Acoustical Analysis of the Mexican Vihuela

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This work studies the tonal generation of the Mexican Vihuela, a 19th century guitar-like chordophone mostly played by Mariachi groups as a rhythmic accompanying instrument. It has five nylon strings attached to a simple wooden bridge, glued to the soundboard. The strings are fixed at the bridge with a sling-knot which results in different boundary conditions compared to the case of a regular guitar-like string termination. Two transverse polarizations of string motion are measured with a high speed camera. Radiated tones are recorded with a dummy head in an anechoic chamber. Finally, radiation patterns are measured with an array of 11 x 11 microphones. All measurements are performed with the vihuela string termination and with a guitar-like bridge applied to the instrument for comparison. The strong beating is found to be caused by the specific string termination. The sling acts as a rigid termination for the perpendicular polarized part of the transverse string motion, but moves freely in the parallel direction. Comparable to effects found in the finish kantele, this leads to two different virtual lengths of the string. Spectra of the radiated tone show string modes of both string lengths resulting in a vivid, chorus-like effect.

1 Introduction

The Mexican vihuela is a chordophone played in Mexican Mariachi music. It shares its name with the Spanish Renaissance guitar *vihuela*, played in Spain, Portugal and Italy by the late fifteenth through to the late sixteenth century. Ancestors of the Mexican vihuela are various types of lutes, brought to what is now Mexico in the early sixteenth century by the Spanish conquistadores [1].

The instrument has an overall length of approx 760 mm and is equipped with five nylon strings, about 490 mm in length, with diameters of 0.7 to 1 mm. The strings are tuned to A_3, D_4, G_3, B_3, E_4 (f_0 = 220, 294, 196, 247, 329 Hz). This is comparable to the guitar tuning without the E_2 string, but A and D are tuned an octave higher than for the guitar. The reentrant tuning is also found in European lutes, the Venezuelan *quatro* or the Hawaiian *ukulele*.

Mexican vihuelas often have gut frets tied around the neck, even though the instrument used in the present work has five metal frets. Chords are played not higher than the fifth fret (an interval of a fourth), meaning the upper range of the neck is not utilized in a normal playing situation; in Mariachi music the vihuela is mainly used to strum chords and to produce a loud, rhythmic sound to compete with the considerably louder trumpet section, violins and several singers. The back of the instrument has a deep convex V-shape (see Figure 1).

(a) Front view (b) Side view

Figure 1: Mexican Vihuela

The bridge is made of a single piece of wood glued to the soundboard. The strings do not run over a saddle on the bridge – as they would for the guitar – but are attached to the bridge with a sling-knot (see Figure 2). This design can also be found in the Turkish *oud* or the Chinese *yueqin* and is denoted as *sling-knot string termination* in the present work.

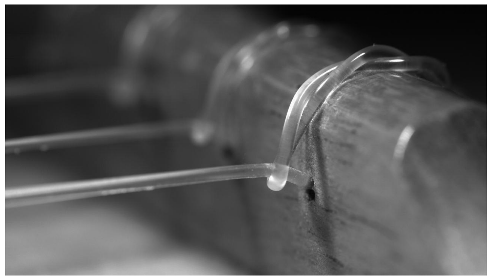


Figure 2: Sling-knot string termination at the bridge.

When a single string is plucked, the instrument produces a tone with easily perceivable beating, even if all other strings are damped. This extraordinary behavior is unexpected for a single string because an ideal string (with rigid boundary conditions) has a harmonic spectrum of possible transverse vibrational modes and thus, no beating could occur. Erkut and Karjalainen [2] show the specific string boundary condition as the main cause for strong beatings in the tone of the Finish *kantele*. Since the string termination of the Mexican vihuela is comparable to the one of the kantele, the main subject of this work is to examine, if the explanation also holds for this instrument.

2 Method

Open string tones are measured in terms of instrument radiation, and string motion in two orthogonal polarizations. Mode shapes of the top plate are determined by knocking at the bridge. To verify the sling-knot string termination as the main cause for the beating an additional measurement series is performed with a guitar-like bridge attached to the top plate of the instrument.

