The influence of the cone parameters on the sound of conical woodwind instruments

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Proponents of the Pulse Forming Theory claim that the reed closing time in wind instruments remains approximately constant over most of their playing range. Another study by Ollivier and Dalmont might provide an explanation for this phenomenon in terms of the geometry of the cone. Specifically, for a Helmholz motion, the ratio N of the cone (which relates its length to the length of the missing part of the cone) is expected to be the same as the ratio of the opening time to the closing time of the reed displacement signal. The objective of this paper is to find out with the aid of simulations via physical modelling whether the geometrical ratio of a cone N_c corresponds to the ratio of the time domain reed displacement signal N_t. For this purpose, two cones which are identical except for the parameter N_c will be taken, and a simulation will be made to obtain the pressure inside the mouthpiece (which, as shown by Ollivier and Dalmont, is in phase with the reed displacement). The ratio N_c of the cone will be compared to the ratio N_t of the obtained signal. Additionally, two lengths of each cone will be simulated, which means that the geometrical ratio N_c will be shortened, expecting the ratio N_t of the time domain mouthpiece pressure signal to shorten accordingly. Results are presented and discussed.

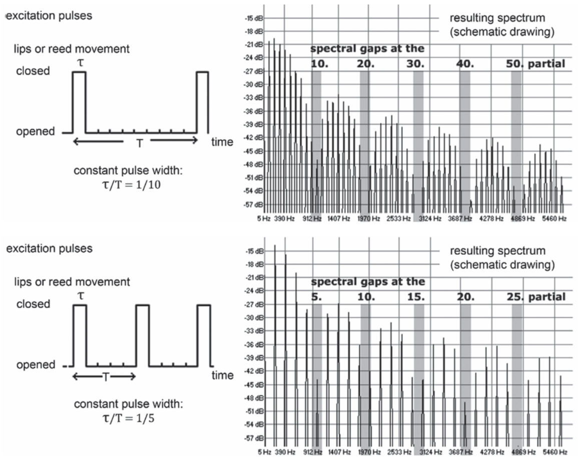

Figure 1: Lips or reed movement and the resulting source spectrum calculated from a closing time of 1/10 of the period (above) and calculated from a closing time of 1/5 of the period (below) (reprinted from [7]).

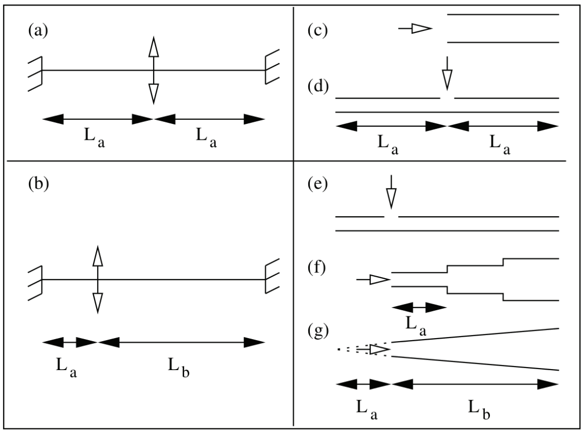

Figure 2: Summary of the formal analogy between the bowed string and woodwind resonators. White arrows indicate the location of the mouthpiece or the bow. A string bowed in the middle (a) is analogous to a clarinet (c) or (d). A string bowed at a location such as $L_a \neq L_b$ (b) is analogous to a cylindrical saxophone (e). If $\frac{L_b}{L_a} = N$ is an integer, it is equivalent to a stepped cone with N cylinders (f). It is approximately analogous to a truncated cone (g) of length L_b, with L_a the length of the missing part of the cone (dashed lines) (reprinted from [9]).

1 Introduction

Researchers have found that the closing time of wind instruments remains approximately constant over their playing range [1] [2] [3] [4] [5] [6] [7]. A recent study conducted by Carral and Reuter [8] measured the sound pressure close to the top of the oboe while being played. Notes of a diatonic C major scale over two octaves were played in three dynamic levels, and both opening and closing times were measured according to [9] and [10]. While the opening time varied among all notes between 0.2 and 3 ms, the closing time always remained within the interval between 0.5 ms and 1.2 ms. Proponents of the Pulse Forming Theory (cited above) have likewise concluded that the fact that the closing time remains approximately constant is the cause of the spectral gaps found in the spectra of wind instruments, as shown in Figure 1.

