Woods for Wooden Musical Instruments

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In spite of recent advances in materials science, wood remains the preferred construction material for musical instruments worldwide. Some distinguishing features of woods (light weight, intermediate quality factor, etc.) are easily noticed if we compare material properties between woods, a plastic (acrylic), and a metal (aluminum). Woods common in musical instruments (strings, woodwinds, and percussions) are typically (with notable exceptions) softwoods (e.g. Sitka spruce) as *tone woods* for soundboards, hardwoods (e.g. amboyna) as *frame woods* for backboards, and monocots (e.g. bamboo) as *bore woods* for woodwind bodies. Moreover, if we consider the radiation characteristics of tap tones from sample plates of Sitka spruce, maple, and aluminum, a large difference is observed above around 2 kHz that is attributed to the relative strength of shear and bending deformations in flexural vibrations. This shear effect causes an appreciable increase in the loss factor at higher frequencies. The stronger shear effect in Sitka spruce than in maple and aluminum seems to be relevant to soundboards because its low-pass filter effect with a cutoff frequency of about 2 kHz tends to lend the radiated sound a desired softness. A classification diagram of traditional woods based on an anti-vibration parameter (density ρ/sound speed c) and transmission parameter cQ is proposed. Also, the effect of plate thickness can be deduced from the resonance frequency of a bending wave in a thin plate, the driving-point impedance of an infinite plate, and the mean frequency interval of resonance modes of a finite plate.

1 Introduction

Wood selection and plate thickness adjustment are the most important parameters in string instrument design. The body of a string instrument consists, in general, of a soundboard (top plate) and a frame board (back plate and/or side plate). This body structure commonly makes up a resonant box with a sound hole (or sound holes). A notable exception is the modern piano, which have a large soundboard on an open frame, rather than a box-hole structure. Traditionally, softwoods such as Sitka spruce and Norway spruce have been used for soundboards (these softwoods may be called *tone woods*) and hardwoods such as amboyna and maple have been used for frame boards or backboards (these hardwoods may be called *frame woods*). Other hardwoods such as grenadilla and ebony have been used for the body of Western woodwinds such as the clarinet and the oboe, while bamboo (a monocot) has been used for the body of Asian woodwinds such as the shakuhachi and the xiao (these hardwoods and monocot may be called *bore woods*). Moreover, hardwoods such as Brazilian rosewood have been traditionally used for the bars in the idiophone family such as the marimba; in this case hardwood serves as tone wood. Just as in human society we put “the right person in the right place,” so in musical instrument design it is essential to put “the right wood in the right place,” especially in string instruments.

Few musical instruments have simple plates as their acoustic radiators. However, the behavior of plates is an important step to understand the box-hole structure in string instruments. The vibration and radiation characteristics of plates are largely dependent on their thickness. In order to achieve high sound quality, the mechanical impedances of the strings and the soundboard must be controlled very carefully in string instruments [1]. If the strings are coupled to the soundboard via a bridge, the bridge impedance also needs to be considered; the elaborate shape of the violin bridge has evolved to meet this need.

The objective of this paper is to provide an overview of why and how wood is used in musical instruments, primarily in string instruments from acoustical viewpoint. A comparison between vibroacoustic properties of wood, plastic, and metal bars [2, 3] will give some clues. Another important quality criterion for instrument woods is the frequency characteristic of acoustic radiation. Tap tones from sample plates or bars will indicate the significant differences between tone woods, frame woods, plastics, and metals. Moreover, based on the acoustical classification scheme which demonstrates the distinctly different functions required by tone woods and frame woods [2], various woods for strings, woodwinds, and percussions will be mapped onto a classification diagram.

2 Why wood?

2.1 Vibration characteristics

Let us briefly consider why wood is selected as the material for string instruments. Table 1 lists all the material properties discussed in this paper, and gives values for three woods common in musical instruments and compares them to a plastic and two metals. Sitka spruce (*picea sitchensis*) is a typical tone wood for soundboards. Amboyna (*pterocarpus indicus*) is a typical frame wood for the body of Japasese shamisen (three-stringed instrument). Also, amboyna is used as tone wood for marimba bars. Bamboo (*phyllostachys bambusoides*) is a monocot and widely used for Asian woodwind bores.

