Acoustical Impedance of the Xiao

Y. Lan and C. Waltham
University of British Columbia, Dept of Physics & Astronomy, 6224 Agricultural Road, Vancouver, BC, Canada V6T 1Z1
yanglan@phas.ubc.ca

The xiao is a Chinese end-blown flute with a history of over a millennium, traditionally made of bamboo, notched at the blowing end, with six or eight finger holes. The tone range of the xiao is two to three octaves. Tones starting from the second octave come from over-blowing, and cross fingerings have to be used for the third octave. Currently most xiaos have difficulties in sounding the higher notes, which also have serious intonation problems. This paper aims at explaining and solving the xiao's problems by studying its input impedance. As an air-reed instrument, the xiao plays at its input impedance minima. We use the transmission-matrix method to model the instrument, and experimentally measure the input impedance to validate the model. For finger hole configurations of 24 tones in the two and a half octaves under test, the model has a maximum deviation of 8 cents from measurements. Then the player's effects are taken into account, and the model is able to predict the tuning of a xiao with any tone hole positions, sizes, and arbitrary bore shape along the symmetry axis. Based on this model, numerical optimizations were applied to find the best configurations. A xiao made of PVC pipe with optimized tone holes shows good tuning results. Modifying the bore shape shifts the frequencies of the impedance minima and can be used for controlling the brightness and volume of the instrument. Our optimizations of the bore shape are ongoing.

1 Introduction

The xiao[1] is traditionally made of bamboo with a node at the blowing end for embouchure. The node is either broken through in the southern style (see the top view of xiao D in Figure 1), or kept with a rectangular or an oval cutting in the northern style (xiao Z, B, K). The styles refer to regions of China. When playing a xiao with the southern style opening, the top end is blocked by the jaw of the player, leaving only the embouchure in the front of the pipe open, similar to playing a xiao with the northern style. An air jet is blown over the edge of the embouchure to excite the instrument. The edge usually has a 30° to 60° angle with the pipe, and has three kinds of shape: "U", "V" and "Arc". The "U" or "V" shape is made by notching from inside of the bamboo pipe and the "Arc" shape is made by cutting off an oblique plane from outside of the pipe. A xiao with southern style opening and "Arc" shaped embouchure edge is similar to a Japanese Shakuhachi[2] in the head part.

The xiao is played vertically, each hand of the player controls half of the six or eight finger holes, and the top finger hole is located at the back (thumb hole). For the first octave, finger holes are opened in sequence to raise a half tone or a full tone (for all upper finger holes, lower holes are kept open, except for a cross fingering[3] note). The second octave comes from over-blowing with the same fingering except for the cross fingering one. For all notes in the third octave, cross fingerings are used. The note played by opening all the lower hand finger holes defines the key of the instrument. All the instruments under study in this paper have a U-shape embouchure edge with eight finger holes and are in key of G, the lowest note D4 (293.7 Hz) is played with all finger holes closed. The lengths of our instruments range from 650 to 980 mm, but they all have two to three pairs of adjustment holes (tassel holes) starting at around 540 mm from the embouchure end.

A good xiao is expected to play in tune for two octaves and a half. The finger hole positions and diameters have a predominant effects on the tunings. With knowledge from experienced xiao makers and practice of making several xiaos, we acknowledge that the xiao makers locates and drill the finger holes (and the lower adjustment holes) using parameters from existing instruments or given by experienced makers, then the instrument is played to test its tunings and finger holes' positions/diameters are slightly adjusted accordingly. However each finger hole controls pitches of at least two notes and sometimes the adjustments cannot correct tuning of every note. In addition, the bore of a xiao is known to have effects on balancing the tuning of notes in different octaves and a conical shape tapering to smaller diameter at the pipe's end is preferred, similar to the Baroque flute. A bamboo pipe near the root part is usually chosen to make the xiao because the tapering will be naturally formed after removing the bamboo nodes and smoothing the inner wall. A more recent technique for tuning a xiao is by adjusting its bore shape, probably learnt from the tuning techniques of the Shakuhachi as an application of the *perturbation* method of Benade [4, 5].