1.1 Relationship between the bowed string analogy and the reed closing time

Ollivier and Dalmont [9], make an analogy between string instruments and conical woodwind instruments: changing the position of the string excitation (where the string is bowed or plucked) is equivalent to changing the length of the truncation for conical woodwinds (where the cone is cut to place the mouthpiece), such as the oboe (see Figure 2). A crucial difference between string and woodwind players though, is that woodwind players cannot control this parameter, since the length of the truncation is fixed. Moreover, the ratio of the reed opening and closing time of a cone excited by a reed is related to the ratio of the used cone to the missing part of the cone: Let $T = t_o + t_c$ be the signal period, t_o and t_c the opening and closing times respectively, L_b the length of the truncated cone, and L_a the length of the truncation. If $\frac{t_o}{t_c}$ or $\frac{t_c}{t_o} = \frac{L_b}{L_a}$, the oscillation is called “Helmholz motion”, in which case the ratio of the durations of the two parts of the signal is determined by the resonator [11]. Assuming a “Helmholz motion” scenario,

$$N = \frac{L_b}{L_a} = \frac{t_o}{t_c} \tag{1}$$

$$t_o = N t_c \tag{2}$$

$$L_b = N L_a \tag{3}$$

The signal period T is related to the total length of the cone (including truncation) by [12]:

$$L_a + L_b = \frac{\lambda}{2} \tag{4}$$

with

$$\lambda = \frac{c}{f} \tag{5}$$

hence

$$t_c + t_o = \frac{2}{c} \cdot (L_a + L_b) \tag{6}$$

and using equations (2) and (3)

$$t_c = \frac{2}{c} \cdot L_a \tag{7}$$

Therefore, t_c is proportional to L_a, and given that for a given instrument L_a is constant, so is t_c.

2 Objectives

The objectives of this paper are as follows:

1. To find out whether the geometrical ratio of a cone N_c and the time ratio of its corresponding mouth pressure signal N_t are the same, as predicted by [9];
2. to compare the closing time t_c of a simulated cone with what was measured for the notes C_4 and G_4 in [8] for the real oboe;
3. to verify that, when the cone truncation L_a of a simulated cone is increased, the closing time t_c is also increased;
4. to verify that, for a shorter closing time, the first spectral gap of the mouthpiece pressure signal is shifted to a higher frequency, as predicted by the Pulse Forming Theory.

3 Methodology

According to [9], the reed opening and the mouthpiece pressure have the same phase, so one can measure the reed closing time by looking at the mouthpiece pressure. The latter can be either measured during performance or simulated with a time domain physical model.

3.1 Instruments

For the reference cone (the Oboe), the cone (half) angle was taken as $\theta = 0.75°$ from what was measured on a real oboe in [13]; the total length of the cone L_T was chosen so as to give a playing frequency that approximates a note C_4 (260Hz) and a note G_4 (390Hz), giving $L_T = 650$mm and $L_T = 440$mm respectively; and the length of the missing part of the cone L_a was chosen to be $L_a = 91$mm, as measured for a real oboe in [13]. In this case, $\kappa = \frac{L_b}{L_T}$, as defined in [13] gives $\kappa = 0.86$ where $L_b = L_T - L_a$ is the used cone, and L_a is the length of the missing part of the cone.

Since the objective of this paper is to measure what happens when the geometrical ratio N_c is changed, a second instrument (which will be referred to from now on as "tárógató-like oboe" or "TLOboe") was designed with the same θ and L_T parameters as the Oboe, but with a $\kappa = 0.75$ for the note C_4 (as measured in [13] for the case of the tárógató), giving $L_b = \kappa L_T = 487.5$mm. A summary and comparison of these four cones is presented in Table 1 and in Figure 3.

$\theta = 0.75°$	Parameter	C_4	G_4
Oboe	L_T[mm]	650	440
	L_b[mm]	559	349
	L_a[mm]	91	91
	D_T[mm]	2.38	2.38
	D_B[mm]	17	11.52
	N	6.14	3.84
TLOboe	L_T[mm]	650	440
	L_b[mm]	487.5	277.5
	L_a[mm]	162.5	162.5
	D_T[mm]	4.25	4.25
	D_B[mm]	17	11.52
	N	3	1.71

Table 1: Geometrical parameters of the simulated cones.

An 8 mm long cylinder is attached to the top of each of the four cones described above, corresponding to the volume of the reed. This ensures that the first sample of the simulated impulse response of the instrument is equal to zero.

