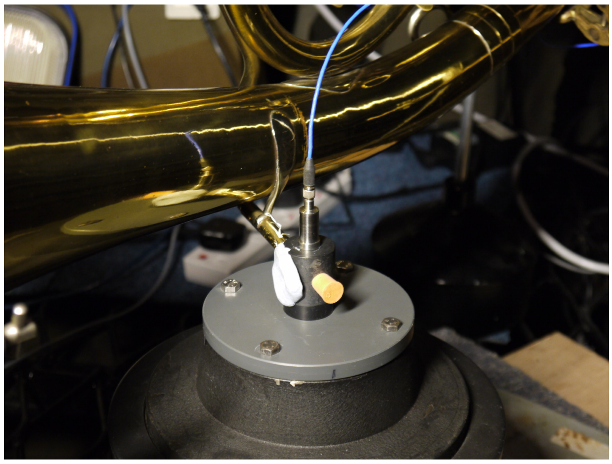

Figure 3: Detail of the horn driver and coupler.

strong standing waves in the air columns, the instruments were driven by a sinusoidal signal with a frequency of 2500 Hz which was well above the bell cutoff for all the instruments. The acoustic signal was supplied by a JBL 2446H horn driver fitted with a small cylindrical coupler which contained a PCB pressure sensor. The coupler, illustrated in Figure 3, had a number of apertures into which the mouthpiece receivers of the brass instruments could be directly inserted. A $\frac{1}{4}$" B&K microphone was used to measure the acoustic pressure on the instrument axis in the plane of the bell exit, while a $\frac{1}{4}$" GRAS microphone measured the radiated pressure on axis 50 cm outside the bell. The signal generation and data acquisition were performed using a National Instruments DAQ card and a Labview program. The experiment was carried out in an anechoic chamber.

3 Experimental results

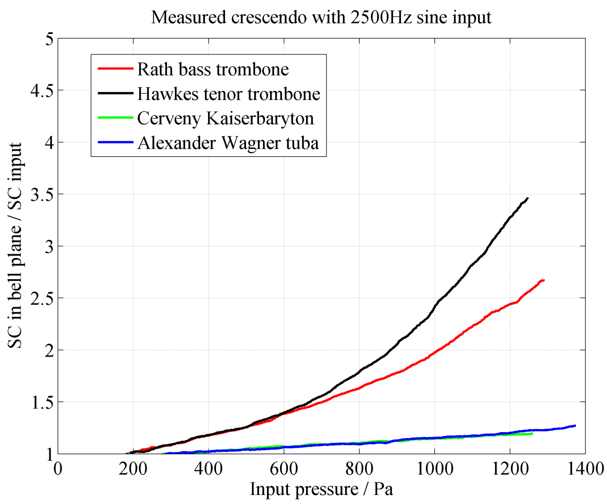

Figure 4: Ratio of spectral centroids measured in the bell plane and at the input for the four instruments studied, as a function of input pressure rms amplitude.

The spectral centroid SC of the pressure signal measured in the bell plane of each instrument during a linear sweep in amplitude is shown in Figure 4. The spectral centroid is normalised by dividing the amplitude-weighted mean frequency in the spectrum by the input frequency of 2500 Hz. To compensate for possible distortion in the generation of the input signal, each value of the spectral centroid at the bell is divided by the corresponding spectral centroid of the measured input pressure; in all the measurements shown the input spectral centroid was close to unity.

For input pressures up to 600 Pa instruments with the same B value have the same rate of increase of SC, but the results diverge above this pressure. Instruments with lower B values have lower SC at 600 Pa than instruments with high B. Below 600 Pa the measurements are consistent with the theoretical prediction that the degree of nonlinear distortion of an input sine wave with a given pressure amplitude which accumulates as it propagates from input to bell is determined primarily by the value of B.

