Dependence of the acoustic power produced by a woodwind on the tonehole size

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It is well known that for a given note the position of woodwind toneholes can be chosen by the maker in a certain portion of the instrument, provided that the radius is properly chosen: a wide radius needs to be located further from the reed than a narrow one. In our work a simplified problem with one radiating source only is investigated: the problem of a short diaphragm at the end of a cylindrical tube, excited by a clarinet-like reed/mouthpiece. We consider tubes of different lengths provided with diaphragms of different radii, in order to keep a fixed playing frequency. Obviously the power radiated by the orifice decreases when the diaphragm radius decreases. But it is not intuitive that when the radius is large enough, the power is found to be almost independent of the radius. Indeed it can be shown that the radiated power depends only on the output flow rate. Moreover energy considerations show that the ratio between the input pressure and the output flow rate does not depend on the length, thus on the diaphragm radius. Finally when losses are ignored, the input pressure depends on the excitation pressure in the mouth but is independent of the tube. At low level of excitation, this simple explanation is confirmed by both numerical calculation and experiment, but experiment confirms the importance of nonlinear effects with flow separation due to sharp corners.

1 Introduction

When designing a woodwind for a given playing frequency, the maker can choose the size and location of a tonehole in a rather large range. If the choice is a very wide hole, the effect is close to that of cutting the tube at the hole location, at least at low frequencies. However it is also possible to choose a narrower hole with a location chosen upstream of the wide hole.

The present paper aims to investigate the following subject: is the above mentioned choice important for the amplitude of the acoustic radiated power? Obviously if the hole has a size tending to zero, the acoustic power radiated by this hole tends also to zero. But what happens when the size of the hole increases? How increases the power? To our knowledge, this question has not yet being treated in the literature. The answer is not necessarily intuitive, because at low frequencies the real part of the radiation acoustic impedance (which is defined as the ratio of a pressure to a flow rate) does not depend on the size of the hole).

In the present paper we consider a simplified problem, with only one radiating orifice: a tube terminated into a constriction of different sizes, excited by a clarinet-like reed and mouthpiece (for flute-like instrument, there would be more than one orifice, because of the existence of the mouth hole, which is a part of the exciting system). The effect of this simplification is discussed in section 2.4. Moreover, the scope of this paper is limited to notes whose frequency corresponds to the first impedance peak.

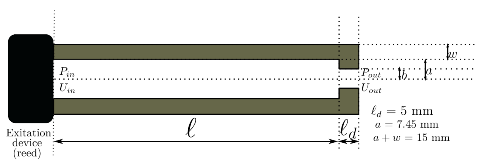

Figure 1: Geometry of the tube with diaphragm. The length ℓ depends on the diaphragm radius b, with a fixed first resonance frequency $f_1 = 250 Hz$.

The geometry is shown in Fig.1. A cylindrical tube of radius[1] $a = 7.45$ *mm* is terminated in a cylindrical diaphragm of length $\ell_d = 5$ *mm*, which is approximately equal to the wall thickness of a clarinet, and of radius b, which is chosen among the following values: 6, 5, 4, 3, 2 *mm* (For a clarinet, the tonehole radius varies from 2 or 2.5mm in the higher part of the instrument to 6mm for the hole which is close to the bell). The case of a tube without diaphragm is also considered. The length ℓ of the tube is chosen in order to keep the first resonance frequency independent of the diaphragm radius, as explained below, equal to 250 *Hz*. Thus ℓ depends on the diaphragm radius, similarly to the case of a tube with a tonehole. Without diaphragm, if the sound velocity in free space at 20°C is $c = 343.4\ m.s^{-1}$, ℓ is equal to 328 *mm*. With a diaphragm, the length ℓ is equal to 323, 317, 306, 286, 237 *mm* for the widest to the narrowest radius, respectively. The tube thickness is $w = 7.55$ *mm*. The tube is excited by a clarinet-like reed and mouthpiece.

In section 2, we present an elementary theoretical analysis, limited to the first harmonic, for reasons explained further. Then we show that complete simulations of self-sustained oscillations lead to more precise results (section 3), and section 4 shows experimental results.

[1]Common output radius of a mouthpiece.