3.2 Physical Model

The double-reed excitation mechanism can be modelled using a single mass-spring system, as in the case of single-reed modelling, and assuming symmetric displacement for both blades. The equation of motion for the reed is given by

$$m\frac{d^2y}{dt^2} + mg\frac{dy}{dt} + k_\alpha y = \Delta p \tag{8}$$

where Δp is the pressure difference across the reed, y is the reed displacement from its equilibrium position, m the mass per unit area, g the damping per unit area and k_α the effective stiffness per unit area of the reed. The methodology proposed in [14] is adopted for the numerical simulations. The coupling of the nonlinear excitation element with the linear resonator is achieved via convolution with the reflection function r_f of the resonator. This was obtained from the input impedance Z_{in} which was simulated using the program VIAS (Versatile Instrument Analysis System[1]) by inputting the bore geometry. The reflection function r_f is then calculated as described in [15].

3.3 Signal Analysis

From the mouthpiece pressure signal it is possible to find out the opening and closing times of the reed. This is done as follows:

The mean value of the pressure over a period must be zero [9]. The pressure $P_{ref} = P_{max} - P_{min}$ is calculated. The time in which the pressure is below P_{ref} is then the closing time, as explained in [10].

Each period of the steady state of the time domain signal was found by looking at the pressure maxima. Then P_{ref} was calculated, and the number of samples that fell above and below P_{ref} were counted and saved as opening samples N_o and closing samples N_c respectively. The opening time t_o and

[1] http://www.bias.at/

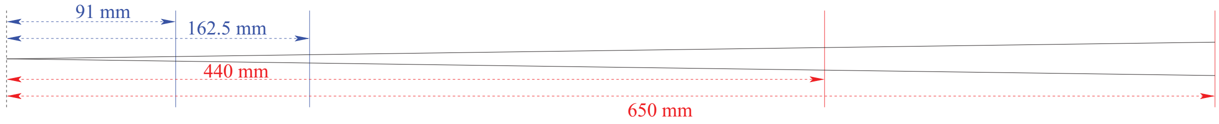

Figure 3: Schematic of the instruments described in Table 1.

	Parameter	C_4	G_4
Oboe	t_c[ms]	0.10 ± 0.01	0.12 ± 0.01
	t_o[ms]	3.94 ± 0.01	2.89 ± 0.01
	$N_c = \frac{L_b}{L_a}$	6.14	3.84
	$N_t = \frac{t_o}{t_c}$	39.4	24.1
TLOboe	t_c[ms]	0.54 ± 0.01	0.56 ± 0.01
	t_o[ms]	3.25 ± 0.01	2.00 ± 0.02
	$N_c = \frac{L_b}{L_a}$	3.00	1.71
	$N_t = \frac{t_o}{t_c}$	6.02	3.57
Real Oboe	t_c[ms]	[0.89, 1.00]	[0.87, 0.94]
	t_o[ms]	[2.87, 2.95]	[1.63, 1.67]
	N_t	[2.95, 3.22]	[1.78, 1.87]

Table 2: Opening time t_o and closing time t_c of the mouthpiece pressure signals calculated from the simulations of the different cones and from measurements on the real oboe, as well as the geometrical ratio N_c and the time ratio N_t.

closing time t_c are calculated from the number of samples as follows: $t_o = \frac{N_o}{f_s}$ and $T_c = \frac{N_c}{f_s}$.

The mean and standard deviation of N_o and N_c throughout the duration of the steady state of the note was calculated for each simulated signal.

4 Results

4.1 Closing time t_c and opening time t_o, and Ratios N_c and N_t

The closing time t_c and opening time t_o, the geometrical N_c and time N_t ratios measured for the simulated mouthpiece pressure signals from the cones described in Section 3.1, as well as those for the real oboe that were presented in [8] are shown in Table 2.

As shown in Table 2, the opening time t_o and closing time t_c of each instrument in both notes did not change proportionally to the playing frequency. But comparing t_c of both instruments, one can see that the t_c of the Oboe is shorter than that of the TLOboe. From this one can conclude that the shorter L_a is, the shorter t_c becomes. It is worth noting that all four cones were simulated using exactly the same reed parameters. Therefore the difference in t_c between the Oboe and the TLOboe cannot be attributed to a difference in reed parameters.

According to equation (7), if the length L_a is known, the closing time t_c can be predicted. However, this prediction (0.5ms for the Oboe, and 1ms for the TLOboe), is five times as much as what was calculated for the Oboe, and twice as much as for the TLOboe. In the case of the TLOboe, the ratios N_c and N_t also differ by a factor of 2, whereas for the Oboe the ratios differ by a factor of 6. The calculation of t_c depends on the reference value P_{ref}.