4 Simulations of nonlinear propagation

Nonlinear wave propagation presents significant challenges for simulation techniques, particularly in shock wave formation, where the inclusion of nonlinear effects within a finite difference simulation of wave propagation produces numerical oscillation at shock fronts as observed by Bilbao [5]. In general, simulating nonlinear wave propagation is complicated by the fact that forward and backward going waves in a duct are not independent of one another. Solutions in the instruments here were performed using numerical solutions of the generalised Burgers' equation for weakly nonlinear wave propagation, a technique that makes the assumption that forward and backward going waves may be well approximated as being independent [6, 7]. Resulting simulations have shown good agreement with experiment for bores of varying cross section and over distances associated with shock formation, particularly in terms of assessing the relative spectral enrichment associated with different bores [3].

5 Simulation results

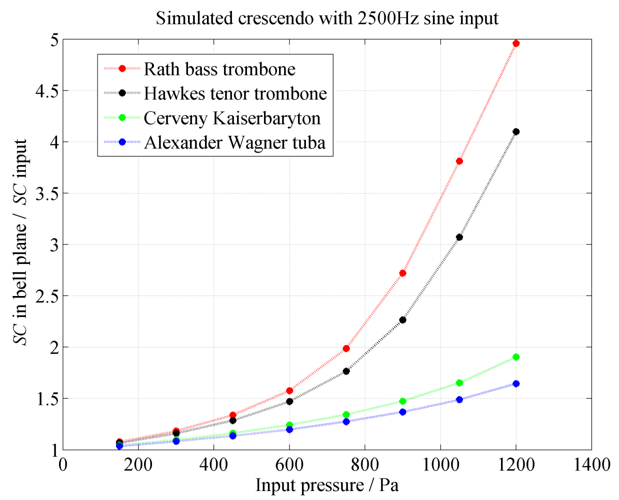

Figure 5: Ratio of simulated spectral centroids in the bell plane and at the input for the four instruments studied, as a function of input pressure rms amplitude.

Results of the simulations are shown in Figure 5. The shapes of the curves of spectral centroid versus input pressure are broadly consistent with the experimental curves shown in Figure 4, although there are quantitative differences which require further investigation. The value of the simulated spectral centroid is primarily determined by the value of B, although a small influence of $D(0)$ is also evident. For instruments with similar values of B, the narrower instrument has a lower spectral centroid, suggesting that this effect is due to viscothermal losses.

6 Influence of input diameter on spectral enrichment

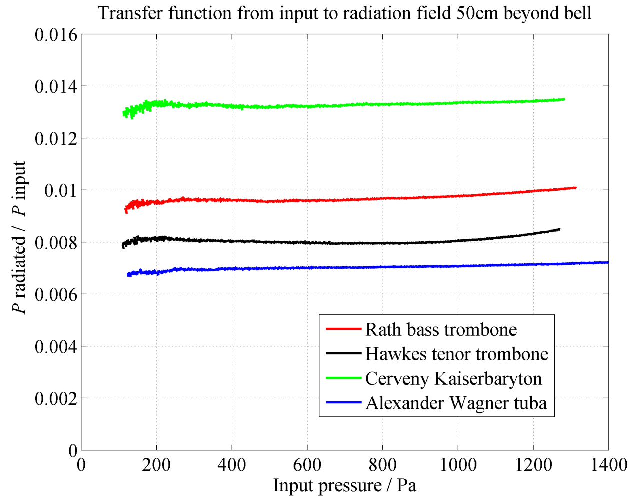

Figure 6: Measured pressure transfer functions between the input of each instrument and a point 50 cm outside the bell on axis.

Figure 6 shows measured transfer functions from input to a point on axis 50 cm outside the bell. Neglecting losses, the transfer function for an instrument with a given relative bore profile should depend inversely on the square of the input diameter $D(0)$. For the Kaiserbaryton (Kb) and Wagner tuba (Wt), with $B = 0.37$ and 0.36 respectively, the ratio of transfer functions is 1.9; for these two instruments $D(0)_{Kb}/D(0)_{Wt} = 1.5$, while $(D(0)_{Kb}/D(0)_{Wt})^2 = 2.2$. For the Rath bass trombone (Rb) and the Hawkes tenor trombone (Ht), both with $B = 0.7$, the ratio of transfer functions is 1.2, while $D(0)_{Rb}/D(0)_{Ht} = 1.2$ and $(D(0)_{Ra}/D(0)_{Ht})^2 = 1.5$. From these experimental results it appears that the ratio of input areas significantly overestimates the change in transfer function; this is possibly a consequence of the neglect of losses.