The strings are plucked with a finger nail near one third of the string length which corresponds to a normal playing position. Throughout this work the polarization of a transverse string vibration in direction perpendicular to the top plate of the instrument is called *perpendicular polarization*. Furthermore the polarization of a transverse string vibration parallel to the plane of the top plate is called *parallel polarization*.

The radiated sound of the Mexican vihuela is recorded with a *Head Acoustics HSU 3.2* dummy head with 44,1 kHz resolution in an anechoic chamber. The dummy head is placed 1 m in front of the instrument in listeners position. Only one channel of the binaural signal is used for the data analysis.

String motions are recorded with a *Vision Research Phantom V711* high speed camera with 49 kHz resolution. For a detailed description of the used high speed camera measurement method see [3]. To maximize the amplitude resolution the video analysis is done with the motion tracking software *MaxTRAQ 2D*, which uses a detection algorithm with sub-pixel accuracy. The camera is placed near one half of the string length in direction perpendicular to the top plate of the instrument, thus it is capable of recording a string deflection parallel to the soundboard. A mirror is attached close to the recorded string with an angle of $\alpha = 45°$ to the soundboard plane. In this way a string deflection in the direction perpendicular to the top plate can be observed through the mirror and both polarizations can be measured in the same recording.

Modal shapes of the top plate are measured with a planar microphone array of 11 x 11 microphones with 48 kHz resolution. The minimum energy method is used to propagate the measured sound pressure levels back to the surface of the instrument (see [4] for a detailed description).

3 Results

3.1 Analysis of the beating

Strong beating can clearly be observed in Figure 3 showing a spectrogram (short-time Fourier transform) of the radiated sound of a plucked *D* string. A corresponding spectrum (discrete Fourier transform) shows distinct double peaks in every string mode frequency area, with the higher mode having the higher amplitude in nearly all cases. Both groups of modes form harmonic spectra with a mean frequency difference of $\Delta f = 8.5$ cents for the first eight measured modes. The frequency axis in Figure 3 enumerates only the values of the higher and stronger modes of each double peak. Figure 4 shows a spectrogram of the radiation of the same note when a regular guitar-like string termination is applied. No beatings are observable, a discrete Fourier transform of the time series shows a harmonic spectrum without double peaks.

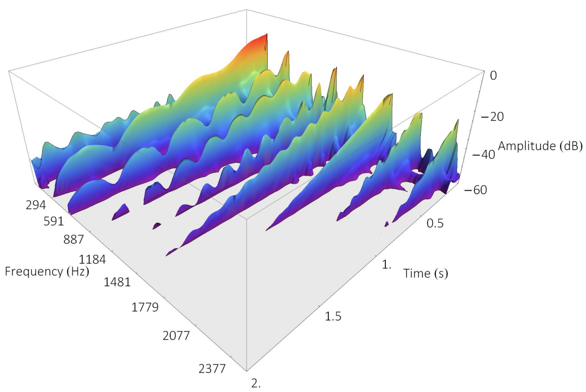


Figure 3: Spectrogram of the radiated sound with sling-knot string termination.

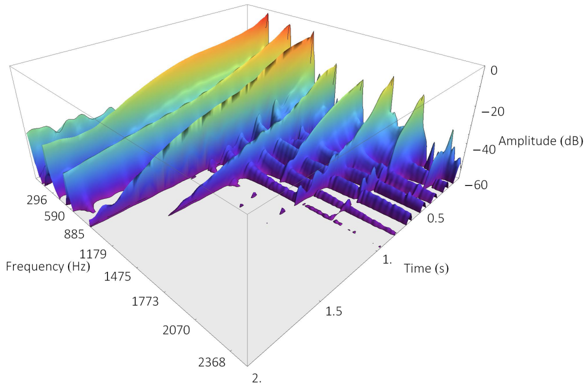
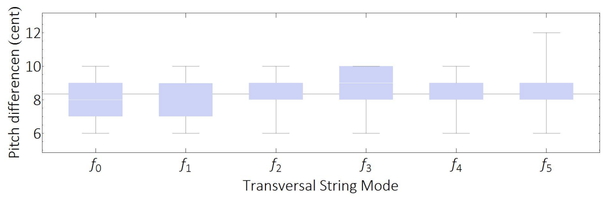
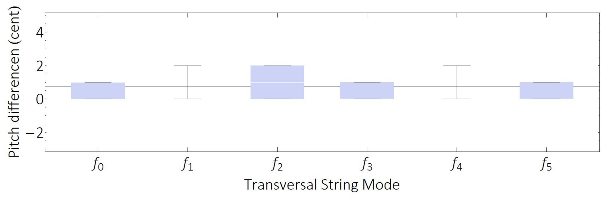


