On the Playability of Wolf Note

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In the musical world, a wolf note is an unpleasant warbling note often found on heavier strings of bowed string instruments, especially the cello. Past research suggested that the wolf note, which is an obvious playability issue, intimately relates to the minimum bow force for the playing of a steady note. This paper explores the correlation between the measured minimum bow force of a cello and subjective judgments of its wolf note by players. Acoustical measurements of the minimum bow force were carried out on the tested cello after making controlled mechanical changes. Psychoacoustical tests on the wolf note with experienced musicians were then employed to investigate the variations of ease of playing induced by these changes. The results strongly suggest a direct link between the measurable acoustical parameter and perceptual preferences, which might inform efforts to improve the quality of bowed string instruments in the future.

1 Introduction

When playing a bowed string instrument, the player does not only rely on their sense of hearing, but also monitors the haptic response provided by the instrument at the same time. Therefore, there are two important aspects of discrimination between bowed string instruments: properties related to the acoustical responsiveness are usually described by “sound quality”, which is normally assessed by the subjective impressions of listeners; and “playability”, which correlates to an extent with the haptic feedback based on the mechanical interactions between the string, bow and the fingers of player, and can only be judged by a player.

This paper explores an obvious playability issue, the wolf note, which is closely related to the minimum bow force for the playing of a steady note. The aim is to investigate the link between acoustical measurements of the minimum bow force and perceived judgments on the wolf note from players. To this end, there are three stages necessary: (i) to correlate small constructional changes on the instrument to acoustical measurements of minimum bow force; (ii) to relate the same changes to perceptual effects on the wolf note; and (iii) to evaluate the correlation between the acoustical changes and the player’s perception. Existing understanding of the nature of the wolf note will be presented at Section 2. A theoretical framework for deducing the minimum bow force, and associated experiments, follow in Section 3. Sections 4 and 5 introduce the acoustical experiments on the minimum bow force and corresponding psychoacoustic tests respectively. The analysis of experimental results is then provided in Section 6.

2 Nature of the wolf note

In the musical world, a wolf note is an unpleasant warbling note typically found about F or F# on the heavier strings of a cello. It is described as an impure and beating-like sound and is a common problem experienced by many players. It is less troublesome on the violin, but is found in the bass viol and to a lesser extent on the viola and the double bass. To musicians, it is difficult to produce a steady tone of good quality when playing at the pitch of the wolf note; rather, the tone is inherently unstable and changes with the seasons, string tuning or setup adjustments. Precautions are always taken to control or minimize this particular note through playing techniques or the use of a wolf eliminator. Thus, the wolf note is one of the specific problems related to ‘ease of playing’ or playability.

Although the wolf note is not popular among musicians, this phenomenon attracts the attention of researchers. During the past century there have been several attempted explanations which contribute to our knowledge of the basic characteristics of the wolf tone. White [1] was one of the first to confirm that the wolf tone often occurs when a played note matches the main body resonance of a bowed string instrument. He also argued that the periodic fluctuation in the intensity of the wolf note is the result of ‘beating’ between two adjacent frequencies.

In 1916 Raman [2] applied optical levers to record the motion of both the top plate and string of the instrument, and raised some doubts on White’s conjecture. He suggested as an explanation of the wolf note that the cyclic intensity variation is due to an alternation of types of forced vibration of the bowed string. As the bow excites the sympathetic resonance, this cyclical alternation repeats, giving the beating sound of the wolf tone.

However, Schelleng considered this qualitative explanation questionable. As an electrical engineer and a fine cellist, he was able to investigate the nature of the wolf note by analogy with equivalent coupled electrical resonant circuits [3]. Schelleng pointed out that the cyclic intensity fluctuation at the wolf note is the result of the beating of two equally forced oscillations. Speaking in terms of frequency, the fundamental resonance peak at the wolf note pitch splits into a pair of peaks. Firth and Buchanan [4] extended the work of Schelleng so as to confirm this theory.

In 1975 Benade [5] gave an argument in terms of the nonlinearity of the friction excitation, which is significant during the wolf note. McIntyre and Woodhouse [6] then measured the transverse waveform of the bridge force and corresponding frequency spectrum of the wolf tone by using a piezoelectric transducer. The results verified Raman’s explanation of an alternation of string oscillation regimes, while the beating between the fundamental pairs described by Schelleng in terms of the frequency-domain view was supported, at least qualitatively, by the measured spectrum of string velocity. By incorporating the nonlinear characteristics caused by the frictional excitation, simulations of oscillograms of wolf notes were developed by McIntyre, Schumacher, and Woodhouse [7] in 1983, which further proved Raman’s explanation.