Both the hole positions/diameters and bore shape adjustments require the maker to test and modify the instrument again and again. It would be very beneficial if tunings can be predicted and the instrument be optimized in a computer. Plitnik el al. used a transmission-line model to calculate the input impedance of the oboe in the 1970s[6], and Ando el al. modelled the input admittance of the Shakuhachi using the same method in the 1980s[2]. Later the method was adapted to be the Transmisson-Matrix Method(TMM) and which has been widely used in various wind instruments like the flute and saxophone[7, 8].

Similar to the flute and other air-reed instruments, the xiao plays near its input impedance minima. Input impedance of the xiao is measured as described in Section 2 and modelled by TMM in Section 3. Section 3 also discusses some irregularities observed in the impedance curves, and their relationships with the woodwind cutoff frequencies[4, 3]. Numerical optimizations on the xiao's tuning is described in Section 4 .

2 Acoustical impedance measurements

We followed the method of Dickens et al.[9] and built an impedance tube for the xiao, made out of brass with inner diameter 7.9 mm, outer diameter 9.5 mm and total length 400 mm. One end of the tube was coupled to an compression driver through a cone and the other end, defined as reference plane, was to be connected to the xiao's embouchure or calibration loads. Four 6 mm microphones were located at 10 mm, 50 mm, 150 mm and 250 mm from the reference plane. Impedance measurements were made at the UBC anechoic chamber[10] with a Presonus 44VSL 24 bit sound card to collect the microphone signals and output a log-scaled swept signal to the driver. We used the full calibrations for the impedance tube as described by

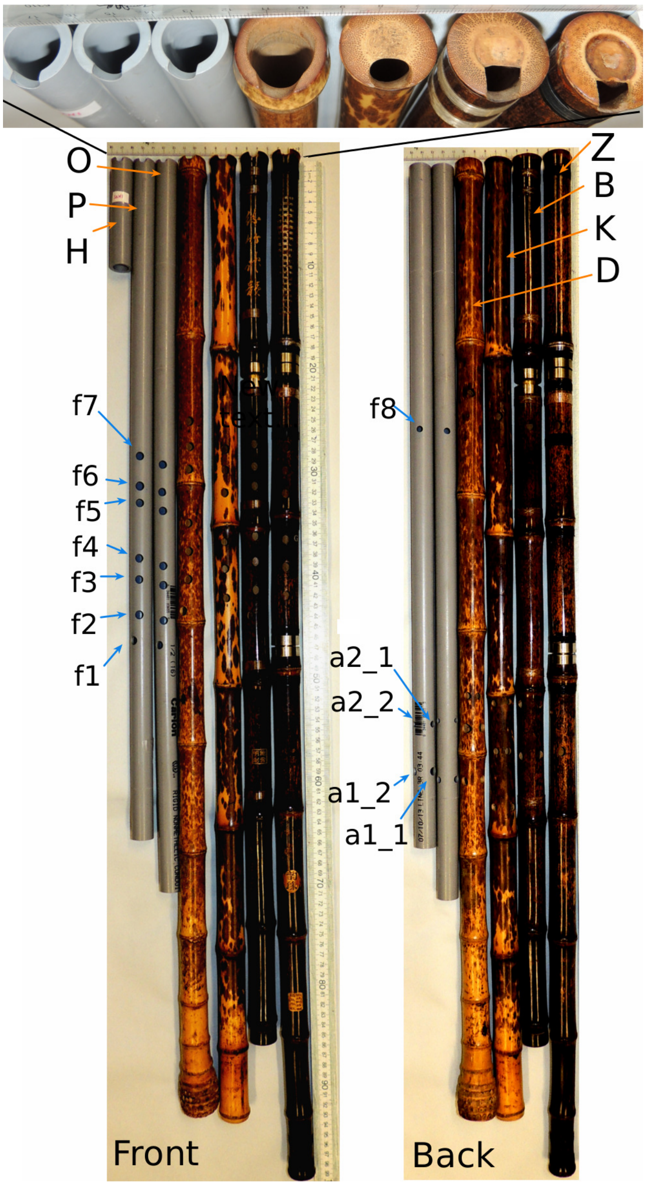

Figure 1: Four xiaos Z, B, K, D made of bamboo. Two xiaos O, P and a xiao head H made of PVC pipes. Holes of xiao P are labelled: f1 to f8 are eight finger holes and a1_1 to a2_2 are two pairs of adjustment holes (tassel holes).

