Modification Design of Harmonic Sound Plate with Equal Width for Metallophone

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The percussion sound of metallophone bar is related to structural modal parameters, in terms of natural frequencies and mode shapes. The harmonic sound plate (HSP) that is a special shape of plate can produce the harmonic sound effect, i.e. the overtone frequencies possess integer ratios with respect to the fundamental frequency. The HSP is potentially applicable to construct a brand new percussion instrument similar to metallophone or xylophone. This work investigates the modification design of HSP so as to make a commercial percussion instrument. The basic design of HSP is first reviewed by showing its percussion sound characteristics relevant to structural vibration modes. The percussion sound spectrum of the HSP reveals three major frequencies with harmonics corresponding to different vibration modes. The prototype of new metallophone consisting of a set of HSPs is also introduced. The initial design concept is to enlarge or reduce the size of HSP proportionally in order to fit the pitch frequency. The metallophone set becomes bulky and not easy to play. Design considerations for the improvement of commercial product of metallophone are discussed. Finite element analysis (FEA) is then adopted to perform sensitivity analysis on geometry parameters and determine final design of HSP geometry such that the HSPs can not only generating harmonic sound effect for different pitch frequencies but also have about the equal width. The conceptual design of new type of metallophone can be finalized and ready for manufacture.

1 Introduction

There are two types of percussion instruments, i.e. tuned and un-tuned. Xylophones and metallophones are typical tuned instruments, while triangles, cymbals and gongs are un-tuned.

The harmonic sound plate (HSP) [1,2] that is a special shape of plate can produce a harmonic sound effect, i.e. the overtone frequencies possess integer ratios with respect to the fundamental frequency. The harmonic sound effect reveals a good sound quality for percussion instrument. Wang et al. [3] also presented the same idea of harmonic sound effect by using the glass material instead of steel to obtain the harmonic sound plate. The use of HSPs in making a new type of percussion instrument is promising. Wang et al. [4] manufactured the prototype of percussion instrument consisting different sizes of HSPs and evaluated its sound quality.

The shape design of metal bar for special sound effect can be carried out by using finite element analysis (FEA) [5]. Petrolito and Legge [6] adopted optimization approach in obtaining special sound effect of xylophone. Bretos et al. [7] tuned the modal properties of wooden bars by FEA as well. Wang [8] summarized the procedure and advantages by applying FEA and EMA for percussion instrument design.

To make a commercial percussion instrument by adopting the HSP, several issues are concerned. Different designs of geometry of HSPs to fulfill different pitch frequencies for musical notes are required. The size of each plate should be in the equal width and suitable for assembly of a set of percussion instrument such as xylophone or vibraphone.This work is to investigate the effect of geometry parameters of HSP on making the set of metallophone and so forth to explore proper geometry design for making the commercial product.

2 Basic design of HSP

This section aims to briefly review the basic design of harmonic sound plate (HSP) [1,2] and discuss the drawback of prototype design [4]. The objectives of design modification are summarized.

2.1 Prototype of HSP metallophone

Figure 1 shows the prototype of metallophone consisting of a series of HSPs, and each HSP's basic pattern is shown in Figure 2. As shown in Figure 1, the new design of metallophone contains Notes from C5 to F7 and arranged just like xylophone in accordance with black and white keys in piano. The width of C5 is about 15.2 cm and 6.5 cm for Note F7. The width of each HSP is gradually decreased due to the elevation of pitch frequency. The different width of each HSP for the prototype design may confuse the player in striking the proper locations during performance. Therefore, to redesign the metallophone with the equal width for each HSP of different musical note is of interest and necessary for commercial product development.

Figure 3 presents the percussion sound characteristics of the HSP for Note E7, and Table 1 summarizes the corresponding frequency comparison results. Some special sound effects are discussed as follows. Figure 3(a) shows the percussion sound response in the time domain due to single stroke on the HSP, and Figure 3(b) reveals the corresponding sound spectrum.

From Table 1, the pitch frequency for E7 is referred to mode 1, i.e. 2637.02 Hz. The special design of HSP reveals three natural frequencies possesses the integer frequency ratios that make the sound spectrum as shown in Figure 3(b). The fundamental frequency is the pitch frequency of the note (E7), and the other two natural frequencies of higher modes are overtone frequencies. The mode shapes of the first three vibration modes are (3,1), (1,3) and (3,3) in terms of rectangle plate modes. They are depicted as shown in Figure 3(b). As one can observe in Table 1, the measured frequencies match well with those target frequencies, in particular the pitch frequency error within 0.34%. The overtone frequency ratios are nearly the integers for 2.02 and 3.03, respectively. This integer ratio phenomenon for overtone frequencies is so called the harmonic sound effect in this work. The percussion sound with this harmonic sound effect is bright and harmony and makes the HSP a good sound quality as a percussion instrument.

