Modeling of a woodwind mouthpiece using a finite-element method and characterization of its acoustic input impedance

B. Andrieux[a], V. Gibiat[b] and J. Selmer[a]

[a]Henri Selmer Paris, 25 rue Maurice Berteaux, 78711 Mantes-La-Ville, France
[b]Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 4, France
bandrieux@selmer.fr

The mouthpiece and the reed are crucial elements providing the excitation of the air column of single reed wind instrument. The amount of knowledge on the mouthpiece behavior and its role on the quality of sound production appears very low when it is considered crucial by the musicians. The present work has been realized in a woodwind factory where the production of high quality mouthpieces is very important. An impedance measurement system based on the Three Microphone Three Calibration Method [Gibiat 1990] (TMTC) is available and of common use by the research team of the factory. The aim of our work has been to verify if it was possible to numerically model the behavior of a mouthpiece and to compare it with an impedance measurement. The mouthpiece has been placed as it is when played by the musician; the reed replaced by the impedance measurement system. To be able to compare the obtained results with a numerical simulation the internal shape of the mouthpiece has been digitalized. The obtained file has then been used as boundary conditions for a numerical simulation. This numerical simulation has used a finite elements solver that leads to the acoustic pressure and acoustic intensity giving easily the impedance in frequency domain. A comparison of the measurements and the simulation shows that it is possible to use the numerical finite elements simulation to test various modifications on the mouthpiece shape. Results showing how differences on the mouthpieces geometry rely to impedance differences and comparison as well as the limits of our protocol will be presented on some examples of real mouthpieces.

1 Introduction

The whole chromatic scale of the notes of woodwind instruments is obtained by placing along the air column (cylinder or cone) holes which are going to allow to obtain various frequencies. By opening holes the musician acts as if he shortened the instrument up to the first open hole (supposing that this one is rather wide) allowing him to play various sounds in pitch [1].

In order to excite the air column all the woodwind instruments have an excitation system, for the saxophone and the clarinet this system is the couple reed-mouthpiece. The mouthpiece, essential for the functioning of the global instrument, allows the synchronization of the oscillation of the reed with the resonance frequencies of the tube and particularly in the case of a conical shaped instrument like the saxophone which is very sensitive at the level of the reed. Between various mouthpieces of different internal geometries the instrumentalist can have a very different feeling.

In this study we wish in the first place, to achieve the impedance measurement on saxophone mouthpieces, geometrical characteristic of air column, in order to obtain a factual measurement of the internal shape of mouthpieces. The mouthpiece has been placed as it is when played by the musician; the reed replaced by the impedance measurement system. We wish to observe if geometrical differences in the chamber can be visible on the impedance measurement.

Then, we use a finite element method in order to calculate the impedance of our mouthpieces. The final goal is to show that the measurements and the computation are correlated.

2 Acoustic impedance and measurement technique

2.1 Impedance and air column geometry

The acoustic waves require a medium to propagate (e.g. air). These waves have then a certain frequency (which depends on the environment) and they lead to a variation of pressure and of speed of oscillation of particles.

Woodwind instruments of tubes are all constituted of an air column (bounded by the structure of the tube) and thus possess proper resonance frequencies. In order to characterize these resonance frequencies we introduce the acoustic impedance notion. The acoustic impedance is defined as the average pressure in the measurement plan over the particular speed the profile of which is supposed constant by excluding the zone of acoustic limit layer in the neighborhood of walls [2]. The impedance is given in the frequency domain, thus we consider that all the signals have been "Fourier transformed".

By characterizing the acoustic impedance we can thus know through its maxima (for a flute they would be minima) the functioning frequencies of a tube, which can be for example a sound tube of saxophone, clarinet or an internal chamber of a mouthpiece.

In the case of measurements on woodwinds mouthpieces we consider the resonance frequencies as an internal geometrical characteristic.

For impedance measurement we use a method derived from the TMTC method [1].

2.2 Impedance measurement within the 4 microphone technique

The impedance measurement system is composed of a brass tube inserted in a plastic tube. Along this tube are placed four microphones. The first one being placed the closer of the load to be analyzed. With the propagation models of the acoustic waves in a tube it is then possible to know the pressure and the particle velocity at the point x=0.