Figure 4: Spectrogram of the radiated sound with regular guitar-like string termination.

Ten measurements of the transverse string vibration of a plucked *D* string are performed for both cases of string termination. Figure 5 shows a box whisker plot of the pitch difference between the two transverse string polarizations. Note that the plots have different scales. In case of the sling-knot string termination intervals between both polarizations lie around 8 cent (a), in case of a guitar-like string termination around less than 1 cent (b).



(a) Sling-knot string termination.



(b) Regular guitar-like string termination.

Figure 5: Pitch difference of parallel vs. perpendicular transverse string polarization in cent. Plucked *D* string, ten measurements.

Corresponding to Erkut and Karjalainen [2], the difference Δl between two virtual string lengths can be deduced from the frequency difference of any of the string modes by

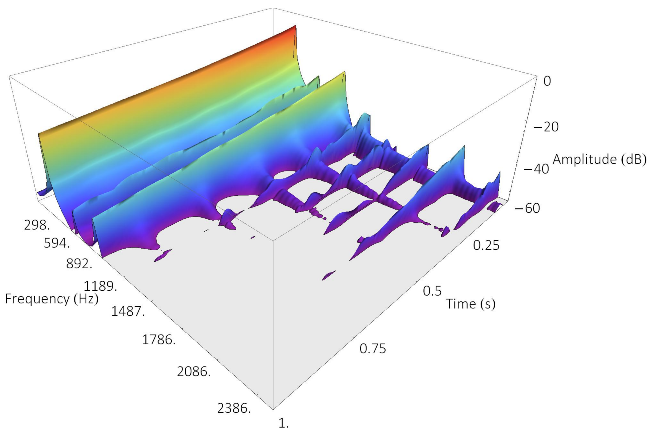
$$\Delta l = \frac{\Delta f_n * l}{f_{n,parallel}} \tag{1}$$

with Δf_n the frequency difference of mode n in Hz, l the string length from nut to sling and $f_{n,parallel}$ being the nth mode frequency of the parallel string polarization. The data predicts a string length difference of approx 2.5 mm, which is equivalent to the distance between the sling and the bridge.

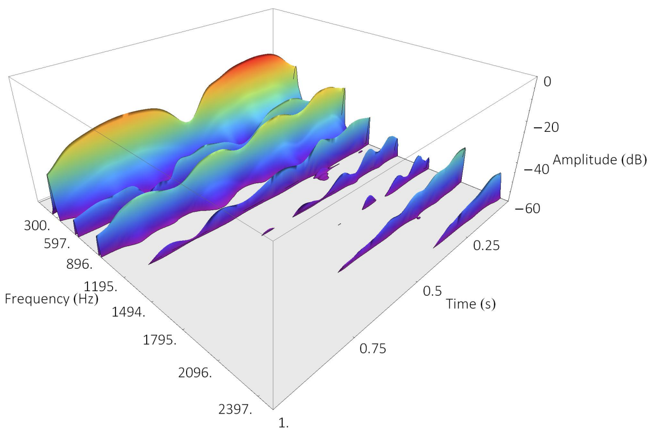
Figure 6 shows spectrograms of two orthogonal transverse polarizations of recorded *D* string vibrations with a sling-knot string termination applied. The parallel polarization has a harmonic spectrum where no beating occurs. In the perpendicular polarization heavy beating is observable, the spectrum (discrete Fourier transform) shows double peaks with the lower peaks being the modes of the parallel polarization.