Figure 1: Prototype of new design metallophone for harmonic sound plate (HSP) [4].

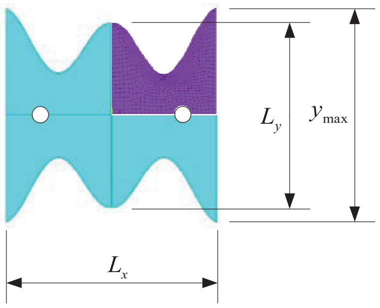
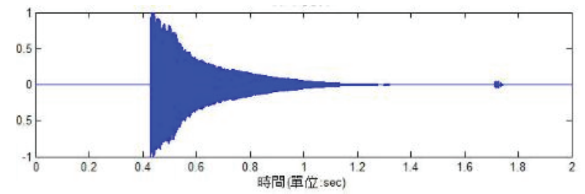
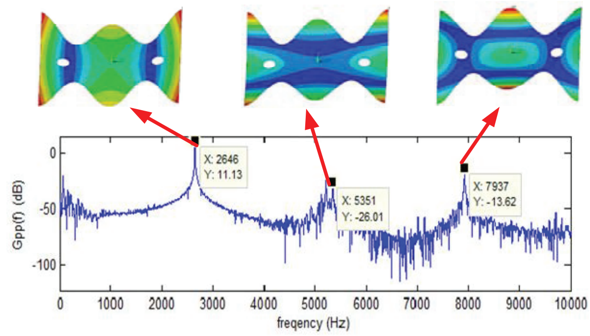


Figure 2: Basic pattern for harmonic sound plate (HSP) [5].



(a) time domain response



(b) auto spectrum of percussion sound

Figure 3: Percussion sound characteristics for Note E7.

Table 1: Modal frequencies of HSP for Note E7.

mode	Target frequency (Hz)	Measured frequency (Hz)	Frequency difference (Hz)	Frequency error (%)	Frequency ratio
1	2637.02	2646	8.98	0.34	1.00
2	5274.04	5351	76.96	1.46	2.02
3	7911.06	7937	25.94	0.33	3.03

2.2 Design consideration

In order to make a commercial product for the new design of HSPs, we have to overcome the drawback for different widths of HSPs. Therefore, the goal for design modification is to carry out the structural geometry optimization, so as to find the equal width for each HSP corresponding to different musical notes. The primary design goal is to find the new design of HSPs with about the same width for two octaves from Notes F5 to F7.

Since the basic geometry pattern for the HSP is found as shown in Figure 2, the variation of geometry parameters are studied to explore the possible shape patterns possessing the harmonic sound effect.

Section 3 will present the sensitivity analysis for geometry variation for the same thickness of HSP that will maintain the percussion sound with harmonics. First, two Notes F6 and F7 are, respectively, considered to find the aspect ratio relations between length and width. From the aspect ratio studies, we found the different thickness of HSP is required to obtain the new design of equal width HSPs that can fulfill the frequency contents, i.e. harmonic sound effect.

3 Sensitivity analysis for geometry

Table 2(a) and 2(b) shows modal frequencies of the original design of HSPs for Notes F6 and F7, respectively. The FEA frequencies agree well with those target frequencies and have the perfect integer frequency ratios. The maximum length (L_x) and width (y_{max}) are listed in Table 3 for Case No. 1, respectively.

For exploring the different length and width aspect ratio effect on the HSP, different lengths of HSP are confined to obtain the corresponding shape of HSP that can meet the frequency ratios properly. Table 3 shows different lengths of HSPs (L_x) and corresponding maximum widths (y_{max}) for the HSPs of F6 and F7, respectively. Each Case No. indicates that the HSP can have the harmonic sound effect. For those cases in Table 3, the quarter of their geometry patterns are depicted in Figure 4(a) and 4(b) for Notes F6 and F7, respectively. Figure 5 shows the aspect ratio trend referred to those data shown in Table 3.

Table 2: Modal frequencies of HSPs for Notes F6 and F7.

