Rapid creation of tuning maps for a clarinet using analytic formulas

W. Coyle[a], J. Kergomard[b] and P. Guillemain[b]
[a]CNRS - LMA, Penn State, 31 Chemin Joseph Aiguier, CNRS - LMA, 13402 Marseille Cedex 20, France
[b]CNRS - LMA, 31 Chemin Joseph Aiguier, CNRS - LMA, 13402 Marseille Cedex 20, France
coyle@lma.cnrs-mrs.fr

One measure that a clarinetist uses to judge an instrument is its ability to play in-tune, over the entire range of the instrument without a large amount of added effort on the players part. Thus, in order to design the most "playable" clarinet we must know the actual playing frequencies, which depend on several control parameters including the blowing pressure, reed opening and the input impedance. We can now rapidly deduce these frequencies (analytically) from the different control parameters and input impedance curve. Four effects are known to influence the playing frequency and are examined separately within the analytic formulas: the flow rate due to the reed motion, the reed dynamics, the inharmonicity of the resonator and the temperature gradient within the clarinet. Numerical simulations have been used to test the validity of the analytic formulas in the first playing register of the clarinet. These numerical simulations have the added ability to distinguish in which register the clarinet is playing depending on the chosen value of reed opening and blowing pressure. This paper will present the "maps" which can be created from the analytic formulas and numerical simulations which show, over the full range of blowing pressures and reed openings possible, the expected resulting playing frequencies for a particular clarinet. These resulting maps could be used by an instrumentalist or manufacturer to better understand the expected tuning homogeneity over the range of an instrument and perhaps aid in the future design of an even more "playable" clarinet.

1 Introduction

The current basic design for the clarinet has been around for quite some time. Small-scale optimizations and adjustments have been made over the years yet we are still generally using the same technology to test the clarinet, human subjective opinion. Though clarinet testers are professionals, highly experienced players that have an abundance of knowledge about clarinets, they still have their own opinions about how their instrument should feel and sound. How then, can a manufacturer know if they are creating a great clarinet?

When a clarinetist wants to purchase a new instrument, they take an average of perhaps four of the same model clarinet on loan. They test each one in a variety of situations and decide at that point, which one works best for them. What if, before the clarinetist was to test an instrument, they could visually see the tuning tendencies of each instrument based on the input parameters of blowing pressure and reed opening? Industry, as well as the average consumer, could use a more scientific way of deciding which clarinet fits their needs. Using analytic formulas which were initially detailed in the conference paper by Coyle et al. [3], maps can be created which allow a user to visualize these tuning tendencies. The maps are specific to each individual instrument and are created using the measured input impedance, clarinet geometry and user chosen reed characteristics.

2 Input Impedance Measurement Information

The entirety of this work uses a wealth of information that can be found when studying the input impedance measurement for the clarinet. The first step in this research is to find the resonance frequencies of the clarinet. The resonance frequency is defined as the location of the measured first impedance peak. This input impedance measurement does not take into account the effects due to the mouthpiece, reed and player but gives us useful information about the body of the instrument, which includes the effects of tone holes, bell and geometry of the resonator – hence resonance frequencies.

This input impedance measurement offers us not only the resonance frequencies of the particular instrument in question (ω_n), it also provides the necessary information to calculate values such as the modal factor F_n (equal to $2 \cdot cl$ for a perfect cylinder [9, p. 427]) and Q_n, the quality factor of the resonator. These modal values are extracted from the input impedance measurements for a given instrument and used throughout the playing frequency prediction methods discussed in this work.