2 Elementary theoretical analysis (power radiated by the first harmonic)

2.1 Simplified linear model of the resonator

For the calculation of the radiated power, two transfer functions of the resonator have to be determined in the frequency domain: the transfer admittance between the output flow rate U_{out} and the input pressure P_{in} for the radiation and the input impedance $Z_{in} = P_{in}/U_{in}$ for the coupling with the excitation mechanism. For these transfer functions, the simplest model of the present section ignores the resonator losses (i.e. both boundary layer and radiation losses). However assuming that $kb << 1$, and therefore assuming that the diaphragm radiates into infinite space as a monopole, the mean radiation power can be deduced from the knowledge of the flow rate U_{out},

$$\mathcal{P}_r = \frac{1}{2}\Re(Z_r)|U_{out}|^2, \tag{1}$$

where Z_r is the radiation impedance. As discussed above, the real part of this impedance does not depend on the diaphragm radius:

$$\Re(Z_r) = \frac{k^2 \rho c}{4\pi}, \tag{2}$$

$k = \omega/c$ is the wavenumber, where ω is the angular frequency, and ρ is the air density. The cross section areas of the tube and diaphragm are denoted $S = \pi a^2$ and $S_d = \pi b^2$, respectively. The model is based on the following (standard) transfer-matrix relationship:

$$\begin{pmatrix} P_{in} \\ U_{in} \end{pmatrix} = \begin{pmatrix} \cos(k\ell) & jZ_c \sin(k\ell) \\ jZ_c^{-1}\sin(k\ell) & \cos(k\ell) \end{pmatrix} \begin{pmatrix} 1 & j\omega M_t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ U_{out} \end{pmatrix}. \tag{3}$$

$Z_c = \rho c/S$ is the characteristic impedance, and $j = (-1)^{1/2}$. The value of M_t, the total acoustic mass of the diaphragm, is not discussed here. When radiation losses are ignored, the radiation contribution can be assimilated as an acoustic mass M_r in this low frequency analysis, with the following value of the output pressure: $P_{out} = j\omega M_r U_{out}$. M_t/Z_c, which involves M_r increases when b decreases. Compressibility effect inside the diaphragm is ignored. This implies $k\ell_d << 1$.

2.2 Transfer admittance of the resonator

If both visco-thermal and radiation losses are ignored, the input impedance can be deduced from Eq. (3),

$$Z_{in} = jZ_c \tan\left[k(\ell + \ell_{eq})\right] \tag{4}$$

$$k\ell_{eq} = \arctan\left[\frac{\omega M_t}{Z_c}\right]. \tag{5}$$

The transfer admittance is given by:

$$\frac{U_{out}}{P_{in}} = \frac{-j}{Z_c}\frac{\cos(k\ell_{eq})}{\sin\left[k(\ell + \ell_{eq})\right]}. \tag{6}$$

The equivalent length ℓ_{eq} a priori is frequency dependent: it decreases when the frequency increases, therefore the diaphragms generate positive inharmonicity. For the case without diaphragm $b = a$, the choice of the total length $\ell + \ell_d$ implies the choice of the first resonance frequency, $f_1 = c/4/(\ell + \ell_{eq})$, where $\ell_{eq} = \ell_d + \delta_d a$ ($\delta_d \simeq 0.7$). For the other cases the same resonance frequency is chosen, then the equivalent length ℓ_{eq} is given by Eq. (5) and the length ℓ is deduced from Eq. (4), with an infinite input impedance:

$$\ell = \frac{\pi}{2k_1} - \ell_{eq}. \tag{7}$$

Then the transfer admittance simplifies in $(-j/Z_c)$, if the following condition is fulfilled:

$$k_1\ell_{eq} << 1 \tag{8}$$

or

$$\omega_1 M_t << Z_c. \tag{9}$$

Considering Eq. (2) and if the condition (8) is fulfilled, i.e. if the diaphragm is not too narrow, its diameter *has no effect on the radiated power* for a given input pressure P_{in}. For instance, if with $k_1\ell_{eq} < 0.1$, the radius needs to satisfy: $b > 5.2$ *mm*.

2.3 Reed and mouthpiece coupled to the resonator

It remains to take into account the excitation mechanism, in order to analyze the relationship between the radiated power and the excitation pressure, which is the pressure p_m in the instrumentalist's mouth. For this purpose, the classical model proposed in [1] is widely acceptable, and is simplified by ignoring the reed dynamics: with the quasi-static nonlinear characteristic $u_{in} = F(p_{in})$, where $p_{in}(t)$ and $u_{in}(t)$ are the inverse FT of $P_{in}(\omega)$ and $U_{in}(\omega)$. When no diaphragm is present the resulting pressure signal $p_{in}(t)$ is a square signal because resonator losses also are ignored (thus all resonances are infinite and with harmonic frequencies). Then it is easily shown that assuming the validity of Eq.(2) the radiated pressure is the derivative of a square signal.

