Perceptual Thresholds for String-Body Coupling in Plucked-String Instruments

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The coupling between the strings and body of a plucked-string instrument has a great influence on its tonal qualities. Strong coupling is advantageous to attain a responsive instrument but over-coupling can lead to unwanted effects and uneven tonal qualities from one note to another. An investigation has been carried out to evaluate the perceptual importance of different levels of string-body coupling in plucked-string instruments. Modal parameters consisting of effective masses, Q values and natural frequencies have been extracted from admittance measurements on a classical guitar and used in a pre-existing model that describes the interaction between body modes and nearby string modes. The model showed that over-coupling leads to fast string decays and significant frequency perturbations that can contribute greatly to an instruments' acoustic signature. Fourier analysis made on the radiated sound from the instrument enables characterisation of the components of the sound. This is useful to validate the coupling model as well as allowing re-synthesised tones to be generated for the use of psychoacoustical tests. Listening tests were carried out to assess the extent of even tonal qualities between notes by highlighting the presence of wolf notes. Extending this work to include other instruments such as the steel-string guitar and banjo helps to clarify the quantitative relationships between the design of an instrument, its mechanical response and its perceived tonal qualities. Ultimately this work aids in understanding what structural changes lead to audible changes in the instrument's sound by considering string-body coupling.

1 Introduction

The level of coupling between the strings and body of a plucked-string instrument heavily influences its radiated sound. Strong coupling is advantageous so that the strings communicate well with the body giving a loud and responsive instrument. But, in the case of the classical guitar, over-coupling leads to some notes sounding 'dead' or out-of-tune. These are the so-called wolf notes. The aim of this work is firstly to evaluate the perceptual significance of over coupled notes on the classical guitar. Then the investigation is expanded to consider the influence of the overall level of string-body coupling on the acoustical signature of instruments of the plucked-string family.

The amount of coupling between the string and body of a stringed-instrument depends on the ratio of the mechanical impedance of the string to that of the body [1]. The dynamics of a system of coupled modes were investigated by Weinreich [2] in relation to coupled piano strings. He showed that a pair of modes will undergo perturbations in frequencies and damping when coupled by a common bridge. Gough [3] used a transmission-line analogue to model the behaviour of a string mode coupled to a body mode showing the same characteristics as Weinreich. More recently, Woodhouse [4] used a modal superposition model to identify string partials on a guitar that might suffer from perturbed frequencies due to strong coupling to the body.

The work done by the aforementioned authors has led the way to assess the *perceptual* significance of the effects of strong string-body coupling on the guitar. In this paper Gough's model is used to describe the interaction of a string coupled to a body mode. The model predicts perturbations in frequency and damping of the coupled modes, demonstrating the potential problems of over-coupling. Measurements of the input admittance of a classical guitar are made and modal parameters consisting of natural frequencies, Q values and effective masses are extracted to use in the model. Psychoacoustical tests are also carried out to find the just noticeable differences (JNDs) in frequency and Q value of components of a guitar tone. The JNDs are inserted into the model to yield over-coupling threshold curves that highlight the wolf notes on a classical guitar.

A comparison is then made between the mechanical responses of a folk steel-string guitar, classical guitar and a five-string banjo. The levels of string-body coupling of these instruments are estimated and related to their design and characteristic tonal qualities. A comparison of the modal parameters of each instrument indicates the parameters that undergo the largest change from one instrument to another.

2 Theory

The model used here has been adapted from Gough [3], where it is part of a more complex model for the overall response of a stringed-instrument. A lossy string is coupled to a single body resonance, where the coupling is characterised by the coupling strength, α.

$$\alpha = \frac{\omega_S^2}{\pi}\left(\frac{2m_S}{M_{eff}}\right)^{1/2}, \tag{1}$$

where m_S is the mass of the vibrating portion of the string and M_{eff} is the effective mass of the body mode. We have restricted the analysis to the fundamental string mode only as this is the most strongly coupled string component. Here Gough's model is used to show the behaviour of coupled string-body modes in terms of the uncoupled frequencies, ω_S and ω_B, and uncoupled Q values, Q_S and Q_B, where subscript S denotes a mode of the string and B a body mode.