3 Analytic Formulas

A clarinet can be described at its simplest, as a cylinder. This cylinder is assumed to be, acoustically, closed at the mouth and opened at the bell therefore, at its base the clarinet can be expected to output frequencies which follow $f_n = \frac{n\pi}{4L}$, where n is odd [6]. There are numerous mechanisms, however, that change this frequency spectrum, as with all musical instruments. There is of course the well known and studied effects of viscous and thermal losses in the bore and radiation losses at the bell [6, 9], but more recently there is research involving gathering analytic formulas that detail the effects of the following: inharmonicity of the resonator, flow created by the reed vibrations, the reed dynamics and a measured temperature gradient in the instrument. The equations for each of these effects are written as a function of non-dimensional control parameters ζ and γ (reed opening and blowing pressure respectively [9, p. 415]) and are presented below. For complete derivation and explanation please see chapter 9 in [9] and [3]. The best way to consider these frequency altering effects is to transform the "length corrections" (as discussed in literature [4, 9]) into a difference in frequency between the resonance (calculated using the instrument's measured input impedance peak frequencies) and the playing frequency (which takes into account all of the loss mechanisms and frequency changing effects).

- inharmonicity

$$N_{cents}Inharm = \frac{-100}{0.06} \cdot \frac{\eta_3}{1 + |1+z|^2} \qquad (1)$$

This effect depends on the heights of the impedance peaks and the difference between the second peak frequency (ω_3) at the resonance (taken from the impedance measurements) and what this value would be in a perfect cylinder ($\omega_3 = 3 \cdot \omega_1$). Here, $\eta_3 = \frac{\omega_3 - 3\omega_1}{\omega_1}$ and $z = \frac{Y_3 - Y_1}{\frac{\zeta(3\gamma-1)}{2\sqrt{\gamma}} - Y_1}$ where the terms $Y_n = 1/Z_n$ are the admittance peak values.

- reed induced flow

$$N_{cents}ReedFlow = \frac{-100}{0.06} \cdot \frac{F_1 \Delta l_0}{2G(\gamma)c} \quad (2)$$

This effect depends on the modal factor of the cylinder F_1 and the empirical function $G(\gamma)$ which is based on the work of Dalmont et al. [4], $G(\gamma) = 1$ if $\gamma < 0.5$ and $G(\gamma) = 2\gamma$ if $\gamma > 0.5$. The term Δl_0 is a known value which describes the equivalent added volume that can be considered a length correction, generally around 10 mm [5].

- reed dynamics

$$N_{cents}Dyn = \frac{-100}{0.06} \cdot \frac{\zeta F_1}{2\sqrt{3}} \frac{q_r}{\omega_r} [1 + \frac{3}{4}(\gamma - \gamma_{th})] \quad (3)$$

This effect depends on the reed characteristics q_r and ω_r as well as the modal value mentioned before F_1. The term γ_{th} is the the pressure threshold of oscillation which depends on the reed opening ζ and the heights of the impedance peaks: $\gamma_{th} = \frac{1}{3} + \frac{2Re[Y(\omega_1)]}{3\sqrt{3}\zeta}$ [9, p. 428]

- temperature gradient

$$N_{cents}Temperature = \frac{100}{0.06} \cdot \frac{9 \cdot l}{4TL} - \Delta T \quad (4)$$

The temperature gradient equation is based on the work done by Noreland [11] and describes a decreasing linear gradient in the instrument from the top of the clarinet (near the mouth) to the bell. The variable l is the location in the clarinet for a particular fingering and L is the total length of the instrument.

4 Comparison with Numerical Simulations

In order to validate these analytic formulas there have been comparisons with numerical simulations that also predict the playing frequencies for the clarinet. Below is a graph that shows the comparison between the numerical simulations that are described in Guillemain et al. [8]. These results were previously presented [2] and are being prepared for publication.

An arbitrary choice for Note was made for these representative figures. Discussed throughout this paper are Notes 1 and 17 of the clarinet with tempered frequencies equaling 146 Hz and 369 Hz respectively (both in the first register of the clarinet). For Figures 1 and 2 (for color see online version) the line at the top in magenta is the temperature gradient effect (this is only