The first importance is weight. The mass of a given plate scales as ρ/c [2, 4], so a 70 g spruce violin top would have to be replaced by a 440 g acrylic top or a 405 g aluminum one, either of which would make for a very heavy instrument. The second parameter that stands out is the quality factor Q. Acrylic is much less resonant than most woods, and aluminum and steel much more resonant. Table 1 indicates the desirability of the intermediate quality factors of wood, and this is explored below from the viewpoint of acoustic radiation.

Before proceeding to this aspect, let us consider the parameters listed in Table 1. Longitudinal wave speeds c [$= (E/\rho)^{1/2}$] are almost the same in dry woods, bamboo, and metals. However, since the density of woods and bamboo is much lower, they have a low acoustic impedance (sound wave resistance) ρc and a high vibration parameter c/ρ. Moreover, woods have an intermediate transmission parameter cQ [2] (acrylic gives too low cQ and metals too high cQ).

Low wave resistance tends to facilitate the radiation via a better coupling to the surrounding air. Therefore, Sitka spruce is to the preferred choice for the soundboard of string instruments. The higher ρc of amboyna wood causes higher reflection of sound within the instrument body; this suggests that amboyna wood is relevant to the frame board in string instruments.

Table 1. Vibroacoustic properties of woods, a plastic, and metals. For orthotropic wood, *E* refers to E_L, *G* refers to G_{LR}, and *c* to c_L, i.e. properties in the direction of the grain.

	Sitka spruce	Amboyna wood	Bamboo	Acrylic	Aluminum	Steel
Density ρ (kg/m^3)	470	870	700	1200	2700	7800
Young's modulus *E* (GPa)	12	20	15	5.3	71	210
Shear modulus *G* (GPa)	1.1	1.6	1.3	1.9	27	83
Elastic modulus ratio *E*/*G*	11	12.5	11.5	2.8	2.6	2.5
Quality factor *Q*	131	155	140	17	980	1370
Longitudinal wave speed *c* (m/s)	5100	4800	4600	2100	5130	5190
Wave resistance ρc (MPa·s/m)	2.4	4.2	3.2	2.5	13.8	40.4
Vibration parameter c/ρ (m^4/kgs)	11	5.5	6.6	1.75	1.9	0.67
Transmission parameter cQ (10^5m/s)	6.7	7.4	6.4	0.36	50.3	71.1
Acoustic conversion efficiency cQ/ρ	1420	855	920	30	1860	912

Schelleng [4] derived c/ρ by supposing that both the stiffness and the inertia of two plates should be the same if their vibrational properties are to be the same. Since the vibration of a wood plate produces sound radiation, c/ρ may be called "radiation ratio" or "sound radiation coefficient" [1]. The higher the c/ρ, the greater the vibration and radiation. A large difference of c/ρ between Sitka spruce and amboyna wood indicates large differences in vibro-acoustical properties between tone wood and frame wood.

2.2 Radiation characteristics

In Fig. 1, tap tones from sample plates of Sitka spruce, maple, acrylic, and aluminum are analyzed in the frequency domain in 1/3 octave bands [3]. The ordinate P_m indicates the maximum pressure level in each 1/3 octave band from 25 Hz to 20 kHz. Consider the general trends; the several peaks and troughs of P_m depend on the sample size. Although Sitka spruce and maple show the strongest response around 1 kHz and similar response below 1 kHz, the response of Sitka spruce is much weaker than that of maple above about 3 kHz. This weaker radiation of Sitka spruce at higher frequencies makes it a preferred tone wood because the emphasis of lower frequencies is desirable to the pitch sensation in Western music. In contrast to Sitka spruce and maple, aluminum shows a much weaker response below 1 kHz and much stronger response above 3 kHz. This characteristic of metal is generally unwelcome in Asian and Western music. Acrylic might yield good radiation, but weak radiation below 400 Hz would make it a poor material for a soundboard.