(a) Note F6 for No. 1

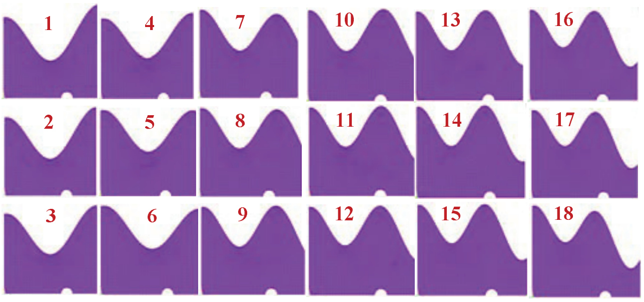
mode	Target frequency (Hz)	FEA frequency (Hz)	Frequency difference (Hz)	Frequency error (%)	Frequency ratio
1	1396.91	1396.77	-0.14	-0.01	1.00
2	2793.82	2793.08	-0.74	-0.03	2.00
3	4190.73	4191.52	0.79	0.02	3.00

(b) Note F7 for No. 1

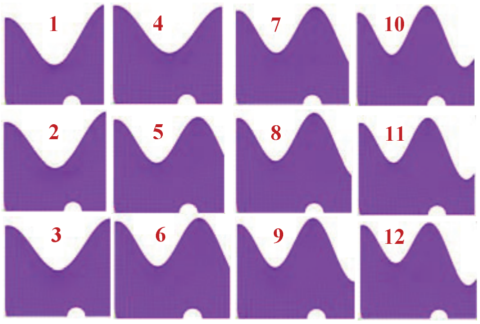
mode	Target frequency (Hz)	FEA frequency (Hz)	Frequency difference (Hz)	Frequency error (%)	Frequency ratio
1	2793.82	2789.82	-4.00	-0.14	1.00
2	5587.64	5583.80	-3.84	-0.07	2.00
3	8381.46	8376.53	-4.93	-0.06	3.00

Table 3: Dimension characteristics of aspect ratios for Notes F6 and F7.

No.	F6		F7	
	L_x	y_{max}	L_x	y_{max}
1	92.55	93.79	65.00	65.58
2	93.00	92.23	66.00	64.66
3	94.00	90.98	67.00	63.31
4	95.00	89.56	68.00	61.51
5	96.00	86.05	69.00	60.85
6	97.00	88.47	70.00	61.26
7	98.00	88.30	71.00	61.69
8	99.00	87.12	72.00	62.30
9	100.00	87.22	73.00	62.64
10	101.00	87.60	74.00	63.47
11	102.00	88.06	75.00	63.22
12	103.00	88.21	76.00	62.90
13	104.00	88.59	-	-
14	105.00	88.99	-	-
15	106.00	89.19	-	-
16	107.00	90.01	-	-
17	108.00	90.34	-	-
18	109.00	93.29	-	-

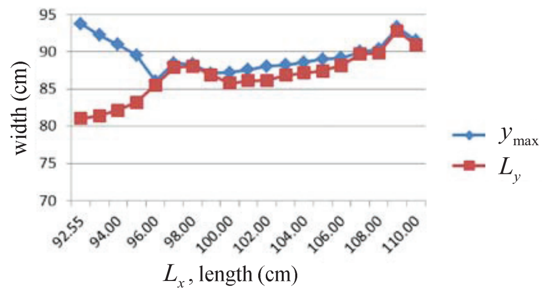


(a) Note F6

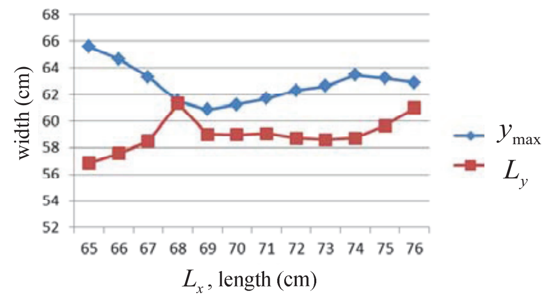


(b) Note F7

Figure 4: Geometry patterns for different lengths of HSP.



(a) Note F6



(b) Note F7

Figure 5: Aspect ratio trend of HSPs for Notes F6 and F7.

Some important findings are summarized as follows. From Figure 4(a) for Note F6, the quarter of geometry pattern of HSPs can be distinguished by the wavelength of sinusoidal wave characteristic. Cases 1 to 6 are about one full wavelength of sine wave, Cases 7-12 are roughly less than 1.5 wavelengths, and Cases 13-18 are larger than 1.5 wavelengths. Similar results can also be observed for Note F7 in Figure 4(b). Cases 1-4 are one full wavelength, Cases 5-8 are less than 1.5 wavelengths, and Cases 9-12 are larger than 1.5 wavelengths.