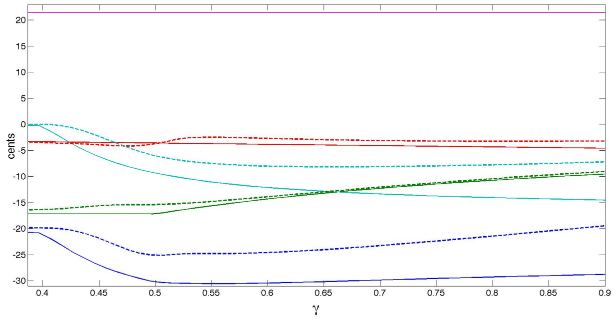

Figure 1: Comparison between numerical simulations (dotted lines) and analytic formula predictions (solid lines) of the playing frequency of the clarinet. Note 1 (tempered frequency (146 Hz), $\zeta = 0.3$

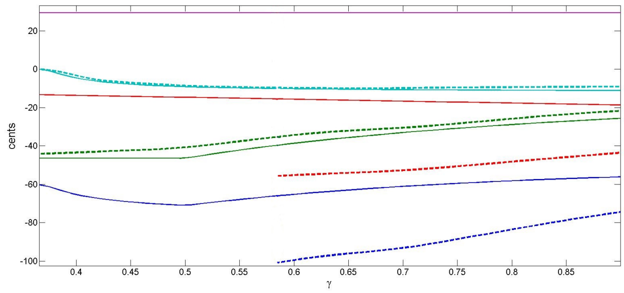

Figure 2: Comparison between numerical simulations (dotted lines) and analytic formula predictions (solid lines) of the playing frequency of the clarinet. Note 17 (tempered frequency 369 Hz), $\zeta = 0.3$

accurate for the analytic formulas since a temperature dependent sound speed profile was not, at this time, used in the simulations). For the others, the dotted lines represent the analytic formula results and the solid lines represent the numerical simulations. The x-axis represents the non-dimensional blowing pressure γ and the y-axis is the difference, in cents, between the resonance frequency and the playing frequency of this particular clarinet. The teal line is inharmonicity, the red is reed dynamics, the green is reed induced flow and the blue line, at the bottom of the graph is the combination of the three effects which gives us the total difference between the resonance frequency of the clarinet and the playing frequency for this particular note at one chosen value of $\zeta = 0.3$ and increasing values of γ.

Considering Figure 1, notice that at low levels of blowing pressure γ the difference is slight between the analytic formulas and the numerical simulations for all of the effects. As blowing pressure increases the difference grows, mainly in the inharmonicity curve. This could be due to the fact that as blowing pressure increases the impedance peaks of the higher harmonics. This is a situation that can be taken into account within the numerical simulations (since we can consider many impedance peaks) however, the analytic formulas only consider the first two impedance peaks and therefore neglects information in the higher ones. This information could be helpful in closing the gap between the two curves.

Figure 2 shows the comparisons for Note 17, a note near the top of the first register and considered to be in the troublesome ‘Throat-tones’ of the clarinet. The curves for inhamonicity and the reed flow effects are extremely close, however in this situation there is discrepancy concerning the reed dynamics effect. In fact, the with the combination of $\zeta = 0.3$ and $\gamma \leq 0.5$ ($\gamma = 0.5$ is the beginning of the beating reed regime) does not allow for playing in this register. This is an issue within the simulations that is still being addressed.

Despite the small differences (realizing that, depending on frequency range, the human hearing threshold can sense an average difference in tones around ±8 cents [7, 10] with 100 cents representing the half-step) the analytic formulas match up well in trend and values to those predicted by the numerical simulations. If the temperature effect was added to the total of the other three effects for the analytic formula predictions we would have, for this chosen value of ζ and increasing γ frequency differences (between resonance and playing frequency) between 0 and 9 cents for Note 1 and 20 and 30 cents for Note 17. Note 17 lies in the throat tones, which are usually quite sharp so it is surprising that we are finding the effects to create an overall negative frequency shift (or flattening effect) however recall that in this work we are comparing to the resonance frequency of the particular clarinet and not the frequencies of the tempered scale.

4.1 Computation Time Improvements

One of the motivations for studying the effects at such a basic level (as with the analytic formulas) is in order to improve computation times. A full scale run of the numerical simulations: clarinet notes 1 - 36, γ values 0 - 1 and ζ values 0 - 1 could take upwards of six full hours on a typical iMac running Matlab whereas the analytic formulas would take no more than 30 minutes on the same machine, which is a significant improvement.