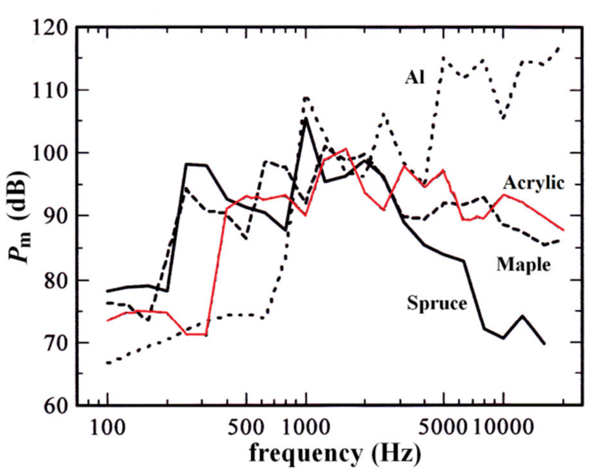

Fig. 1. Acoustic radiation characteristics of tap tones from the sample plates of Sitka spruce and maple and from bars of acrylic and aluminum (after Fig. 12 in Ref. [3])

The large difference of radiation characteristics between wood and metal at higher frequencies (above around 2 kHz in Fig. 1) is attributed to the relative strength of shear and bending deformations in flexural vibrations. As the frequency of the flexural vibration increases, the shear deformation component in the flexural deformation increases in wood, but the bending deformation component is still dominant in isotropic materials like metals. The shear effect, which is indicated by higher values of the elastic modulus ratio *E*/*G* (= E_L/G_{LR} in wood) in Table 1, causes an appreciable increase of loss factor at higher frequencies [3]. Although *E*/*G* of the Sitka spruce example is lower than that of the amboyna wood example, Sitka spruce can often have a much higher value (> 15) than that of amboyna wood (which can be < 8) [3].The stronger shear effect of Sitka spruce compared to maple, acrylic, and aluminum seems to be relevant to sound radiation from the soundboard because its low-pass filter effect with a cutoff frequency of about 2 kHz gives the radiated sound a desired softness [5].

3 Classification diagram of woods

The vibration parameter c/ρ is insufficient alone to acoustically classify wood properties. Another important characteristic for instrument woods is good transmission of vibration [2, 6, 7]. For the back plate of the violin, and the body plate (and neck) of the shamisen, good transmission is needed to make their sounds [2]. If the attenuation (or damping) is relatively weak, the characteristic acoustic transmission is the reciprocal of the attenuation constant α of the longitudinal wave. The solution of the lossy wave equation gives

$$\alpha^{-1} = 2Q/k = 2cQ/\omega, \quad (1)$$

where k and ω are the wave number and angular frequency, respectively [8]. Wood properties, such as *c* and *Q*, are usually measured by observing the first-mode bending vibration of strip-shaped sample plates with the free-free boundary condition [9, 10]. Wood properties such as *E* and α are almost frequency independent over the frequency range of 300 Hz to 1 kHz [11]. Thus it is appropriate to choose a wood sample with a first mode at about 500 Hz and measure the "transmission parameter."

The acoustic conversion efficiency (ACE) proposed by Yankovskii [12] has also been used to characterize acoustic materials [10, 13]. This ACE is the ratio of acoustic energy radiated from a beam to the vibration energy of the beam