The solutions to the coupled equations given by Gough are

$$\Omega_\pm^2 = \frac{\omega_B^2 + \omega_S^2}{2} + \frac{i}{2}\left(\frac{\omega^2}{Q_B} + \frac{\omega^2}{Q_S}\right) \pm \left[\left(\frac{\omega_B^2 - \omega_S^2}{2} + \frac{i}{2}\left(\frac{\omega^2}{Q_B} - \frac{\omega^2}{Q_S}\right)\right)^2 + \alpha^2\right]^{\frac{1}{2}}. \tag{2}$$

Expanding the square-root in Eq. (2) and collecting real and imaginary terms, β and γ respectively, simplifies the equation into the following form

$$\Omega_\pm^2 = \frac{\omega_B^2 + \omega_S^2}{2} + \frac{i}{2}\left(\frac{\omega^2}{Q_B} + \frac{\omega^2}{Q_s}\right) \pm (\beta + i\gamma)\ . \tag{3}$$

Note that both β and γ contain the coupling strength. The frequencies and Q values of the coupled modes, $\omega^*_\pm$ and $Q^*_\pm$ respectively, are then given by

$$\omega_{\pm}^{*} = \left(\frac{\omega_B^2 + \omega_S^2}{2} \pm \beta\right)^{1/2}, Q_{\pm}^{*} = \frac{\omega^2}{\frac{1}{2}\left(\frac{\omega^2}{Q_B} + \frac{\omega^2}{Q_S}\right) \pm \gamma}. \quad (4)$$

The model produces typical coupling curves [2-4], showing the perturbations in frequency and Q value of the coupled modes as a function of frequency, ω. If we consider the response of this coupled system near the frequencies of the two uncoupled modes we can predict the behaviour of the system as a function of coupling strength.

Uncoupled parameter values for the second body mode of classical guitar BR2 and a nylon string were inserted into the model. Varying the 'impedance matching term', m_S/M_{eff}, yields the following graphs that show the values of the perturbed frequencies and Q values of the coupled modes as the coupling strength is increased. Note that for the string and body modes chosen here the coupling strength is $\alpha/\omega^2 = 0.05$.

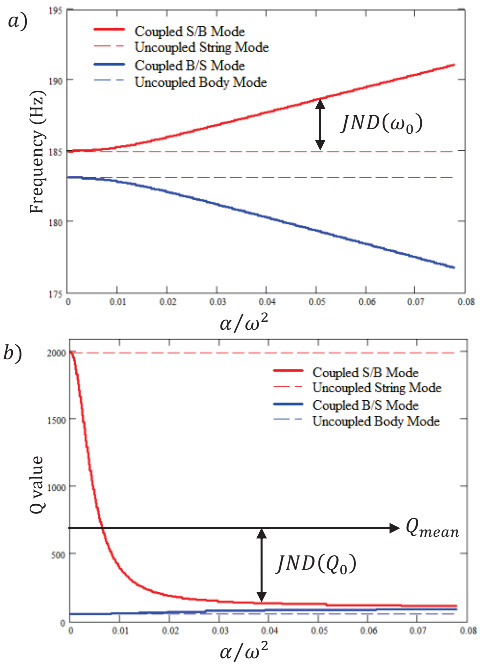

Figure 1: Results from the model showing a) frequencies and b) Q values of the coupled modes as a function of α. Definitions of JNDs also shown. $\omega_B = 2\pi\, 183.1\ s^{-1}$, $\omega_S = 2\pi\, 185.0\ s^{-1}$, $Q_B = 52$, and $Q_S = 2000$.

Figure 1a shows that, as a result of this coupling, the frequency of the fundamental 'string' component, labelled 'Coupled S/B mode', deviates from its natural uncoupled frequency and grows increasingly inharmonic as the coupling strength is increased. Figure 1b shows a dramatic fall in the Q value of the 'string' mode that will result in a fast decay of the fundamental in the radiated sound. Therefore, for low coupling values the 'string' component is likely to sound in-tune and have a long sustain (high Q), but low coupling also results in a quiet sound. As the coupling strength is increased the note gets louder but the fundamental of the note gets increasingly out-of-tune and decays faster (low Q). These are the unwanted aural effects of wolf notes on the classical guitar.