Due to the overwhelming decrease in computation time it is much more useful to implement the analytic formula predictions when studying these individual effects and when doing a full scale run of predictions, for every value of γ and ζ as well as for each note of the clarinet.

5 Using analytic formulas for tuning maps

A recent publication by Almeida et al. shows an example of tuning map that was made by experimental measurements with an artificial mouth and clarinet [1]. This work will show similar maps made from the analytic formulas discussed in this paper. Shown here are tuning maps for the same Notes 1 and 17 discussed earlier, for the full range of input parameters ζ and γ.

The color scheme is made so that black represents no sound and as the parameters lead to more “in-tune” playing the lighter the colors become, with white representing the “most in-tune” (with the expected impedance peak, resonance frequency) at -20 cents for Note 1 shown in Figure 3 and -35 cents for Note 17 in Figure 4. Notice that the threshold of oscillation γ_{th} is shown clearly in Figure 3 especially as there is a point in γ below about 0.35 where there is no sound, despite the choice for ζ. These maps show clearly, in the white regions, what combination of parameters are necessary to play the instrument as in-tune as possible. Recall that the effect of the temperature gradient is not included here.

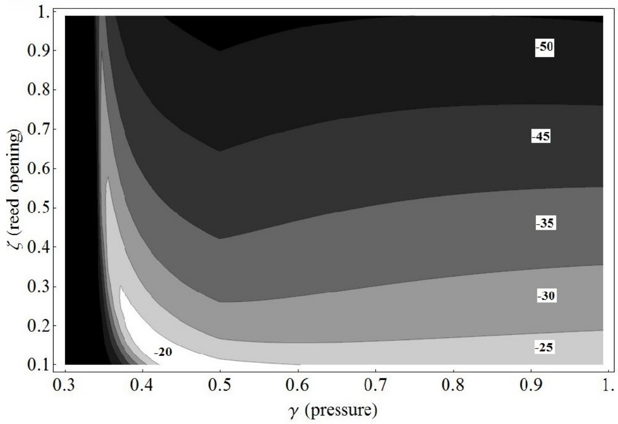

Figure 3: Tuning “Map” for Note 1: Difference between the playing frequency and the resonance frequency of a clarinet, in cents as a function of blowing pressure γ and reed opening ζ.

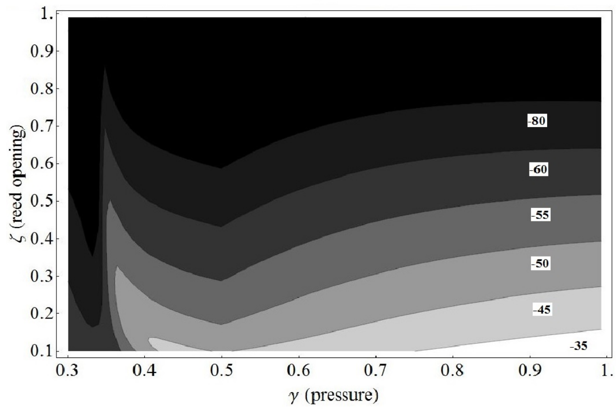

Figure 4: Tuning “Map” for Note 17: Difference between the playing frequency and the resonance frequency of a clarinet, in cents as a function of blowing pressure γ and reed opening ζ.

6 Conclusions and Future Work

The analytic formulas have been shown to be a quick, relatively accurate method to predict the playing frequencies for the clarinet given its input impedance measurement and other user chosen characteristics. The maps show clearly the playing regions where the clarinet should play “in-tune” based on the input note, blowing pressure and reed opening. The hope is that these maps could help manufacturers and consumers alike by giving the first concrete scientific measure of a great clarinet.

In order to further validate these formulas there have been measurements with artificial mouth and experienced musicians alike (using pressure/force/displacement sensor equipped 3-D printed mouthpieces) and this data is in the process of being analyzed. There are also surveys that have been sent to professional musicians as well as various university clarinet professors in order to validate the assumptions and support motivations for this project.

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