Considering two microphones we can write the equations which govern the functioning of the system following the hypothesis: only the plane waves are present in the neighborhood of the measurement plan. It is then possible to say that what is measured by the first sensor respectively the second deducts from the values of pressure and speed in the measurement plan and to write [1]:

$$S1 = \alpha p + \beta \rho c v$$
$$S2 = \gamma p + \delta \rho c v$$

(1)

P and v are the pressure, the particle velocity in the measurement plan, ρ the density of the air and c the speed of sound. The coefficients are then easy to determine in the

case of a cylindrical tube but are unknown if the geometry is not perfectly cylindrical and have to take in account the influences of microphones.

With :

$$y = \frac{S2}{S1} \qquad (2)$$

$$Z = \frac{p}{v} \qquad (3)$$

The impedance Z is written function of two coefficient A and B and y_0 the ratio of the signals S2/S1 determined with the tube closed by an infinite impedance. We can write the impedance as:

$$Z = \frac{(Ay + B)}{(y - y_0)} \qquad (4)$$

Z is the acoustic impedance unknown and to determine.

A and B are coefficients that can be calculated analytically or determined by two calibrations on known impedance. They take into account the speed of the waves and can be analytically determined according the density of the air, the temperature and the distance which separate the microphones of the measurement plan. In the calculation appears an indecision when the distances inter microphones correspond to ½ wavelength. In order to avoid this inconvenience we can move closer as much as possible the microphones (what harms in the precision in low frequency) or use several couples of microphones, what we used here.

Theoretically, we can, with our impedance measurement system reach 8000Hz; indeed a second indecision appears when the distance from the first microphone to the measurement plan is equal to ¼ of the wavelength and thus closer the first microphone is from the measurement plan more it is possible to go up in frequency. In reality the measurement is often subject to interferences, and we consider that it is correct until 5000Hz. In the case of impedance measurement on complete instruments the range in frequency is sufficient.

3 Impedance measurement on mouthpieces

3.1 Measurement by playing side

For the mouthpiece impedance measurement we use an adapter in flexible resin and brass allowing to join the table of the mouthpiece. To have at best the functioning of the mouthpiece we choose that the conduct between the tube of impedance and the mouthpiece is equivalent to the surface left free by the reed in normal playing condition.
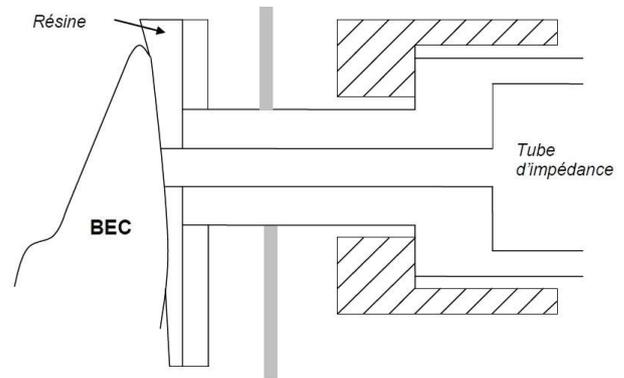

Figure 1 : Adaptation of the mouthpiece

We are measuring the impedance of the mouthpiece connected to a cylindrical cavity.

3.2 Impedance measurement

We measure the impedance of mouthpieces taken out of production, mainly alto saxophone S80 C*. We obtain for a mouthpiece the impedance plot presented below. What we measure and calculate is a mouthpiece connected to a cylindrical cavity, thus it is normal that we observe the following resonances.
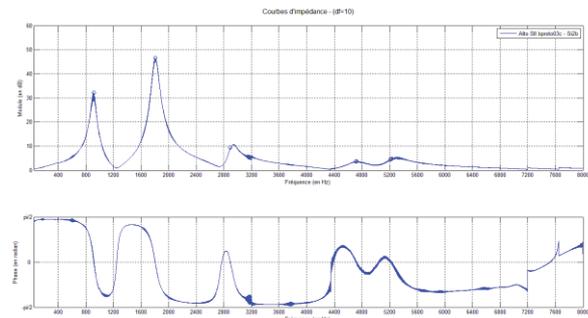