3 Deducing the minimum bow force

3.1 Minimum bow force

To attempt to understand the quantifiable meaning of the wolf note, the minimum bow force is a possible approach. The first systematic study of the maximum and minimum limits on the normal force between the string and the bow was made by Schelleng. The minimum bow force is determined by the condition that the string is no longer able to stick to the bow hairs throughout the time required to sustain Helmholtz motion: a minimum force is needed to cover energy dissipation from the vibrating string to the body as the string's termination moves with the body and bridge vibrations. Of the two force limits for steady bowing, only the minimum bow force is strongly influenced by the vibrational characteristics of the body of bowed string instruments. Therefore, we might expect different instruments or different notes on the same instrument to show alternative values of minimum bow force.

The playing conditions related to the bow force and bowing position at a certain bowing velocity are shown schematically in Figure 1 in a version of the famous Schelleng diagram [8]. The red line marks the upper limit for bow force required by the Helmholtz motion, while the black solid and dashed lines denote possible values of the minimum bow force. The amplitude of the bow force is schematic only. As mentioned previously, the minimum limit would be expected to vary as suggested by the solid and dashed lines for different instruments or different notes on the same instrument.

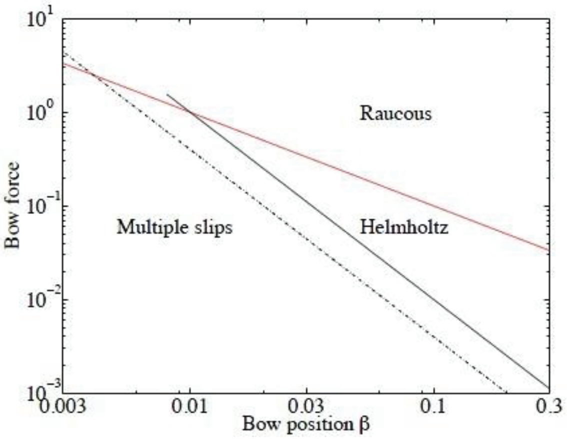


Figure 1: Schelleng's diagram. Two alternative positions of the minimum bow force line are indicated by the solid and dashed black lines.

During playing at the wolf note, the minimum bow force increases when the body motion grows, because the rate of energy loss from the string also grows. The more quickly energy transfers to the instrument body, the higher the minimum bow force value. When the minimum bow force exceeds the real bowing force, a second slip occurs in the middle of the sticking period of the Helmholtz motion. The string then slips twice or more per period. The second slip, which is in the opposite phase to the original one, grows as the original slip gets smaller. Therefore, the energy of the body vibration goes back into the string and the minimum bow force value falls down again. A new Helmholtz motion is built up when the second slip replaces the original one entirely. The minimum bow force varies as this energy alternation between the bowed string and the instrument body repeats, resulting in the annoying wolf note. Taken at face value, the minimum bow force seems a good candidate for being involved in quantitative judgments of wolf note severity.

3.2 Calculation of the minimum bow force

Following the previous discussion, a theoretical framework for deducing the minimum bow force will be given in this section. Schelleng derived the minimum bow force based on Raman's model with three assumptions: the string motion is an idealized Helmholtz motion; the bowing point is very close to the bridge; and the instrument body and bridge behave as a simple mechanical dashpot. In other words, the bowing position as a fraction of the string length β is taken to be sufficiently small, and the assumed dashpot rate R has a constant value.

However, the resulting formula for calculating the minimum force limit cannot be made quantitative because of the uncertainty of the dashpot rate R. To obtain a better approximation to the minimum bow force, Schelleng's calculations were extended by Woodhouse [9] by addressing this problem without the restrictive assumption for R. The equation can be written in terms of the following parameters: f_0 is the natural frequency of the string, β the bowing position as a fraction of the string length, Y_0 the characteristic admittance of the string, Y_1 the measured admittance of velocity at the bridge, v_b the bow velocity, and μ_s and μ_d respectively the coefficients of sticking friction and sliding friction. The condition of the minimum bow force is then expressed as

$$f_{\min} = \frac{2v_b}{\pi^2 \beta^2 {Y_0}^2 (\mu_s - \mu_d)} [\max_t \{ \mathrm{Re} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} Y_1(2n\pi f_0) e^{2n\pi i f_0 t} \} + \mathrm{Re} \sum_{n=1}^{\infty} \frac{Y_1(2n\pi f_0)}{n^2}] \qquad (1).$$