However a problem occurs: such a signal corresponds to an infinite power. The reason is that Eq.(2) is not valid at higher frequencies. Therefore for the elementary analysis we use the approximation of the first harmonic, and will refine the calculation in the next section. Near the threshold, the higher harmonics are very weak, and this is particularly true when inharmonicity occurs (because of the diaphragms). In Ref.[2], the following relationship was found for the first harmonic:

$$P_{in}(\omega_1) = p_M\sqrt{\frac{Y_1 - A}{3C}} \tag{10}$$

p_M is the closure pressure, proportional to the reed stiffness and to the reed opening at rest; $Y_n = Z_c/Z_{in}(n\omega_1)$ is the dimensionless input admittance. Eq. (10) implies that Y_1 is real, and therefore the value of the operating frequency can be deduced. A and C are the first and third orders of the Taylor expansion (around $p_{in} = 0$) of the nonlinear characteristic $F(p_{in})$. Because the resonator losses are ignored, $Y_1 = 0$ and the input pressure $P_{in}(\omega_1)$ at frequency f_1 does not depend on the radius of the diaphragm; this is also true for the output flow rate $U_{out}(\omega_1)$ as well as for the radiated power. We summarize the hypotheses of this result: the diaphragm is wide enough (condition (8)); no resonator losses are considered; and the input acoustic pressure is reduced to a sine signal. The value of the power radiated by the first harmonic is:

$$\mathcal{P}_r = \frac{k^2\rho c}{8\pi}\frac{1}{Z_c^2}p_M^2\left(-\frac{A}{3C}\right). \tag{11}$$

This nonlinear relationship is written in dimensionless quantities in [3]. The simplest control parameters that can be defined with such a model are the mouth pressure p_m and the reed channel opening area S_c. Corresponding dimensionless quantities are $\gamma = p_m/p_M$ and $\zeta = Z_c S_c\sqrt{2/(\rho p_M)}$, respectively. The values of the polynomial coefficients are:

$$A = \zeta\frac{3\gamma - 1}{2\sqrt{\gamma}};\ C = -\zeta\frac{\gamma + 1}{16\gamma^{5/2}}. \tag{12}$$

Notice that the oscillation threshold $\gamma = 1/3$ ($A = 0$) does not depend on the diaphragm as well, and that the Taylor expansion of the function $F(p_{in})$ is valid for a non-beating reed only, approximately for $\gamma < 1/2$.

2.4 Generalization to a side hole

The generalization of the elementary approach to a unique side hole is possible, with some restrictive hypotheses, because it is impossible to treat in a simple way the general case of a complete lattice of toneholes. Considering a tube of total length L, and one tonehole located at a distance ℓ_{down}, and ℓ from the tube end and entry, respectively. When it is closed, the frequency is $c/(4L)$, and when it is open, it is $c/(4L_1)$, with $L = \ell + \ell_{down}$ and $L_1 = \ell + \ell_{eq}$. We suppose that there is one semi-tone between the two frequencies, i.e. $L_1 \simeq 0.94L$. At low frequencies, the tube portion downstream the hole is an acoustic mass, $\rho\ell_{down}/S$, in parallel with the acoustic mass of the hole M_h and that of the tube. Consequently we can use the model given by Eq. (3), replacing the mass M_t by M_{dh} given by:

$$\frac{1}{M_{dh}} = \frac{1}{M_h} + \frac{S}{\rho\ell_{down}}. \qquad (13)$$

In order to use the same analysis than for a diaphragm, M_{dh} needs to be small; actually $M_{dh}^{-1} > M_h^{-1}$, therefore if the acoustic mass of the hole is sufficiently small, the analysis applies. Moreover the flow rate U_{out} entering the mass M_{dh} is the sum of the flow rates going out of the hole and of the tube. If the distance ℓ_{down} is smaller than the wavelength, the total flow rate radiates as a single monopole in the far field, and Eq. (1) remains valid. Therefore the elementary analysis remains valid if the hole is wide enough.

2.5 Taking losses into account

When taking losses into account, we observe a small decrease of the input impedance at frequency f_1 when the diaphragm radius decreases, thus a small decrease of the ratio $p_M/P_{in}(\omega_1)$ (see Eq. (10)). Another effect of the losses is a shift of the oscillation threshold. The same equation (10) shows that because $\Re\left[Y_{in}(\omega_1)\right]$ is positive, the value of the threshold value of the coefficient A, and therefore that of the mouth pressure γ, increases. Strictly speaking, a consequence is that at very low excitation levels, where the power is a quasi-vertical function of the excitation level, the effect of the diaphragm radius is very strong for a fixed γ. But the practical consequence is small because the power remains weak. Notice that near the threshold the sound is really sinusoidal, therefore the previous analysis is relevant.