Figure 2 : Impedance measurement on a S80 C* alto saxophone mouthpiece

Some geometrical parameters are well known by the musicians to have a strong effect on the acoustic functioning (height of the baffle, shape of the sound room for example).
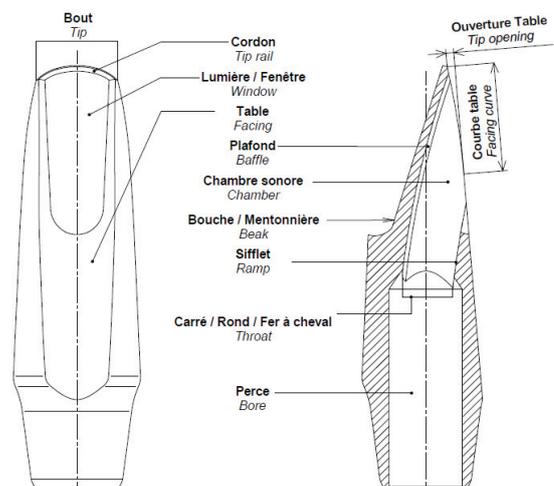

Figure 3 : Description of a mouthpiece

We wish to measure the relevance of our measurement. We verify the repeatability of measurements by successive measurements. We, then, want to know the impact of a

modification on the geometry of the chamber on the impedance plot.

3.3 Impact of internal geometry modifications

Having verified the repeatability of our measurements on the same mouthpiece, we modify some parameters on the chamber such as the width of the square or the height of the baffle (parameters known for musicians as having an impact on the acoustic functioning of the mouthpiece). We observe the modifications of the geometry by plotting the ratio of the impedance of the mouthpiece modified on the impedance of the reference mouthpiece.
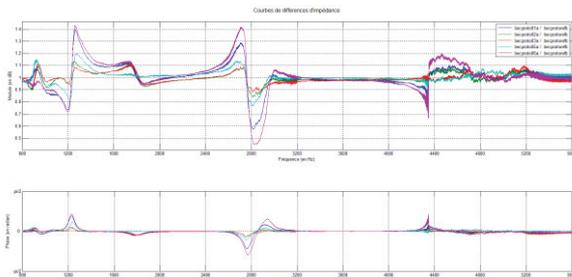

Figure 4 : Plot of ratio of prototype mouthpieces on reference mouthpiece

Each of the plot above represents the ratio of a mouthpiece modified geometrically over a reference impedance. It is interesting to note that all the mouthpieces present a more or less important modification. The mouthpiece presenting the most important geometrical modification is the one who presents the biggest difference on the impedance. We are not capable of knowing which difference will imply such or such plot but we know that the geometrical modification on the chamber have a measurable and observable impact on the impedance measurement.

Geometrically identical mouthpieces are measured in order to have an idea of the repeatability of our measurements. We obtain smaller differences than those obtained with geometrically different mouthpieces.
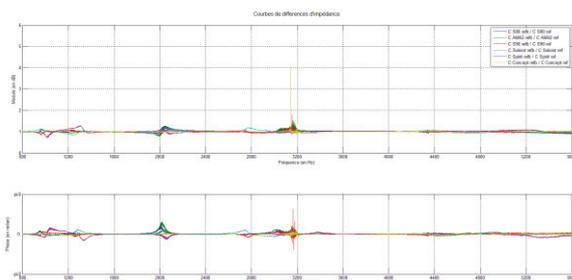

Figure 5 : Measurement on identical mouthpieces

The first part of our study concerned the impedance measurement, we want then to model by a finite element method the mouthpieces in order to correlate what we have found by measurement with what it is computed.

4 Finite element computation on mouthpieces

We want to mesh the internal air column of the mouthpiece and then to apply the right boundary conditions in order to compute the input impedance of this mouthpiece.

We chose to use Code_Aster for our computation. Code_Aster is a free finite element computation code developed by E.D.F. (Electricité de France). This code is able to compute acoustic fields on geometrical structures (Helmholtz equation).

We use the graphical interface in order to achieve the pre and post processing (Salome-Meca). This interface allows us to import tri dimensional files and to mesh our geometry. It is then possible to plot the calculated fields using post treatment tools.

4.1 Finite element method and acoustics

We have seen before that the parameters important to access to for acoustics and impedance measurement are the pressure field and the local particle velocity. We use the acoustic module of Code_Aster to be able to have the Helmholtz equation resolution.

$$\Delta p' = \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} \tag{5}$$

$$\nabla^2 \hat{p} + k^2 \hat{p} = (\nabla^2 + k^2)\hat{p} = 0 \tag{6}$$

The computation is done on the chosen mesh and we chose in our study minimal size of mesh in order to have 10meshes in one wavelength for the maximum frequency.

4.2 Meshing

We use a mesh composed of triangles and tetrahedral elements. We use in the first time an automatic generated mesh, we will be able to modify this mesh to refine it in the zones that we consider critical.