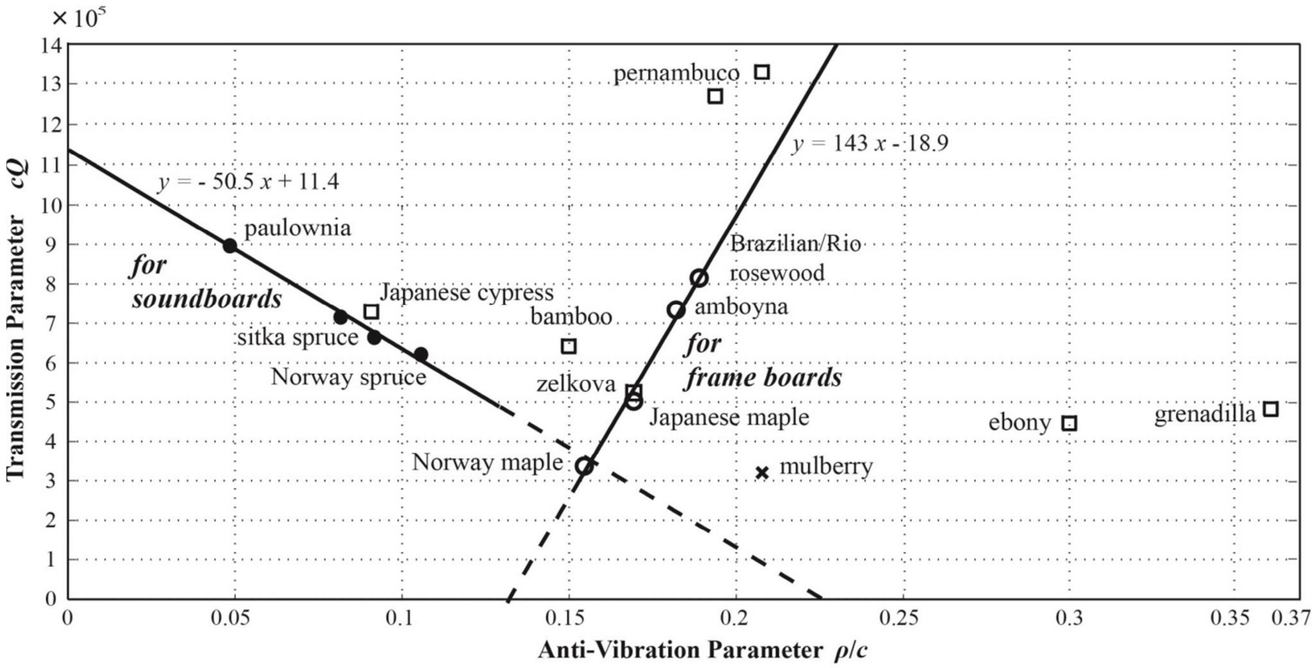

Fig. 2. Classification diagram of traditional woods for string instruments and for other instruments. ●: tone woods for the soundboard; ○: frame woods; ×: traditional wood for the Satsuma biwa; □: traditional woods for other instruments (see the text). The regression line for soundboards is expressed by $y = -50.5x + 11.4$, where $x = \rho/c$ and $y = cQ/10^5$. The regression line for frame boards is expressed by $y = 143x - 18.9$.

and is proportional to cQ/ρ. Thus ACE is simply the vibration parameter (radiation ratio) c/ρ multiplied by Q. Therefore, ACE has the same meaning as the radiation ratio, and it is different from cQ. It is thus proposed to use cQ to characterize the vibration transmission characteristic of wood.

A classification diagram of tone woods and frame woods is shown in Fig. 2 based on ρ/c (the abscissa) and cQ (the ordinate). Since ρ/c is the reciprocal of the vibration parameter and means resistance to vibration, it may be called the "anti-vibration parameter." A clear separation is seen between tone woods [paulownia, sitka spruce, and Norway spruce (*picea abies)*] and frame woods [Brazilian/Rio rosewood (jacaranda, *dalbergia nigra)*, amboyna wood, Japanese maple (Japanese kaede, *acer sp.*) and Norway maple (*acer platanoides*)]. See Table 1 and Ref. [2] for physical properties of these woods.

Also, Fig. 2 includes some other woods: mulberry (Japanese kuwa, *morus alba*) for the best quality Japanese lute, the Satsuma biwa [2]; ebony (Japanese kokutan, *diospyros spp.*) [13] whose properties are similar to grenadilla (*dalbergia melanoxylum*) for best quality clarinets; pernambuco (*Guilandia ecbinata*) [14, 15] for the violin bow; zelkova (Japanese keyaki, *zelkova serrata*) [2, 16] for the shell of the Japanese big drum, wa-daiko; bamboo (ma-dake) (see Table 1) for the shakuhachi body; Japanese cypress (Japanese hinoki, *chamaecyparis obtuse*) [16] for the stage floor for playing the Noh (percussive sound of player's stamping feet is an important element in the Noh play). These woods have different physical properties that cannot be explained well by the quality criteria for string instrument woods, though Japanese cypress and zelkova can be excellent substitutes for Sitka spruce and Japanese maple, respectively. Mulberry and ebony are very heavy and hard, and show very low vibration transmission. Pernambuco is also heavy, but shows very high vibration transmission. Bamboo shows characteristics intermediate between tone woods and frame woods.