7 A proposed spectral enrichment parameter $E(B, D_0)$

The goal of the work presented here is to derive a simple empirical formula which encapsulates, at least to a first level of approximation, the effects of the geometrical structure of a brass instrument on the extent of spectral enrichment arising from nonlinear propagation in the instrument. The geometrical information is expressed by the dimensionless brassiness potential parameter B and the input diameter $D(0)$. The present work confirms previous evidence that B is monotonically related to the degree of spectral enrichment expected at a given frequency and input pressure level in that instrument [3], but the functional form of that dependence is far from obvious. The present study suggests that measurement and theory are in broad agreement for input pressures up to around 600 Pa, so it is proposed that a test of an instrument with a sine wave input at frequency 2500 Hz and input pressure 600 Pa should be taken as a benchmark. The simulation results presented in Figure 5 show that, for the four instruments studied and the benchmark excitation, the relationship between B and the spectral centroid SC_{bell} of the pressure signal measured in the bell plane can be represented approximately by the expression

$$SC_{bell} - 1 \propto B. \tag{3}$$

The transformation from the signal at the bell plane to the signal in the radiation field 50 cm beyond the bell plane requires an estimation of the effect of the radial scale of the instrument. This depends on the transfer function (TF) from input to radiation field. It was shown in the previous section that postulating $TF \propto D(0)^2$ exaggerated the radial dependence of the transfer function. The influence of losses can to a first approximation be taken into account by postulating $TF \propto D(0)^n$ with $n < 2$. A reasonable fit to the spectral centroid data was found for a choice of $n = 1$.

Combining the postulated dependence of spectral enrichment on $D(0)$ with the dependence on B postulated in Equation 3 leads to the formula

$$E = 1 + \alpha B/D(0). \tag{4}$$

where the spectral enrichment parameter E is an estimate of the spectral centroid 50 cm from the bell of an instrument with input diameter $D(0)$ and brassiness potential parameter B, for a sinusoidal input with frequency 2500 Hz and a fixed radiated pressure amplitude. The value of the coefficient α in this empirical formula can be found by fitting to the measured or simulated spectral centroids of the radiated pressure signals.

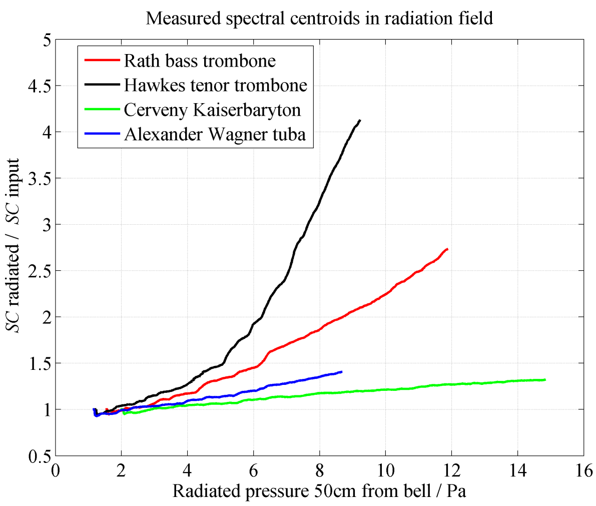

Figure 7: Spectral centroids of pressures measured 50 cm from the bell of the four instruments versus rms radiated pressure amplitude.

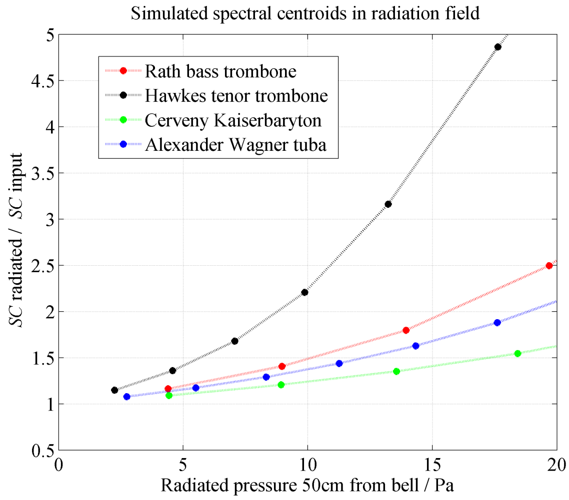

Figure 8: Spectral centroids of pressures simulated 50 cm from the bell of the four instruments versus rms radiated pressure amplitude.