Dickens et al., except for changing their 97 m resistance load to a brass tube of 2880 mm (measured by a tape) in length and of the same diameters as the impedance tube. At first, the measured impedance curves show ripples of about 55 Hz interval. The ripples come from inaccurate measurement on the calibration tube's length L, and were removed by adjusting L in the calculation of the tube's theoretical impedance.

After the above calibrations, what's measured is the impedance at the reference plane with a circular area of diameter 7.9 mm, which is poorly matched to our xiaos' smaller U-shaped openings. To get the real input impedance at the the embouchure, the discontinuity needs ultimately to be modelled by the multimodal theory[11]. For now we used end-corrections instead at the discontinuities. A test measurement was made on an open tube with inner diameter 6.4 mm, outer diameter 7.9mm and length 999.0 mm. The measurement matched well with the theoretical impedance of the tube, see Figure 2. For the theoretical calculation, an end correction -3.78 mm is applied at the discontinuity. For measurement on the tube with it far end closed, the end correction is -3.22 mm.

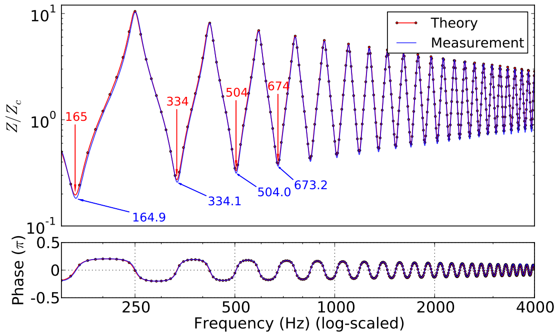

Figure 2: Theoretical and measured impedance of an open cylindrical pipe with inner diameter smaller than that of the impedance tube.

Impedance of the xiao is measured by blocking the top part of the pipe and attaching the U-shaped opening to the impedance tube (see Figure 3). Modelling clay (blue) was used to fill the gaps. The measurement results are shown in the next Section together with the modelling results.

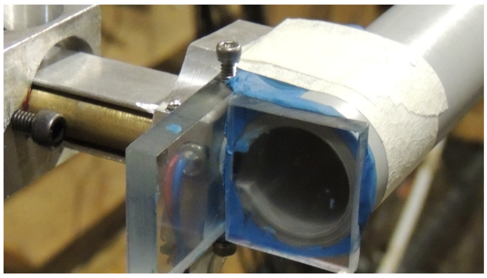

Figure 3: A PVC xiao under impedance measurement.

3 TMM model of the xiao

The xiao's body part from 100 mm below the embouchure to the pipe end can be easily modelled by the TMM . The special geometry of xiao's embouchure is not a standard element in the TMM and need to be tested experimentally.

3.1 Model of the xiao head

A xiao head of inner diameter 15.2 mm, wall thickness 2.8 mm and pipe length 100 mm was made of a PVC pipe (see H in Figure 1). The U-shaped embouchure hole was formed by cutting from one end of the pipe with a 7.9 mm drill, the pipe was mounted with a 45° angle to the drill to form an oblique edge. The outside U-shape is 4.5 mm in length and the area is equivalent to a 6.06 mm circle. The U-shape's length extends to 7.3 mm at the inside of embouchure and is equivalent to a 8.06mm circle. Then the embouchure hole was modelled as a short conical waveguide located at the geometrical center of the inside U-shape.

To model the xiao embouchure part, the equivalent T circuit[12] of a side hole was used with the following modification: the conical waveguide was used in this T circuit with an end-correction t_{emb}. Then a transfer matrix[7] was used to calculate the impedance at the outside U-shaped opening. The xiao head with its end closed was measured and modelled, and an end-correction of $t_{emb,c} = 3.52$ mm is used. For the xiao with open end, radiation impedance at the end is accurate enough for the imaginary part[13], but the real part depends on the geometry of the end[14]. and can be written as:

$$\Re(Z_{rad}) = x\,ka^2 Z_c \tag{1}$$

Here, x is a coefficient depending on the flange condition and sharpness of the edge, k is the wave number, a is radius of the opening and Z_c is the characteristic impedance. The value of x affects the amplitude of impedance curves. For our flute head with its end open, fitting results was obtained as $x = 0.212$ and $t_{emb,o} = 3.72$ mm. Measurement and modelling results of the xiao head are shown in Figure 4.