4 Frame woods

Since tone woods have been studied long [1, 2, 4, 5, 10, 14], frame woods are mainly described in this paper.

4.1 Woodwind instruments

Because the sound of woodwinds is produced by the resonance of the air column enclosed by the instrument body, the body wall material is not primarily important from the acoustical viewpoint. For example, bamboo has been used for longitudinal end-blown flutes, e. g. the Japanese shakuhachi and Chinese xiao, and African blackwood (grenadilla) has been used for the clarinet and the oboe.

As shown in Table 1 and Fig. 2, the physical and acoustical properties of bamboo are between those of tone wood (spruce and paulownia) and frame wood (rosewood, amboyna, and maple). However, the properties of bamboo significantly vary from the inside to the outside of the culm wall. For example, the density ranges from about 600 kg/m^3 at the inner regions that contain a small amount of fibers (about 15 %) to about 1000 to 1200 kg/m^3 at the outer surface that contains large amount of fibers (about 60 %) [1, 17]. The variation of Young's modulus E (= E_L) and tensile strength with position in culm wall is almost correlated with that of density. The value of E varies from about 5 GPa at the inside to about 17 GPa at the outside [17]. Also, the innermost and outermost surfaces of the culm wall are formed by heavily thickened and lignified parenchyma cells (called the pitch ring) and by the cortex with a wax-coated layer (called the epidermis), respectively [1].

Bamboo is readily available in Asia, and its dense and fibrous outermost surface is suited for woodwind body and playability. In addition, the thick root end of bamboo, which is used to make the shakuhachi, strengthens the wall rigidity. On the other hand, slender dark-stained bamboo pipes are usually used for free-reed mouth organs (Chinese *sheng*, Korean *saeng*, Japanese *sho*, and Indonesian *khaen*) [18]. The much smaller diameter of these organ bores probably supports the wall rigidity.

Ebony and grenadilla, hard woods traditionally used for the clarinet and oboe bodies, are plotted in Fig. 2. From their positions (very weak vibration generation and transmission, particularly extraordinarily high ρ/c), ebony and grenadilla should be really suited for reed instrument bodies which have very high acoustic pressures in the bore.

Pernambuco shows values of anti-vibration parameter a little higher than that of Brazilian rosewood, but shows values of transmission parameter much higher than that of Brazilian rosewood. This extraordinarily high cQ suggests the property necessary to the violin bow.

4.2 Membranophones

Wood frames (or shells) for drums generally have negligible effects on the sound from the membranes stretched with high tension at both sides. However, it has been well known that the most suitable shell material for a Japanese traditional wooden drum (called "wa-daiko", where "wa" means "Japan" and "daiko" or "taiko" means "drum") is keyaki (zelkova). Although the reason is not clear, it is said that zelkova has a kind of *toughness*. In Fig. 2 zelkova is plotted very closely to Japanese maple. However, Japanese maple is never applied to the wa-daiko shell. An appropriate physical or acoustical parameter should be explored to represent the toughness of wood.

According to Ono et al [19], a sharp spectrum peak of the tap tone from the shell of a keyaki wa-daiko is detected at 360 Hz when the membranes are stretched, but that peak is shifted to 260 Hz when the membranes are removed. This result shows that the shell of a keyaki wa-daiko can elastically deform during the attachment of the membranes with high tension. Ono [19] explains that a material with a large sound speed c exhibits high elastic deformability. However, Norway spruce and paulownia have a very large c value of 5300 m/s and zelkova has 4180 m/s [2, 6]. Norway spruce and paulownia are not used for the drum shell. Zelkova seems to have a high recovery force in response to strong membrane tension.

It should be also noted that zelkova is used for tone plates in Japanese temples. These tone plates are hung from the roof and hit by a wooden hammer to call the time. In this case zelkova serves as tone wood.

4.3 String instruments

The Japanese Satsuma biwa, which is almost totally made of mulberry, is a unique string instrument as inferred from the position of mulberry in Fig. 2. Mulberry lies far off the quality criteria for Western string instruments, given by the two regression lines. Mulberry lies in between Norway maple and ebony, and has a very high value of the anti-vibration parameter and a very low value of the transmission parameter.