The model demonstrates a compromise that must be reached by guitar makers. Using their experience they must produce a strongly radiating instrument that is free of audible wolf notes. Here we are interested in calculating the threshold for over-coupling in the classical guitar, so we must first measure the thresholds of these two unwanted effects by carrying out listening tests.

We define $JND(\omega_0)$ as the just noticeable difference in frequency of a harmonic fundamental of a guitar tone. Relating to Figure 1a this is the minimum perceivable deviation of the solid line from the dashed line. The frequencies of the coupled modes repel one another, so this deviation can sharpen of flatten the 'string' component depending on the value of ω_S relative to ω_B. Next is the just noticeable decrease in Q value of the fundamental. This effect occurs for all notes on the instrument, the important point is that the effect is more pronounced at frequencies near a strong body resonance. Therefore Q value reduction produces a wolf note when it is so severe that the note stands out as sounding 'dead' compared to its neighbouring notes. We first define Q_{mean} as the mean Q value of fundamentals of the five nearest notes to a strong body resonance. Then $JND(Q_0)$ is defined as the just noticeable decrease in Q value from Q_{mean} (see Figure 1b). These definitions are used to define the coupled parameters that produce a wolf note, ω_{JND} and Q_{JND}.

$$\omega_{JND} = \omega_S \pm JND(\omega_0) \begin{cases} + \, for\ \omega_S > \omega_B \\ - \, for\ \omega_S < \omega_B \end{cases}, \quad (5)$$

$$Q_{JND} = Q_{mean} - JND(Q_0).$$

The above parameters are then used in the model to show the perceptual limit of increasing the coupling strength until a wolf note is produced.

Re-arranging Eqs. (4) for α gives two expressions, one for the coupling strength needed to produce a given perturbation in frequency and the other for a given perturbation in Q value. Then replacing $\omega^*_\pm$ with ω_{JND} yields the minimum coupling needed to produce an out-of-tune note, and replacing $Q^*_\pm$ with Q_{JND} gives the minimum coupling needed to produce a 'dead' sounding note. Sections 3 and 4 of this paper explain how the physical and psychoacoustical parameters of this model were measured.

It is interesting to compare frequency perturbations from the model with the inharmonicity due to string stiffness. The model predicts a sharpening of about 3.6 Hz or 24 cents to the fundamental of $F^{\#}_3$ for the modal parameters shown in Figure 1. This agrees with measurements of the radiated sound of this note. String stiffness sharpens the partials of a string approximately proportional to the square of the partial number [5]. For the same note on this instrument the inharmonicity due to string stiffness is not as severe until well above the 20th partial. This shows that the inharmonicity due to string-body coupling is much more significant than the effects of string stiffness on notes near strong body modes on the classical guitar.

3 Input Admittance

The uncoupled modal parameters of the body are extracted from its mechanical response, with the strings attached but heavily damped. The form of mechanical response measured here was the input admittance measured at the bridge of the instrument, i.e. where the strings transfer their vibrations to the soundboard. The input admittance of the body is defined as its velocity divided by the excitation force as a function of frequency, both measured at the same point (or as close as practically achievable). In this case the velocity was measured by integrating the signal from an accelerometer attached to the bridge and the force was supplied and measured using a small impact hammer. Note that the admittance is the inverse of impedance, so the characteristic impedance of a string multiplied by the admittance of the body gives an estimate of the coupling level of that string-body system.

The admittance of a guitar body shows a series of peaks that represent its various modes. Each mode is modelled as a simple harmonic oscillator [6], with a natural frequency, Q value and effective mass. These parameters are estimated by fitting the sum of the responses of the oscillators to the admittance curve (details of parameter estimation can be found elsewhere [7]).

The admittance 'seen' by the strings at the bridge is not the same for all strings [8]. It is therefore important to extract parameters from the admittance measured at the relevant location to use in the model. This is demonstrated below by measuring the input admittance at different string positions along the bridge of a classical guitar.