One example for the minimum bow force deduced note by note from a measured admittance function on a cello for bow speed 0.1 m/s and $\beta = 0.1$ is shown in Figure 2. The four curves show minimum bow force values for each of the four cello strings, plotted over a two-octave range from the open note of each string. The yellow vertical lines denote successive octaves C3, C4 and C5, while grey lines mark semitones within the frequency range 65 to 1000 Hz. The peaks in this plot indicate the problematic notes which are likely to cause difficulties for playing on the cello. Particularly, the high peak around note F3 at 173 Hz on the C string curve marks the wolf note on the tested cello.

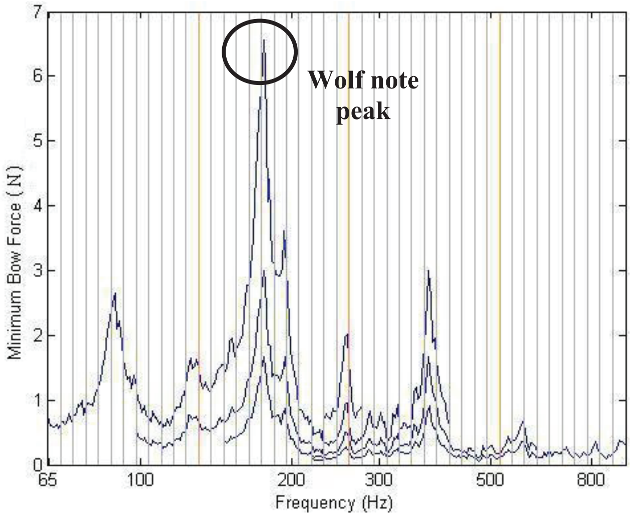


Figure 2: Minimum bow force as a function of played note frequency on the tested cello

There is no doubt that some high peaks can have such implications. Because the wolf note is the most striking manifestation of note-by-note variation from the minimum bow force plot, the following sections will explore the extent to which the wolf note peaks revealed by such plots correlate in detail with playability problems.

4 Acoustic measurements

4.1 Experimental setup

This section concentrates on the change of minimum bow force induced by small physical changes to the tested instrument. In two respects the cello is more suitable for this kind of playability experiment, compared to the violin. First, in the cello the wolf note is generally more severe. Second, it is much easier to make the measurements and to make controlled changes in behaviour because both the size and weight of the cello are considerably larger than those of the violin.

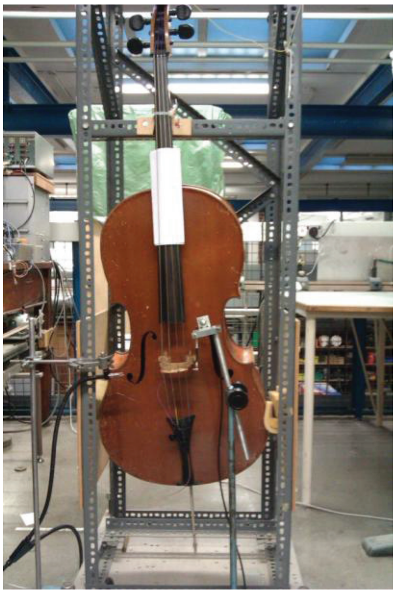


Figure 3: Cello set up for input admittance measurements.

Preliminary playing tests were used to select one cello which is of decent quality and showing a clear wolf tone. In order to derive the minimum bow force plot, we first measure the frequency response function. The acoustical measurements were undertaken on a cello held in a test fixture as seen in Figure 3. To closely mimic the holding method used by players, the tested cello was held by a steel support frame with a firm base and steadied firmly by soft foam pieces from two sides. Its endpin was fixed in one hole of the center metal strip and neck fastened by a cable tie to another strip above. We slightly angled the cello so that both the input force and response velocity were collected from the bowing directions on the nearby strings during admittance measurements.

The common set up for hammer tests was used. An input force was applied to the A string corner of the cello bridge by a miniature instrumented hammer (PCB 086D80) and a laser-Doppler vibrometer (Polytec OFV-056) was adopted to measure the velocity response at the C string corner. The main benefit of using a laser vibrometer here is its ability to conduct non-contact measurement of the response of the cello bridge. Then the transfer function of velocity, which is the input admittance of the cello body, could be obtained by Fourier analysis of the force and output signals. On the basis of the input admittance, the minimum bow force can be calculated using Eq. (1).