Obviously vibrational energy from the parallel polarization is present in the perpendicular polarization but not the other way round. A possible explanation is the following: The boundary condition for the parallel polarization is given by the bridge and the string is not aware of the sling. In the case of the perpendicular polarization the string is terminated by the sling, which is moved by the transverse vibration in parallel polarization and with its frequency. Thus, the boundary condition for the perpendicular polarization is modulated with the frequency of the parallel polarization.

The spectrum af the radiated sound implies the coupling from string to top plate to be much stronger for the perpendicular polarization as for the parallel polarization. This is due to the high stiffness of the bridge body, comparable to the guitar bridge [6] and in contrast to the violin or the banjo, where the flexible bridge construction is designed to support a rotary motion of the bridge [7, 8].



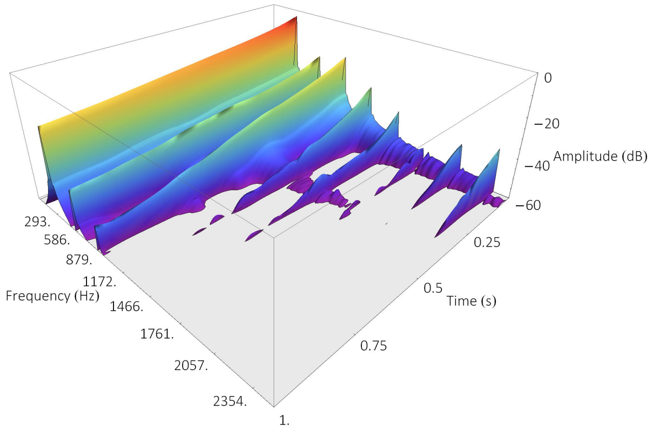
(a) Sling-knot string termination, parallel polarization of transversal string deflection.



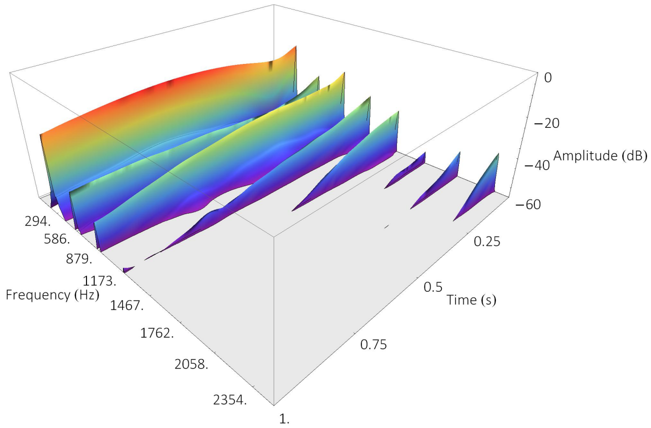
(b) Sling-knot string termination, perpendicular polarization of transversal string deflection.

Figure 6: Spectrograms of transversal string deflections obtained by high speed camera recordings. The string is attached to the bridge with a sling-knot (see Figure 2). *D* String, played *forte*.

For comparison, the behavior of the transverse string vibration with a regular guitar-like termination is presented in Figure 7. Both polarizations have the same mode frequencies and no beating appears.



(a) Regular guitar-like string termination, parallel polarization of transversal string deflection.



(b) Regular guitar-like string termination, perpendicular polarization of transversal string deflection.

Figure 7: Spectrograms of transversal string deflections obtained by high speed camera recordings. The string is attached to a bridge with a guitar-like termination. *D* String, played *forte*.

3.2 Torsional vibration

With the high speed camera measurement method it is possible to measure the torsional vibration of a string. Figure 8 shows a time series of the transverse (parallel) vs. the torsional deflection of a plucked *E* string. The signal of the string torsion is enhanced for reasons of clarity. The fundamental frequency of the torsional vibration is 2.7 times higher than its transversal counterpart. The appearance of the torsional vibration strongly depends on the way a string is plucked. The string documented in Figure 8 is picked with the finger tip. In this case the skin twists the string torsionally before releasing it. After release the torsional motion vanishes within 100 ms. When picked with a finger nail the torsion is not measurable. The appearance of beatings is independent of the way how the strings are plucked. A possible further influence of the torsional vibration to the tone generation is not subject of the present work.