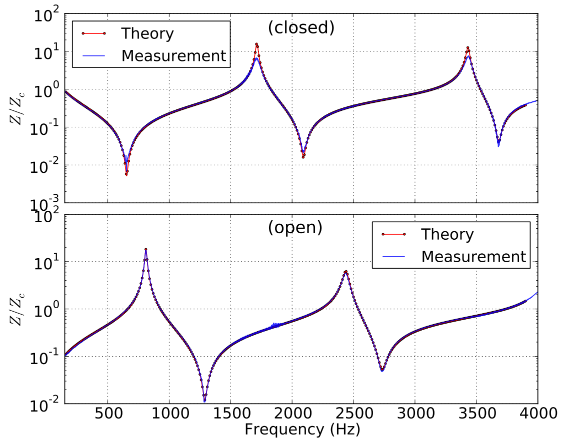

Figure 4: Impedance of the xiao head H.

3.2 Model of the whole instrument

Now the input impedance of the xiao can be modelled by adding the body part in TMM. Similar to the xiao end, real parts were added at the side holes' radiation impedance and inner length correction as Eq. (1). We obtained $x = 0.48$ for the xiao side holes. Results of measured and modelled impedance of xiao P for two representative fingerings are shown in Figure 5 (the red and blue curves).

The measured impedance curve cannot be directly used for predicting the playing frequencies of air-reed instruments, because the player's effects are not considered in the measurements. The player's effects include: covering ratio of the embouchure hole, radiation impedance at the embouchure, velocity of air-jet, temperature and air-content changes inside the pipe. To the authors' knowledge, these effects have not been well modelled yet. Dickens used an empirical formula for the flute embouchure (Eq. (7.2) in [7]). We found the formula also works for the xiao and the embouchure effects can be expressed by a single formula:

$$Z_{emb}\big(f(m)\big) = \big(2.937log(m) - 11.6284\big)Z_h. \tag{2}$$

Here $f(m)$ is the frequency of midi number m and Z_h is the radiation impedance of a hole on the pipe with the same area as the embouchure. Finally, the minima of $Z + Z_{emb}$ should predict the playing frequencies of xiao. The xiao plays at the first few minima by changing velocity of the air-jet.

To validate the model, the xiao P was played by one of the authors (Y. Lan), each note was played at moderate volume and recorded for 5 s. The xiao was played normally and intentional pitch adjustment at the embouchure was avoided. The playing frequencies were measured right after the impedance measurements in the same anechoic chamber to avoid temperature change. The SPL of the notes were averaged to 1Hz resolution and plotted together with the admittance curve in Figure 5.

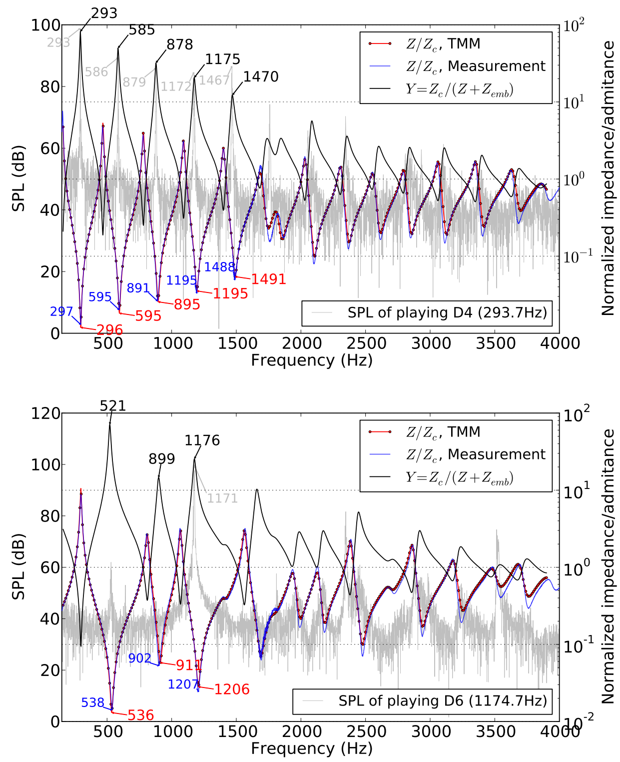

Figure 5: Impedance, admittance and SPL spectrum of xiao P. Upper: playing D4 with all finger holes closed, lower: playing D6 with cross fingering oxxx-ooxo.