The measured and simulated spectral centroid curves for the radiated sound, plotted against the radiated pressure amplitude, are shown in Figures 7 and 8 respectively. At a reference radiated pressure amplitude of 6 Pa the simulated spectral centroid values are given in Table 2. Fitting Equation 4 to the result for the Rath trombone, with $B = 0.7$, $D(0) = 14.0$ mm and $E = SC = 1.25$ yields the value $\alpha = 5$ mm. The formula for spectral enrichment is then

$$E(B, D(0)) = 1 + 5B/D(0). \tag{5}$$

with $D(0)$ in mm.

Values of E for the four instruments are listed in Table 2. Comparison with the results of the simulation shows that the rank order of spectral enrichment predicted by the calculation of E agrees with that derived from the simulations, although the enrichment of the tenor trombone is underestimated.

Table 2: Simulated radiated spectral centroids and spectral enrichment values.

Maker	Instrument	SC (radiated)	E
Rath	bass trombone	1.25	1.25
Hawkes	tenor trombone	1.54	1.30
Cerveny	Kaiserbaryton	1.13	1.15
Alexander	Wagner tuba	1.20	1.22

Contours of equal spectral enrichment E are shown in Figure 9, in which the four measured instruments are represented by circles on a $(B, D(0))$ plot. As an example of the utility of such a contoured plot it can be noted that, although the Kaiserbaryton has a slightly higher value of brassiness potential parameter B than the Wagner tuba, the Wagner tuba has a larger enrichment parameter than the Kaiserbaryton. This predicted difference in musical behaviour is confirmed by the measured and simulated spectral centroid plots in Figures 7 and 8.

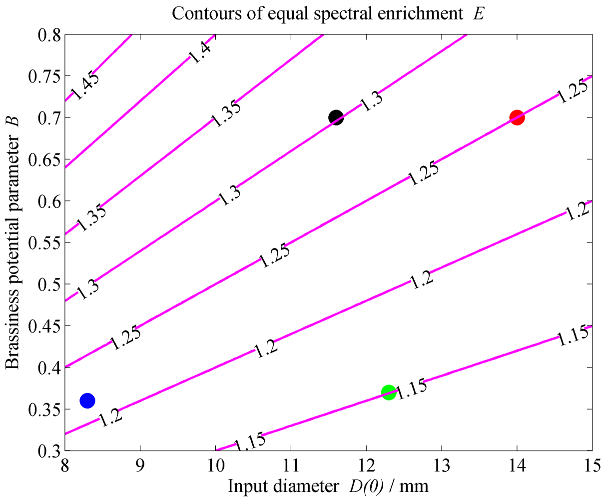

Figure 9: Representation of the four instruments studied on a $B, D(0)$ plot. Red circle: Rath bass trombone; black circle: Hawkes tenor trombone; green circle: Cerveny Kaiserbaryton; blue circle: Alexander Wagner tuba. Magenta lines are contours of equal spectral enrichment $E(B, D(0))$.

8 Conclusion

The benchmark test of spectral enrichment used in the present study involved the injection of a 2500 Hz sine wave into the mouthpiece receiver of the instrument. The spectral enrichment parameter E is thus defined for this idealised situation, and does not necessarily bear any direct relationship to the much more complicated situation in which a player's lips and a mouthpiece are the source of the injected sound wave. Nevertheless, the predictions of the behaviour of the four studied brass instruments based on the enrichment contours in Figure 9 are broadly in accord with the playing and listening experience of musicians. Many further experimental tests and simulations, based both on sine wave excitation and on lip excitation, will be required to establish whether the concept of spectral enrichment proposed here is of genuine musical utility, and to refine the acoustical basis of its definition.

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