From Figure 5, the increase of length L_x will result in the decrease of maximum width $y_{\max}$, but the maximum width becomes increasing with the increase of length over some limit of the length. For the case of Note F7 as shown in Figure 5(b), the smallest $y_{\max}$ is at the length of 69 cm. This phenomenon indicates that there is the limit of widths to obtain the HSP fulfilling the frequency ratio requirement. For Note F7, the widths of HSP for the thickness of 3mm in the case study can only be in the range between 60.85 cm and 65.58 cm. The similar observations can also be bound for Note F6. This may conclude that different thicknesses of plates should be considered to design the equal width of HSPs for different pitch frequencies of musical notes.

4 Final modification design

According to the aspect ratio study in the previous section, the strategy to redesign the HSP is to test for different thicknesses of plates such that the final design of HSPs can be obtained to meet the requirement of harmonic sound effects.

The natural frequencies of simply-supported rectangle plate can be expressed as follows [3]:

$$f_{mn} = \frac{\omega_{mn}}{2\pi} = \frac{\pi}{2}\left[\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2}\right]\sqrt{\frac{D}{\rho t}}. \quad (1)$$

where L_x, L_y, and t are the length, width and thickness of the rectangle plate, respectively. (m,n) are mode number in the x and y directions, respectively. ρ is the density, and D is the plate rigidity as follows:

$$D = \frac{Et^3}{12(1-\nu)}. \quad (2)$$

where E and ν are Young's modulus and Poisson's ratio, respectively. From above equations, one can derive the relation for the plate natural frequencies of mode (m,n) as follows:

$$f_{mn} \propto \frac{t}{L^2}\sqrt{\frac{E}{\rho}}. \quad (3)$$

For the same musical note, the fundamental frequency of the plate must be the same, i.e. $f_{mn}^{N} = f_{mn}^{O}$. The dimensional scale factor R to transform the original (O) to new (N) thickness for the redesign of geometry shape can be derived:

$$R = \frac{L_N}{L_O} = \sqrt{\frac{t_O}{t_N}}. \quad (4)$$

Equation (4) provides a quick evaluation of different thicknesses design. The fine tune on the structural natural frequencies of HSP can then be analyzed accordingly.

Table 4 shows the dimensions for the final design of metallophone set consisting of two octaves for Notes from F5 to F7. The plate thicknesses are chosen for those available from 1.0 mm to 4.0 mm. The maximum widths of

HSP are in the ranges between 75.0 mm and 81.0 mm. The about same widths of HSP design for different musical notes are achieved and ready for manufacture.

Table 4: Final Design of metallophone set for Notes from F5 to F7 (unit: m).

Note	L_x	y_{max}	L_y	t
F5	0.0730	0.0786	0.0648	0.0010
F♯/G♭	0.0703	0.0773	0.0632	0.0010
G5	0.0675	0.0750	0.0608	0.0010
G♯/A♭	0.0882	0.0800	0.0792	0.0015
A5	0.0855	0.0783	0.0774	0.0015
A♯/B♭	0.0853	0.0804	0.0803	0.0015
B5	0.0793	0.0765	0.0751	0.0015
C6	0.0916	0.0810	0.0800	0.0020
C♯/D♭	0.0885	0.0786	0.0772	0.0020
D6	0.0884	0.0779	0.0763	0.0020
D♯/E♭	0.0882	0.0774	0.0719	0.0020
E6	0.0764	0.0768	0.0672	0.0020
F6	0.0729	0.0782	0.0647	0.0020
F♯/G♭	0.0703	0.0771	0.0630	0.0020
G6	0.0672	0.0747	0.0607	0.0020
G♯/A♭	0.0881	0.0787	0.0771	0.0030
A6	0.0852	0.0770	0.0747	0.0030
A♯/B♭	0.0838	0.0796	0.0794	0.0030
B6	0.0786	0.0764	0.0749	0.0030
C7	0.0920	0.0805	0.0797	0.0040
C♯/D♭	0.0887	0.0780	0.0768	0.0040
D7	0.0908	0.0781	0.0769	0.0040
D♯/E♭	0.0868	0.0766	0.0711	0.0040
E7	0.0761	0.0766	0.0666	0.0040
F7	0.0725	0.0779	0.0642	0.0040

5 Conclusions

This work presents the structural design modification for harmonic sound plates that require equal widths for different musical notes. The motivation and requirement of equal width design of HSPs is addressed for manufacturing commercial products of metallophone sets. The aspect ratio of maximum length and width of HSP is studied to explore the possible shape designs for HSPs. The geometry patterns for different lengths of HSPs are then obtained and shown the reality for some limitation of aspect ratios. Therefore, different plate thicknesses are considered to perform optimum design of HSPs such that a complete set of HSPs to make a metallophone is finalized. The brand new percussion instrument by adopting the HSPs can be ready for manufacture.

Acknowledgments

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