In the upper figure, the SPL spectrum of tone D4 has obvious harmonics located at approximately integer multiples of the playing frequency (293 Hz). The harmonics are generated by non-linear effects of air-jet at the embouchure[15], and will not necessarily match the maxima of the admittance curve. However, alignment of the admittance maxima affects the amplitude of the harmonics. To play a brighter tone color and larger volume, a xiao should have the admittance maxima (impedance minima) aligned by integer multiples.

In the lower figure, results of a cross fingering oxxx-ooxo playing D6 is shown (starting from the upper most thumb hole f8 to the lowest finger hole f1, x means the hole is closed and o means open). This fingering is also used for C5 (523.3 Hz, the model predicts 521 Hz for xiao P) in the first octave. So naturally impedance minima of this fingering will not be harmonically aligned and C5 of our xiaos has obviously different tone color with other notes.

For three PVC xiaos made and finger hole configurations of 24 tones in the two and a half octaves under test, the model has a maximum deviation of 8 cents from measurements in Z. The accuracy of Z_{emb} of xiao is still under improvement, but for now the minima of $Z + Z_{emb}$ predict the playing frequencies of all the notes at least within 20 cents.

3.3 Irregularities in impedance curves and woodwind cutoff frequencies

It is known that an open or closed cylindrical or conical pipe has harmonic impedance curves (see the typical open pipe impedance in Figure 2). However impedance of pipes with open side holes is not always harmonic. While studying the impedance and cavity mode of a guqin[16], it was recognized that the irregularities in impedance curves come from resonances happening at shorter segments of the pipe. The resonances were validated by as below.

Sound pressure of the longer PVC xiao O was calculated by TMM and shown in Figure 6. The contour shows the real part of pressure, red for positive and blue for negative. The pressure is normalized to the embouchure pressure, and input impedance at the embouchure is superimposed. At about 1.2 kHz, a pattern can be seen between the pipe end and the adjustment hole a1_1. Input impedance at a1_1 was calculated by TMM and plotted in the narrow box, it correctly shows minima at around 1.2 kHz. This may explains why the xiao O can be played at the first three and the fifth impedance minima, but cannot be played at the fourth. In contrast, xiao P has a shorter segment formed by the pipe end to a1_1. As the impedance in the top of Figure 5 indicates, xiao P with all finger holes closed can be played at the first five and the seventh impedance minima, but not at the sixth one.

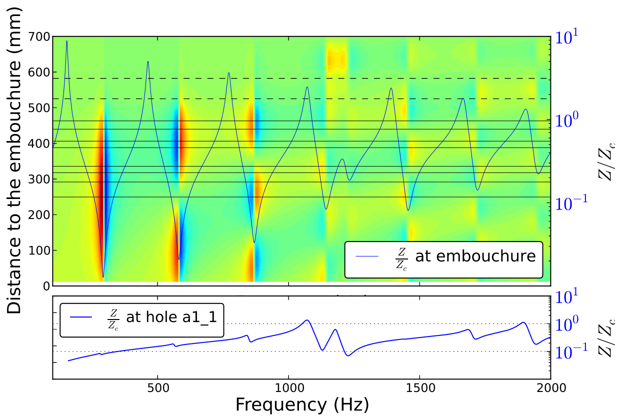

Figure 6: Pressure distributions and impedance curves of xiao O with all finger holes closed. The horizontal lines indicates hole locations, solid line for closed holes and dashed lines for open holes.

The resonances were further tested by the cross fingering oxxx-ooxo of xiao O, (see Figure 7). A pattern shows at about 900 Hz and position of the three closed holes. Input impedance at f8 is plotted below the contour, it has minima at about 500 Hz and 900 Hz. Blowing at f8 can exactly sound the two resonances.

Based on the observed relationship of impedance irregularities and locations of resonances along the pipe, we can state that any irregularities in the impedance curve have a corresponding resonance at a shorter segment S of the pipe, formed by the pipe end with an open hole, or formed by two open holes. The resonances of segment S can only be excited by blowing at either of its end (for example, at Figure 7, the resonance at about 900 Hz cannot be played at the emboucure).