We use the internal air volume contained in the mouthpiece with a cylindrical part led by our cylindrical adapter in order to be in the same configuration as for the measurement.
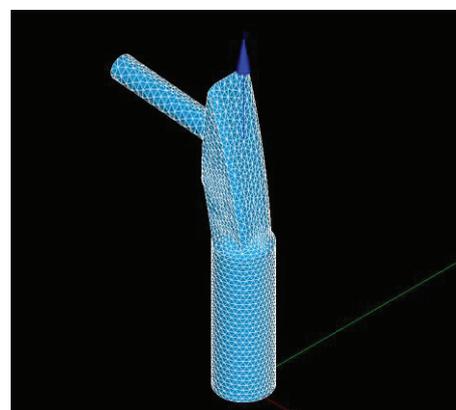

Figure 6 : Meshing of the air with the adapter

4.3 Boundary conditions

Using acoustic functions of Code_Aster allow us to:

- Impose a speed equals to zero on the boundaries of the mesh (corresponding to the walls of the tube and then the walls of the mouthpiece)

- Choose the boundary conditions at the input and the output to have a pressure equals to zero at the output and a plane wave at the input

- Choose a group of nodes at the input on which we will take the values of the pressure and velocity fields in order to compute the acoustic impedance

Once a basic script is done with the Eficas utility we use Python functions to go farther in our model. The condition of speed equals to zero in walls will show itself insufficient, we will apply after a wall admittance.

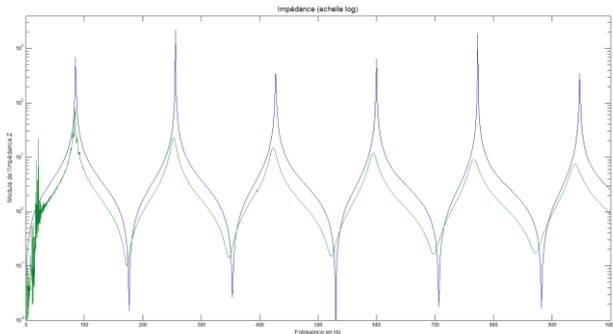The validation of the computation is done on a 1 meter cylindrical tube.

Figure 7 : Measured impedance (green plot) computed impedance (blue plot)

Without the losses on the walls we find the general shape of the impedance plot (modulus) for a cylindrical tube, we don't have the diminution of amplitude in modulus and we have a difference in frequency.

4.3 Losses on the walls

In order to get close to our measurement we have to add the wall losses in the tube. We find in the literature that adding the viscous layer on the wall allows to get close to the real situation [2]. We can add that viscous layer by adding a wall admittance. That wall admittance represents the viscous and elastic losses due to the layer placed close to the wall. The losses are function of the frequency, of the angle of incidence of the sound wave and of the characteristics of the considered fluid (in our case: air).

$$\frac{1}{Z_{parois}} = 0,5(1-i)\frac{\omega^2}{\rho c^2}[l_{vor}\sin^2\theta_i + (\gamma-1)l_{ent}] \quad (7)$$

$$l_{vor} = \sqrt{\frac{2\mu}{\omega\rho}} \quad (8)$$

$$l_{ent} = \frac{l_{vor}}{\sqrt{Pr}} \quad (9)$$

ρ, μ, γ are respectively the air density, the dynamic viscosity and the isentropic coefficient of the air.

ω, θ_i, c are respectively the pulsation, the incidence angle and the sound speed in the air.

In the case of our 1 meter-cylindrical tube we consider grazing incidence on the walls of the tube (we take an angle equals to 90° with the normal). We still have a gap, we try to correct it by adding a small portion of a tube of 0.6 times the output radius (Rayleigh correction).

We try to take in account the thermic losses on the walls [3]:

$$\varepsilon = \frac{1}{R}\sqrt{\frac{c_0}{4\pi f}}[\sqrt{l_v} + 0.402\sqrt{l_h}] \quad (10)$$

$$\rho c = \frac{\rho c_0}{1+\varepsilon} \quad (11)$$

The values of l_v and l_h are function of the temperature. Knowing the radius of the conduct we have access to the variation of sound celerity function to the frequency. With this data we don't get closer to our measurement plots. We don't take this in account. For the moment, we limit ourselves to a comparative study on different geometries.