3 Numerical simulation of the power for a complex signal

With the help of the numerical tool developed in [4] sound synthesis can be realized to obtain the input pressure in both the non-beating and beating reed regimes, considering a sound with several harmonics. The simulation uses the computed input impedance, and the modal expansion is determined by optimization. The number of resonator modes is fixed to 4, corresponding to contributions below approximately 3 kHz. The reed dynamics is taken into account with one reed mode. Realistic value of the reed natural frequency is chosen to be $f_r = 2341$ Hz and the quality factor $Q_r = 0.8$.

In order to reach the steady-state regime for different values of the excitation pressure γ, the computation is done considering a step function for $\gamma(t)$. The simulation provides the spectrum of the input pressure p_{in}, then the transfer functions are calculated by using Eq. (6). The number of harmonics considered here are only limited by the Nyquist frequency ($Fe/2 = 22050Hz$). Obviously the model of the resonator is not suitable at very high frequencies, but the contribution of the highest harmonics to the radiated power is very small. Fig. 2 shows the results for the diaphragms of different radii. In the beating reed regime and before extinction, the radiated power ratio between the two extremes configurations doesn't exceed 3 dB for a sound power level that approaches 95 dB.

Notice that the playing frequency is slightly increasing when the pressure γ increases, about 30 cents between the oscillation and extinction thresholds because of the inharmonicity effect. The main difference with the simplest analytical result (11) is probably the effect of the threshold shift who distinguishes the different curves, especially for small radii. We do not discuss here the influence of the spectrum.

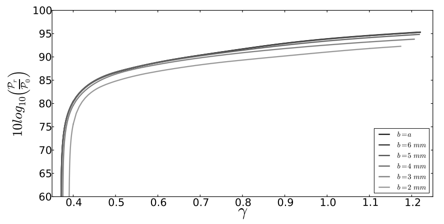

Figure 2: Radiated power computed for the six configurations. The line darkness is proportional to the considered diaphragm radius.

4 Experiment

Experiments were carried out with a static pressure controlled device provided with artificial lips to handle the reed aperture as explained in [5]. No sound was obtained for the narrowest diaphragm ($b = 2$ mm). The power is not directly measured in an anechoic room but a pressure measurement is realized and primary reflecting surface were covered with acoustic foam to minimize interferences. A half inch Bruel & Kjaer microphone was located at 50 cm from the radiating orifice and the RMS sound pressure level is extracted with a 50 ms integration time and represented in figure 3. Measurement of the static pressure in the downward cavity of the mouthpiece is performed with a Endevco piezoresistive pressure transducer. Acquisition were carried out at a 20kHz sampling frequency for a fixed 2 minutes duration, and divided into two parts for the increasing and decreasing linear pressure ramps.

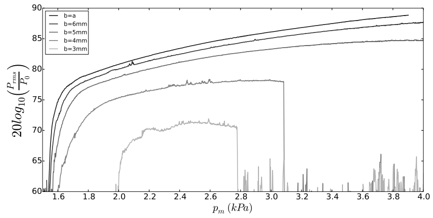

Figure 3: Measured sound pressure level radiated at a distance of 50 cm from the diaphragms. The self-sustained oscillations are managed with an artificial mouth and a controlled static pressure. Only the slowly decreasing pressure ramps (3 $kPa.min^{-1}$) are represented to focus around the threshold region. No sound was able to be emitted with the 2 mm diaphragm.

Our interest is focused on the relative values between the different cases, not on a quantitative comparison between theory and experiment which implies a determination of the parameters of the mouthpiece and reed. Moreover the measurements are limited to a pressure measurement, and this is proportional to the total acoustic power only if the radiation is really that of a monopole. At low excitation levels, Fig. 3 shows that for the three higher values of the radius, $b = 5, 6, 7.45$ *mm*, the difference in radiated pressure is close to 2 *dB*, while it is much more important for narrower diaphragms, around 15 *dB* between $b = a$ and $b = 3$ *mm* : this value is much larger than that found by simulation.

These two features can probably be explained by the existence of nonlinear losses due to the generation of turbulence at sharp edges of the hole. Using Eq. (6) and the approximated formula given in Ref. [6], it is possible to find an order of magnitude of the acoustic velocity at the tube end (thus of the acoustic Mach number), and to confirm this hypothesis. This phenomenon was investigated in Refs.[6, 7].

Further experiments will be done with diaphragms having rounded corners, in order to investigate these effects.

5 Conclusion

What is the effect of the tonehole radius on the radiated power for a given playing frequency? A general investigation could be very difficult. After the present work, a first answer could be the following: no, if two conditions are fulfilled: i) the radius is not too small; ii) the excitation level is not too high.

Acknowledgments

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