The closing time t_c, opening time t_o and the time ratio N_t of the simulated Oboe obtained here can be compared to what was measured in [8] for the real oboe. Although the geometry of the simulated Oboe is probably not exactly like that of any real oboe, an attempt was made to reproduce a realistic geometry (see [13]). The closing time of the real oboe is almost 9 times longer than that of the simulated Oboe. Some reasons for this could be that the tone holes have an effect on the closing time, or that the geometry of the real oboe is not as simple as assumed. Interestingly, the time ratio N_t of the real oboe is closest to the geometrical ratio N_c of the TLOboe. Another fact worth noting is that, in order to simulate the instruments described in Table 1, an 8mm long cylinder was added at the top. The effect that this might have in t_c is unclear.

4.2 Spectra

The spectral envelopes of the mouthpiece pressure signals for the two instruments (Oboe and TLOboe) simulated here for notes C_4 and G_4 are shown in Figure 4.

The first difference that can be seen in the spectra of these two instruments is that the Oboe has a wider lobe at the low frequencies than does the TLOboe. Indeed all harmonics have a higher amplitude on the Oboe than on the TLOboe. This is true for both notes. This will result in the Oboe having a brighter sound colour than the TLOboe.

According to the Pulse Forming Theory, in the case of the oboe and other wind instruments, the reed closing time remains constant as the frequency changes, and hence some harmonics are not excited or very weakly excited, resulting in a spectrum that resembles a $\frac{sin(x)}{x}$ function, whereby the areas where the harmonics have a high amplitude are called formants [3], [7]. The term formant in this case is used because the frequencies that fall in those areas are always present, independently of which note is played. One way of estimating the closing time t_c of recordings of real instruments has been to find the frequencies of these gaps.

In the case of the instruments presented here, we would have to find where the gaps in frequency occur. Table 3 shows the frequencies of the first four frequency gaps. All frequency gaps of the Oboe occur at a higher frequency than those of the TLOboe.

From Table 3 one can see that there is at least one frequency gap that occurs in both notes: For the case of the Oboe these are at 3 and at 5.7kHz, and for the case of the TLOboe these are at 2.4 and at 3.5kHz. Interestingly, the inverse of the second Oboe common frequency and of the first TLOboe common frequency gives us a value that is close to the closing time t_c calculated and shown in Table 2 (0.18ms and 0.42ms respectively).

4.3 Cylinder approximation

According to [9], when the geometrical ratio N_c is an integer, the cone can be approximated by a series of

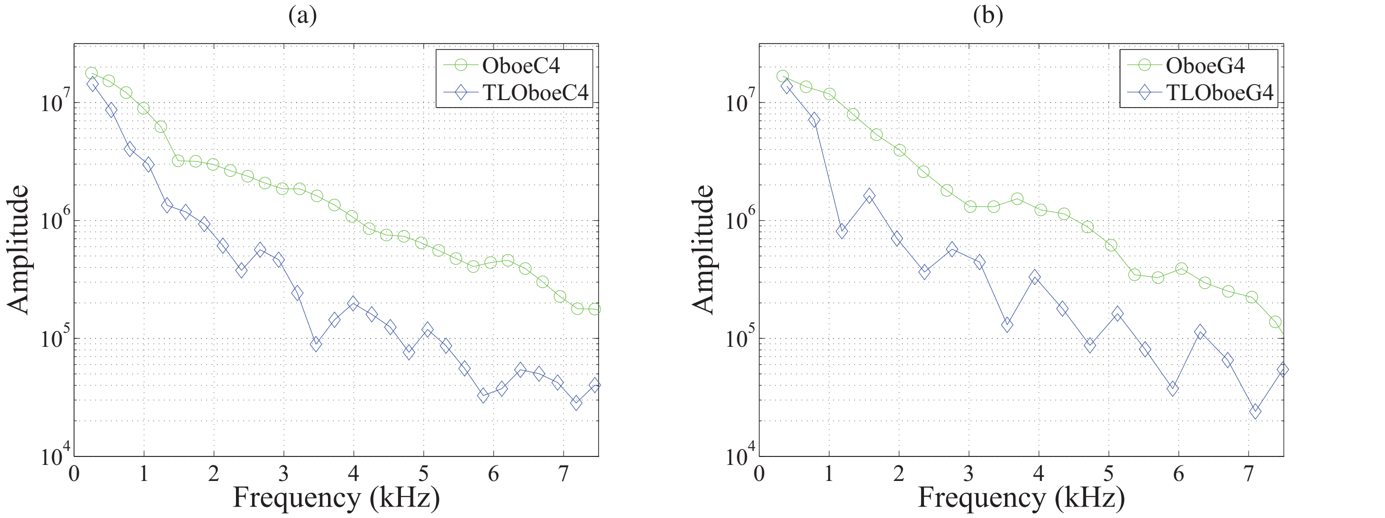

Figure 4: Spectral envelope of the Oboe and TLOboe playing the note (a) C_4 and (b) G_4.