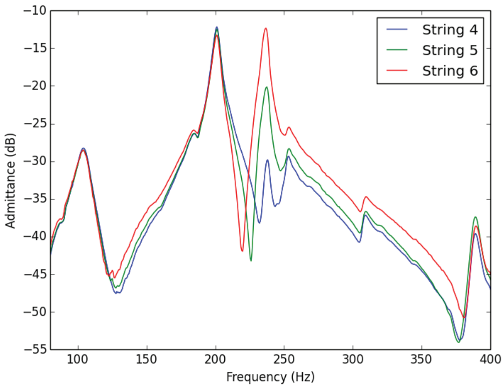

Figure 2: Input admittance of guitar BR1 at three string positions on the bridge. String 4 is closest to the centre of the bridge, string 6 is closest to the edge of the bridge.

The admittance curves in Figure 2 show a small frequency band (80-400 Hz) containing only a handful of low-order modes, which are the most strongly coupled to the strings [9]. Each curve is the magnitude of the complex admittance averaged over five measurements. Due to the symmetry down the vertical axis of the soundboard of this instrument, the admittances at strings 1-3 have a similar shape to strings 6-4.

Notice the large discrepancies in the height of the third peak, this is explained by considering the shape of this mode. The first two lower-frequency peaks are named T(1,1) modes due to their shape [10], an example is shown in Figure 3a. These modes show a consistently high peak in the admittance along the bridge because the bridge lies on the anti-nodal region of this mode. The third peak however is a T(2,1) mode, its shape is shown in Figure 3b. This time the edges of the bridge lie near anti-nodes of this mode but the middle strings terminate near a nodal line. The effective mass increases considerably at a nodal line [9] and so the middle strings are weakly-coupled to the T(2,1) mode (see Eq. (1)).

Notes with fundamentals near the third peak are played on strings that are weakly coupled to this mode, so this mode does not pose the threat of a wolf note. We can therefore narrow down the search for wolf notes to notes near the first two body modes.

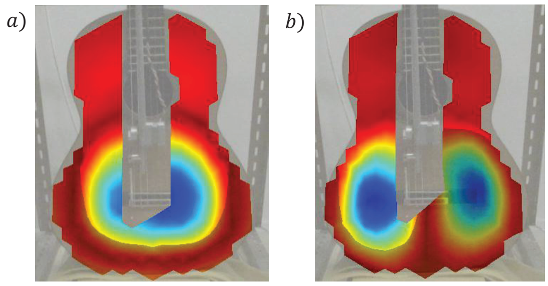

Figure 3: a) T(1,1) and b) T(2,1) mode shapes of a classical guitar top plate. Vibration measurements made using a scanning laser vibrometer.

4 Listening Tests

4.1 Experimental method

The guitar tones used for psychoacoustical evaluation were constructed in the following way. The radiated sound of a plucked note, D_3, was recorded on to a computer using a microphone and soundcard. The string fundamental is characterised using three parameters, namely frequency, Q value and initial amplitude. These are extracted using a combination of a large FFT and a series of short-term FFTs in such a way that amplitude errors due to leakage and damping is minimised [11]. The original fundamental is then filtered out of the sound using a basic Fourier filter. This allows reconstruction of the fundamental with slight adjustments to its parameters before it is added to the filtered tone using additive synthesis. Two listening tests were carried out using this guitar tone, the first measured $JND(\omega_0)$, and the second measured $JND(Q_0)$.

The type of listening test chosen here is the three-alternative forced-choice (3AFC) listening test using a 3-down 1-up adaptive procedure [12], this is defined here as the just noticeable difference. The test consists of many trials, for each trial the listener hears three tones in random order. Two tones are the same, called the reference tone, and the other is different, called the modified tone. The difference between the reference and modified tones is called the modification. The task of the listener is to identify the odd-one-out of the three tones, i.e. the modified tone. After three consecutive correct answers the modification is decreased; this is the down response sequence. After one incorrect answer the modification is increased, called an up response (hence 3-down 1-up). A down sequence followed by an up response, and vice versa, is called a turning point. The test lasts for a total of eight turning points and the result is defined as the mean of the last six turning points.