However, the poor vibrational properties of mulberry seem to match the playing style of the Satsuma biwa, in which the string is strongly struck with a large triangular wooden plectrum (*bachi* in Japanese) made of hard boxwood (Japanese tsuge, *buxus sempervirens*). Striking the Satsuma biwa strings yields characteristic impact tones because the top plate of low resonance nature is simultaneously struck by the stroke of the plectrum.

A mechanism has been invented to compensate for the poor resonance nature of the mulberry body. This mechanism is called the "sawari" (means "gentle touch"), which allows strings to vibrate against the neck or frets, creating a reverberation with a high-frequency emphasis [6, 7]. The biwa frets are very wide compared to those of the guitar and are usually made of hard magnolia (Japanese hohnoki, *magnolia obovata*) for generating subtle sawari effects. A few variants of this sawari are seen as "jawari" (in Hindi) on the Indian sitar and tambura, and as "bray pins" on medieval European harps.

5 Effects of plate thickness

Although it is difficult to determine the spectral frequency of the plate radiation, the frequency f_B (in Hz) of a bending wave in a thin uniform plate is given by [20]

$$f_B = 0.0459 h c_{PL} k_n^2, \qquad (2)$$

where h is the plate thickness, c_{PL} is the longitudinal wave velocity in an infinite plate [$c_{PL} = c/(1 - \nu^2)^{1/2}$, where ν is Poisson's ratio], and k_n is the wave number corresponding to the normal modes of vibrations that depend on the boundary condition of the plate. Therefore, h (as well as ρ and E) is important to determine the vibrational properties of woods, particularly of tone woods. The thinness of top plate in the violin, cello, and guitar largely contributes to decrease their resonance frequencies [6].

Also, the mechanical input impedance Z_m of a thin infinite plate (defined as the ratio of force to the velocity normal to the plate at the driving point) depends on the plate thickness as well as the characteristic impedance ρc. For bending waves in this plate, Z_m is surprisingly a real, frequency-independent constant [8]:

$$Z_m = 4[3(1 - \nu^2)]^{-1/2} \rho c h^2. \qquad (3)$$

Plate thickness is important to meet two conflicting requirements. First sufficient vibratory energy must be transmitted from the string to the soundboard to produce an audible tone. But secondly, the energy should not be transmitted so rapidly that the string vibration dies down quickly resulting in a sound resembling that of a thud [21]. In order to achieve high sound quality, the mechanical impedances of the strings and the soundboard must be controlled very carefully in string instruments [1].

In the violin, a bridge stands on the top-plate surface and forms a termination of the strings. The careful design of bridge geometry (pattern, shape, and thickness) realizes a desirable impedance range for the sound quality of the violin [22]. At the same time, very subtle thickness adjustments of the top and back plates are essential to create the tonal excellence.

Plates provide fairly regularly spaced vibration modes. For a plate of area S, thickness h and sound speeds c_x and c_y in each of the orthonormal directions, the mean frequency interval Δf is as follows [23]:

$$\Delta f \sim (c_x c_y)^{1/2} h / 1.5S \qquad (4)$$

This equation yields values of 73 Hz and 108 Hz for violin top and back plate dimensions respectively. For real violin plates, the bass end of the frequency spectrum is "blocked" by the curvature [23], which makes for a larger spacing. The bracing of a guitar top plate has the same function. The plate thickness is elaborately adjusted in these situations.

Firth [24] notes that for the clarsach, a small Scottish harp, the modes of the bare soundboard are about 50 Hz apart. This spacing grows to about 100 Hz with the addition of the stiffening string bar (which goes a long way to removing the effect of the anisotropy of the soundboard wood), and then to almost 200 Hz for the completed soundbox. Comparable values were found for a modern concert harp soundbox [25].

6 Conclusion

In this paper we have surveyed how wood is chosen and works in musical instruments. Physical properties of Sitka spruce, amboyna wood, bamboo, acrylic, aluminum, and steel are discussed. Various woods are acoustically classified based on the anti-vibration and transmission parameters. Particular interest has been paid to frame woods in woodwind instruments, membranophones, and string instruments. Finally, vibroacoustical significances of the plate thickness are described.

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