3.3 Radiation patterns

Figure 9 shows different radiation patterns of the top plate. The air resonance mode is situated at 133 Hz (a), which is similar to the Spanish Renaissance vihuela [5] and below the fundamental frequency of any of the strings. At 241 Hz (b) the top plate couples to the Helmholtz mode.

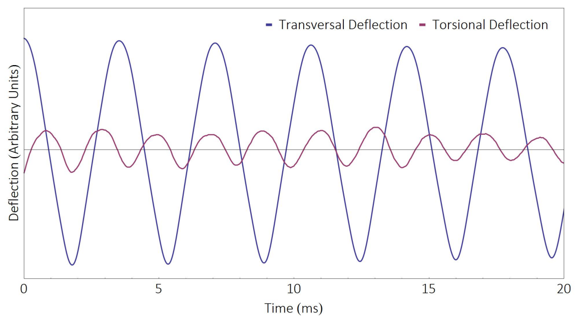
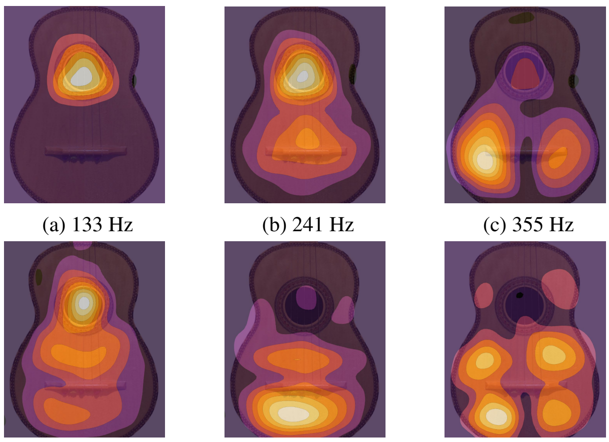


Figure 8: Time series of a transversal vs. the torsional vibration of a plucked *E* string. The data is obtained from a high speed camera recording.

The mode shape at 355 Hz (c) would emphasize a rocking motion of the bridge with a low impedance presented to the string in parallel polarization. However only the open E_4 string has a mode (f_0 = 329 Hz) in this region. The radiation patterns at 479 Hz (d) and 750 Hz (e) are obviously divided by the bridge. At 969 Hz (f) the top plate is attracted to a motion pattern that twists the bridge body.



(a) 133 Hz (b) 241 Hz (c) 355 Hz

(d) 479 Hz (e) 750 Hz (f) 969 Hz

Figure 9: Modal shapes for a number of low-frequency modes of the vihuela top plate.

4 Conclusion

The tonal generation of a Mexican vihuela is investigated with respect to the heavy beating perceptible in the radiated sound. Radiated tones of all open strings are recorded in an anechoic chamber. Two transversal string polarizations are measured by means of a high speed camera method. Radiation patterns of the top plate are measured with a microphone array. Two series of measurements are performed, one with the typical sling-knot string termination and one with a guitar-like string termination.

The measurements clearly confirm the assumption that the strong beating of the vihuela tone is caused by the specific string termination at the bridge. The sling acts as a termination point for the perpendicular polarization of the transverse string vibration but is not present for the parallel polarization. This results in two effective string lengths with two slightly detuned harmonic spectra. The question how the two polarizations interact can not be finally answered yet. The parallel polarization is observable in the perpendicular polarization but not the other way round. A possible explanation is the following: The boundary condition of the perpendicular polarization is modulated with the frequency of the parallel polarization. Thus the vibration in perpendicular polarization is driven by the parallel polarization. This leads to beating already present on the string. Comparison of the radiated spectra and the string motion spectra implies that the coupling of vibrational energy from string to the top plate is much stronger in the perpendicular polarization. Torsional motion of a string is only observable if the string is plucked with the finger tip. Plucking with a finger nail does not excite a measurable torsion of the string. The appearance of beatings is independent of the way the string is plucked. Since the string termination design is similar to the ones of the Turkish *oud* or the Chinese *yueqin* the described effect should be present in the tone generation of these instruments as well.

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