This type of test is widely used in musical acoustics and further details can be found elsewhere [5, 12-14]. Three people with a musical background took both tests. Future testing will be carried out with groups of larger numbers.

4.2 Results

The first of the two tests measured $JND(\omega_0)$. Here the fundamental had fixed initial amplitude and Q value for both the reference and modified tones so the modification was a change in frequency of the fundamental. The average between all listeners was $JND(\omega_0) = (1.7 \pm 0.4)$ Hz, which is equivalent to about 20 cents. A comparable experiment in the literature was reported by Moore [13]. He measured $JND(\omega_0)$ for a continuous tone comprising of 12 partials with equal amplitude. His result was a little over three times the result reported here. This discrepancy is attributed to the fact that the fundamental here was 10dB louder than the mean of the next 11 partials, whereas in Moore's experiment all 12 components had the same amplitude. Therefore the higher order partials here had a smaller affect in masking the fundamental.

In the second listening test the fundamental had a fixed initial amplitude and frequency for both the reference and modified tones. The modification here was the Q value of the fundamental and the average result was $JND(Q_0) = (351 \pm 62)$ from $Q_{mean} = 687$. Similar experiments could not be found in the literature for comparison.

5 Coupling Threshold Curve

The physical and psychoacoustical parameters were inserted into the model to generate two coupling threshold curves. These curves show the minimum coupling needed to produce a wolf note due to frequency and Q value perturbations. The two curves were then combined to give a single coupling threshold curve, that is, the minimum of the two individual curves. Plotted on the same graph as the threshold curve in Figure 4 are points showing coupling strengths between a body mode and fundamentals of nearby notes, given by Eq. (1). The points that lie above the coupling threshold curve are therefore classed as perceivable wolf notes.

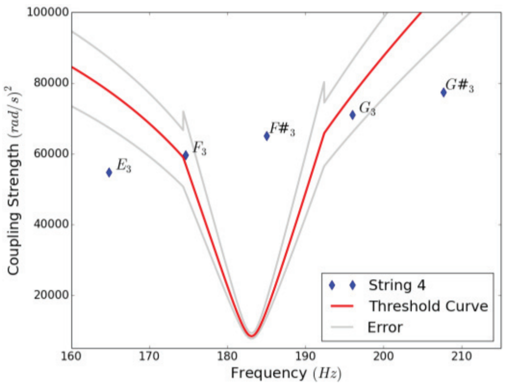

Figure 4: Coupling threshold curve for second body mode of guitar BR2.

Notice that the shape of the curve in Figure 4 changes between 174-192 Hz. This is the region where the threshold for Q value reduction is less than the threshold for frequency perturbation. The general shape of the curve shows the sensitivity to frequency placement of the body mode, e.g. an increase of 3 Hz to the frequency of the body mode would shift the wolf notes from F_3 and $F^{\#}_3$ to $F^{\#}_3$ and G_3. But these body modes are susceptible to relatively large frequency shifts due to atmospheric changes (+/- 5% [15]). Therefore the following method could be used as an indication of the number of wolf notes on a guitar. Simply read the two frequencies where the line joining the points of the graph intercept the threshold curve and convert this frequency band into cents. This is an estimate of the number of wolf notes, e.g. in Figure 4 this line is 182 cents long so will always include either one or two wolf notes.

6 Comparison between Instruments

6.1 Mean Coupling Level

We have found inconsistencies in tonal qualities between notes on the classical guitar by highlighting the presence of wolf notes. This was done by looking at the fine detail of the string-body coupling. Now we take a step back and look at the influence of the level of string-body coupling on the acoustical signature of plucked-string instruments. Here we show the coupling level as the mean characteristic impedances of the strings on an instrument multiplied by the mean level of its admittance within a certain frequency band (80-1,000 Hz).

The nylon strings (NS) of the classical guitar require less tension, T, to get up to playing pitch than steel strings (SS) because they have a smaller mass per unit length, μ. The SS of the banjo and folk guitar are essentially the same with small differences in playing length. The characteristic impedance, $Z_0 = \sqrt{T\mu}$, for each set of strings was estimated using parameters from the literature [14]. The mean characteristic impedances of the NS on the classical guitar, SS on the folk guitar and SS on the banjo are $Z_{C0} = 0.4$, $Z_{F0} = 0.8$ and $Z_{B0} = 0.7$ respectively in units of kg/s.