	C_4	G_4
Oboe	1.5	3
	3	5.7
	4.5	8
	5.7	10
TLOboe	0.8	1.2
	1.3	2.4
	2.4	3.5
	3.5	4.7

Table 3: Frequencies (in kHz) of the spectral gaps of the two instruments for both notes.

cylinders of increasing diameter, as shown in Figure 2. This ratio for the TLOboe playing the note C_4 is $N_c = 3$. The input impedance of the cone as described in Table 1 (without the 8mm long cylinder at the top) and of its cylinder approximation is shown in Figure 5. The peaks of the impedance of the cylinder approximation are harmonic, whereas those of the cone are not (as expected for a cone impedance [12]). An interesting fact is that every fourth harmonic of the cylinder approximation is an antiresonance (which is what would be expected in the spectrum of a string), and it coincides with an antiresonance of the cone impedance. The inverse of that frequency is just under 1 ms, which is close to the closing time found for the real oboe in [8].

Quoting Heptner [5]:

> "The resonance curve of the conical air column is not the cause of the formation of formant regions. This curve has obviously an influence on the amplitude of each harmonic component, but each effective length of air column has its own resonance curve."

Further on, he says:

> "It can only be because the "exciter", the reed, does not offer these frequencies to the "consumer", the resonator"[2].

[2]Translation by one of the authors

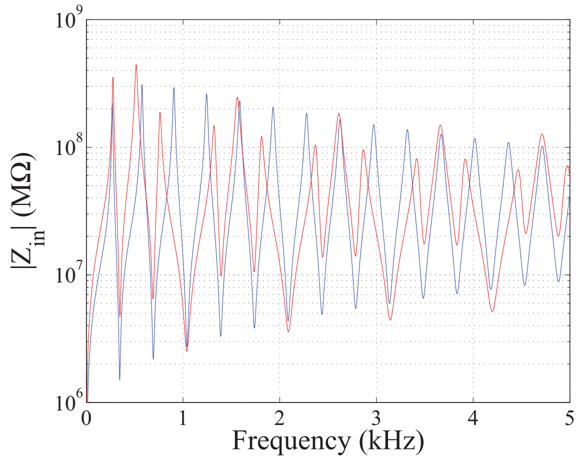

Figure 5: Input impedance of the cone corresponding to the TLOboe note C_4 (blue) and of its cylinder approximation according to [9] (red).

Since Heptner discarded the fact that the air column could be responsible for the frequency gaps in the spectrum, he concluded that they must come from the reed [5] . Figure 5 shows clearly that there are harmonic frequencies on the resonance curve of the air column where a harmonic component cannot be built upon. Moreover, if the length of both the cone and its cylinder approximation are reduced in such a way that the ratio N_c remains an integer, the antiresonances fall in the same frequencies as those shown in Figure 5, and they coincide on both impedances. It is therefore the authors' opinion that the geometry of the cone has more influence on the closing time t_c and therefore the formants of wind instruments than was granted by Heptner [5].

5 Conclusions and Future work

Two cones were taken (Oboe and TLOboe), whereby the only geometrical difference between them was the length of the missing part of the cone L_a. Two different lengths (total length of the cone L_T) of each cone were taken, that correspond to a first resonance that is close to the notes C_4 and G_4.

The closing time t_c of both Oboe and TLOboe was approximately the same for both notes. Since all cones were simulated using the same reed parameters, the difference seen here of t_c between Oboe and TLOboe cannot be attributed to the reed, as opposed to what other researchers like Heptner [5] have claimed. It appears that the shorter L_a, the shorter the closing time t_c. The geometrical ratios N_c do not match the time ratios N_t. The closing time of the real oboe does not match with what is predicted for the Oboe. The shorter L_a, the higher in frequency the spectral gaps are shifted, and the brighter the sound becomes. The discrepancies found here, compared to what is predicted by both the Pulse Forming Theory and the Analogy with the bowed string theory, might be explained by the fact that those two theories assume that the reed movement is a square signal, which is not the case in a real instrument. Further investigations regarding the closing time and its dependency on the cone parameter L_a are planned, and include: Simulations with more than two lengths of cone L_b, measurements with cones manufactured with the same simulated cone dimensions, and comparison of these with signals from real oboes played with staples of two different lengths, so as to match the same L_a parameters of the simulated cones.

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