The above statement can be related to the woodwind

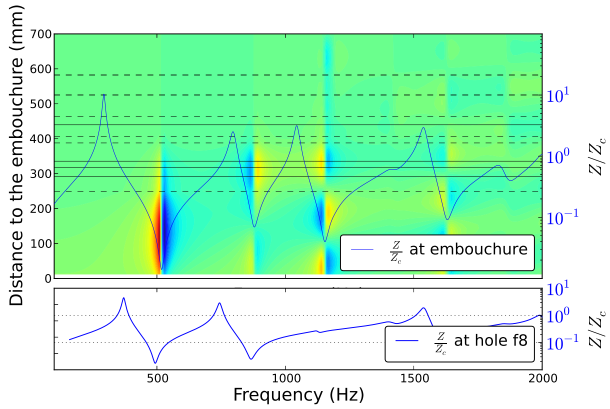

Figure 7: Xiao O with fingering oxxx–ooxo, see the caption of Figure 6 for details.

cutoff frequency f_c[4, 5]. Above f_c, the impedance curve's amplitude reduces and its number of minima increases, interval of the minima correspond to standing waves of the whole instrument (as if the holes were closed)[3]. The bottom of Figure 5 shows that at fingering oxxx-ooxo, xiao P has $f_c \approx 1.5$ kHz. However, above f_c, xiao P with this fingering can still be played at about 1.6 and 2.5 kHz, which is the third and fifth impedance minima of the embouchure segment. All other impedance impedance minima cannot be played at the embouchure, but some of them can be played at the open holes of their segment, traced by their resonance locations in the pressure contour (not shown). So above f_c the woodwind is not equivalent to a whole pipe with all holes closed, although the standing waves were also observed to extend to the whole pipe in the contour.

4 Numerical optimizations

An optimization algorithm (L-BFGS-B[17]) was used to optimize the xiao based on the playing frequencies calculated by the TMM. The optimization works by minimizing the total error, which is the sum of squares of the notes' deviations from the equal temperament[8]. At first, hole positions and diameters were used as the variables for the optimization. Xiao O is a halfway optimization result when Z_{emb} had not been taken into account, and it has bad tunings. Xiao P was made using an earlier optimization result on January 2014.

The playing frequencies of xiao P was measured as described in Section 3.2 and plotted in Figure 8. Also plotted are the playing frequencies of xiao Z and B. Z and B are of moderate quality and they have no sign of delicate tuning by adjusting the bore shapes. But their tunings are better than the xiao K and D made by Y. Lan when he was an inexperienced maker.

Our latest optimization result on the hole positions/diameters with cylindrical bore have all 24 notes within deviations of ± 15 cents. Then we include the bore shape optimization by describing the bore by N conical segments, and the deviations reached ± 10 cents. Some preliminary results on bore shape optimization on alignments of the impedance minima are given in Figure 9. We got the typical bore shape of the Shakuhachi[18], shown as shrinking from the embouchure and enlarging near the end. This type of bore shape has also been adopted by some xiao makers.

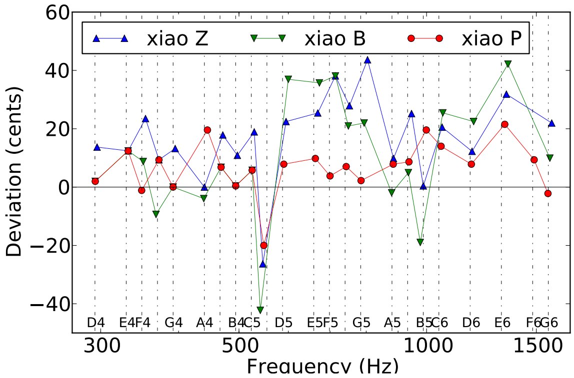

Figure 8: Playing frequencies of the xiaos versus the deviations from equal temperament.