The input admittances at the bridge of each instrument were measured and are plotted in Figure 5 in the range 80-1,000 Hz. Plotted on the same graph are the mean coupling levels, shown as horizontal lines.

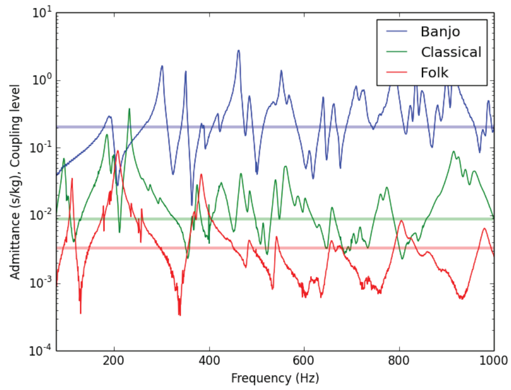

Figure 5: Admittance of banjo, classical guitar and folk guitar along with mean coupling levels (horizontal lines).

The admittance and coupling levels of these instruments can easily be related to their design. The thin membrane soundboard of the banjo is lighter and significantly less stiff (more flexible) than the wooden-plate soundboards of the guitars. This means the impedance mismatch in the banjo is relatively small and energy is transferred from string to soundboard at a high rate. For the heavier and stiffer (more rigid) soundboards of the two guitars, the coupling is weaker resulting in a lower rate of energy transfer and a longer sustain. The soundboard of the folk guitar is thicker and has more struts attached to its underneath than the classical guitar soundboard. This makes it stiffer to be able to withstand the higher tensions of its strings. So the admittance, hence also coupling strength, tends to decrease with increased mass and stiffness of the soundboard.

Relating the admittance curves of Figure 5 to the problem of wolf notes, it seems that wolf notes are less prominent in the folk guitar. Whereas perceivable perturbations in frequencies and Q values of string components on the banjo are likely to occur on so many notes that it is part of the instruments' characteristic sound.

6.2 Modal Parameters

Frequency response functions containing the eight lowest-frequency modes were fitted to each admittance curve in Figure 5 and the modal parameters extracted. The mean and standard deviation of their Q values and effective masses are shown in Table 1.

Table 1: Mean, $\bar{x}$, and standard deviation, σ, of Q values and effective masses of eight lowest-frequency modes.

	Q value		Effective Mass (kg)	
	$\bar{x}$	σ	$\bar{x}$	σ
Folk	44	12	2.10	1.13
Classical	36	22	0.51	0.41
Banjo	70	23	0.05	0.03

The difference in Q values between the two guitars is not significant, though the difference between these Q values and that of the banjo is likely to cause a large difference in tonal quality [16]. There are clear differences between the effective masses of the three instruments. This reiterates the main differences between the mechanical responses of these instruments as this parameter affects the general level of the admittance [7]. Changes of this magnitude will easily cause audible changes in the radiated sound [16].

These findings support the notion that the effective masses of body modes have a substantial influence on the tonal qualities of plucked-string instruments [8-10, 15, 16]. The sound radiation properties of these instruments are also likely to have a significant effect on tonal quality, but these properties have not been measured here.

7 Conclusion

A model describing the interaction between coupled modes was used to show the behaviour of the vibrations of a single string mode coupled to a body mode of a classical guitar. The model showed frequency and Q value perturbations as unwanted by-products of strong coupling. Perceptual thresholds for these effects were measured and, along with parameters of a body mode and a string, were fed into the model to yield a coupling threshold curve. This curve was plotted along with coupling strengths between various notes and a body mode to highlight wolf notes on the instrument.

The analysis of string-body coupling was made more general by comparing the coupling level of different instruments within the same family. The results suggest that although frequencies and Q values of body modes do influence the tonal qualities of an instrument, the effective masses have more influence on its acoustical signature.

Acknowledgments

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