4.3 Computation on mouthpieces

We compute the impedance of an internal geometry of a saxophone alto S80 C* mouthpiece. We obtain those results:
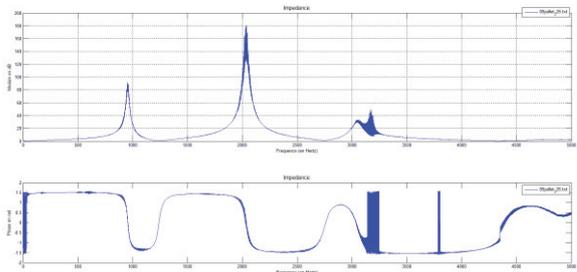

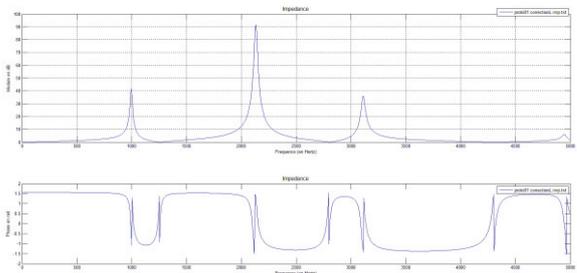Figure 8 : Measured impedance on a S80 C*

Figure 9 : Computed impedance on a S80 C*

Except some absurd points on the plot we have something close (in the global shape) to what we measured. To obtain a closer plot we will have to find another model, in the first time we will use the results to make a comparative study.

It is interesting to note that we have now access to the pressure and velocity fields on the whole mesh. We can imagine to use these informations for another study.
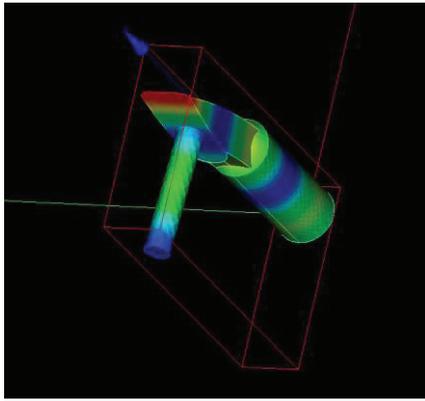
Figure 10 : Pressure field for one frequency

We follow the same approach as for the measure, some of the geometries of the prototypes are drawn in CAD and we compute the ratio of the impedance of the prototypes mouthpieces over the impedance of our reference mouthpiece.
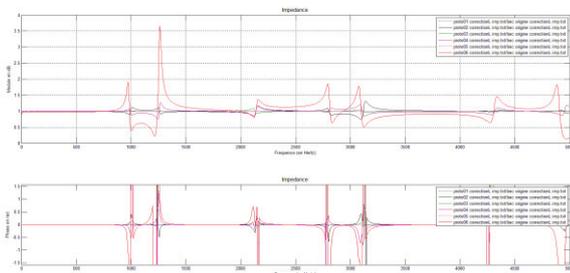

Figure 11 : Ratio of the impedance of the prototypes over the reference mouthpiece

We don't get exactly the same result that we have by our measurement technique but we identify zones located in frequency where the modification in geometry have a strong impact on the impedance plot. The 3 zones are located at the same positions with the measurement technique and with the calculated method

4.4 Results

We are now able to find geometrical differences in the chamber of woodwind mouthpieces measuring the acoustic impedance or computing this acoustic impedance by a finite element method. We observe also that modifications in this chamber imply modifications in the impedance plot (located in frequency). The resonances observed are characteristics of the mouthpiece connected to the conduct, a reliable comparative tool.

We also know that in this study we only consider the internal geometry of the mouthpiece, a small part in the global acoustic functioning of the mouthpiece. The chamber is a part of the information explaining if a mouthpiece work or no for the musician. Many other parameters will affect the feeling of the musician. The parts linked directly with the reed for example have also a strong impact (tip rail).

We could imagine a more complex system taking in account acoustics and mechanics. We can model a mouthpiece with its reed. We can model the reed like a spring system with parameters that can be found in publications.

5 Conclusion

We are able, with our impedance measurement system, to measure the acoustic impedance by the side played by the musician. That point is fundamental because to our knowledge these works had never been done. We are also able to observe differences in internal geometry by acoustic impedance measurement.

We model our mouthpieces by a finite element method with a GNU code (Code_Aster) in order to find what we measured. Differences in internal geometry observed by measurement are observed by computation. We can imagined new models integrating the reed, an important point in the feeling for the musician.

It will be interesting to go further in order to correlate the opinion of musicians with the internal geometry of the chamber.

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Acknowledgements

References

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