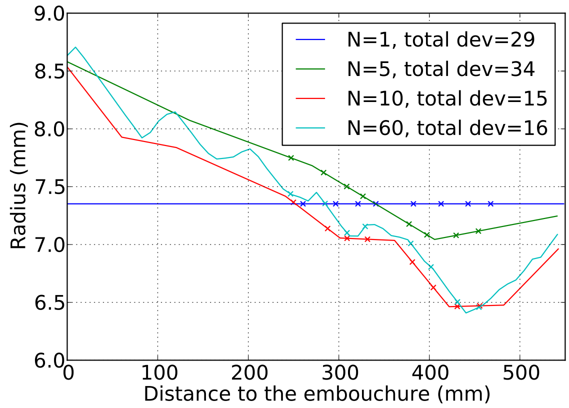

Figure 9: Optimization results of the bore shape, × means the finger hole locations. total dev=$(\sum d_i^2)^{\frac{1}{2}}$ where d_i is the deviation of the i^{th} impedance minima to the harmonic frequency in cents. d_i includes the first, second and fourth harmonics of all finger holes, cross fingering notes are not taken into account.

5 Conclusions

The xiao's acoustic impedance has been measured and a TMM model has been built considering the xiao's special embouchure. The TMM model has also been used to calculate the pressure and resonances of the xiao, irregularities in the impedance curves and their relationships with the cutoff frequencies have been examined. Numerical optimizations on hole positions/diameters has produced a xiao with good tunings. Bore shape optimizations are ongoing and some preliminary results has been obtained.

6 Acknowledgement

The authors thank Alan Thrasher and the UBC Chinese Orchestra for help with playing the xiao, Murray Hodgson of UBC Mechanical Engineering for sharing the anechoic chamber and the Natural Sciences and Engineering Research Council (Canada) for financial support.

References

[1] A. R. Thrasher. *Xiao*. Grove Music Online. Oxford U. Press. Accessed 12 May 2014.

[2] Y. Ando et al. Measuring and calculating methods of shakuhachi input admittance. *Journal of the Acoustical Society of Japan (E)*, 6(2):89–101, 1985.

[3] J. Wolfe et al. Cutoff frequencies and cross fingerings in baroque, classical, and modern flutes. *J. Acoust. Soc. Am.*, 114(4):2263–2272, 2003.

[4] A. H. Benade. Mathematical theory of woodwind fingerholes. *J. Acoust. Soc. Am.*, 31(11):1564–1564, 1959.

[5] A. H. Benade. *Fundamentals of Musical Acoustics*. Dover Books on Music Series. Dover Publications, 1990.

[6] G. R. Plitnik et al. Numerical method for calculating input impedances of the oboe. *J. Acoust. Soc. Am.*, 65(3):816–825, 1979.

[7] P. Dickens. *Flute acoustics: measurements, modelling and design*. PhD thesis, PhD Thesis, University of New South Wales, 2007.

[8] A. Lefebvre. *Computational acoustic methods for the design of woodwind instruments*. PhD thesis, McGill University, 2010.

[9] P. Dickens et al. Improved precision in measurements of acoustic impedance spectra using resonance-free calibration loads and controlled error distribution. *J. Acoust. Soc. Am.*, 121(3):1471–1481, 2007.

[10] C. E. Waltham et al. Acoustic imaging of string instrument soundboxes. In *Proc. Meet. Acoust.*, volume 19, page 035004. Acoustical Society of America, 2013.

[11] V. Pagneux et al. A study of wave propagation in varying cross-section waveguides by modal decomposition. part i. theory and validation. *J. Acoust. Soc. Am.*, 100(4):2034–2048, 1996.

[12] C. J. Nederveen et al. Corrections for woodwind tone-hole calculations. *Acta Acustica united with Acustica*, 84(5):957–966, 1998.

[13] J. P. Dalmont et al. Radiation impedance of tubes with different flanges: numerical and experimental investigations. *Journal of sound and vibration*, 244(3):505–534, 2001.

[14] J. M. Buick et al. Investigation of non-linear acoustic losses at the open end of a tube. *J. Acoust. Soc. Am.*, 129(3):1261–1272, 2011.

[15] N. H. Fletcher et al. Harmonic generation in organ pipes, recorders, and flutes. *J. Acoust. Soc. Am.*, 68(3):767–771, 1980.

[16] C. E. Waltham et al. Vibroacoustics of the guqin. In *This conference*.

[17] R. H. Byrd et al. A limited memory algorithm for bound constrained optimization. *SIAM Journal on Scientific Computing*, 16(5):1190–1208, 1995.

[18] Y. Ando. Input admittance of shakuhachis and their resonance characteristics in the playing state. *J. Acoust. Soc. Japan (E)*, 7(2):99–111, 1986.