4.2 Measurement procedures

In an attempt to relate small physical changes to differences in the minimum bow force plot, two sets of acoustical experiments were carried out: one based on adding three different clips to the cello bridge, and another based on a clip with adjustable moment of inertia. The two experiments concentrated on the C string response because this string is most susceptible to the wolf note around F and F#.

For the first experiment, three different clips were used for making changes to the cello. Figure 4 shows the clips using in this experiment. The weights of clips 1, 2 and 3 are 1.6 g, 4.3 g, and 6.2 g respectively. The third clip is a wooden clothes peg. The measurement setup with clip 3 attached to the bridge is shown in Figure 5.

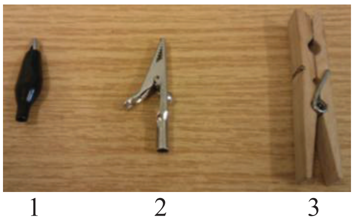


Figure 4: Three different clips used in measurements.

Figure 5: Measurements in progress with the clip 3.

The results of the first set of tests showed that the amplitude of the wolf note peak was sensitive to the moment of inertia of the cello bridge, and it also revealed that finer gradations of variation would be desirable. Consequently, a new clip was designed for making geometrical changes of the cello in the second set of experiments. Figure 6 shows this new clip 4, made from a peg and an adjustable mass which can rotate 360° around the connecting bolt.

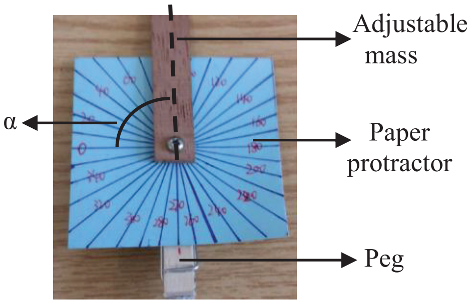


Figure 6: Clip 4 used for measurements

The exact value of the angle α between the center line of the added mass and 0° which is perpendicular to the peg can be read out by means of a paper protractor. The total moment of inertia of the clip and the bridge varies with this angle so that the amplitude of wolf note peak changes. The weight of this clip is the same as that of clip 3. During the measurement, the input admittance was measured with this clip on the bridge, with ten different setting angles from 80° to 225°.

5 Psychoacoustic tests

The player-testing study mainly focuses on the following question: Can the player tell the difference in ease of playing after various small changes are made to the cello? The extent to which change across the experiments can be detected by playing tests was used as an indicator for the playability research.

Two sets of playing experiments were conducted corresponding to the two series of acoustical measurements. Six participants in the experiment have an average playing experience of twenty years. They were instructed to play and assess the cello used by the experimenter in acoustical experiments. Modifications were then made using the four clips described in Section 4. The strings were tuned optimally and other set-up details were adjusted properly prior to the experiment. The experiment took place in an acoustically dry room. Since the bow might influence the playing techniques used by players, all participants were asked to use their own bows.

During the experiment, three test subjects were first allowed to get used to the test cello in the bare state, with no added clip. They were asked to find the wolf tones (or other problematic notes) in the tested cello. These problematic notes and their other comments on difficulties of playing were recorded by experimenters. As soon as participants were familiar with the tested cello, players were required to put on an eye mask as shown in Figure 7 in the second session of the experiment. Participants were not allowed to take off the eye mask during the whole process of this session, so that they had no idea what kind of change had been made on the cello to be tested.

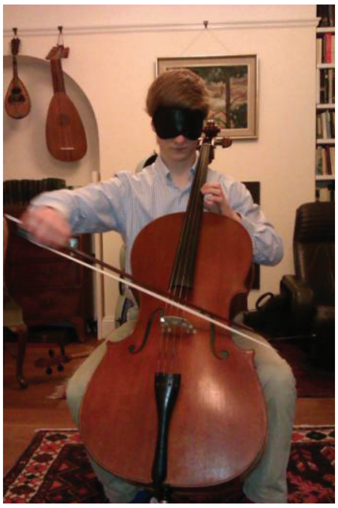


Figure 7: Psychoacoustic tests in progress

In the first set of playing tests, either one of the clips 1, 2 and 3, or no clip, was added on the bridge on every trial. Each time the experimenter made a change, the test subjects were asked to play in the vicinity of the problematic notes they found earlier, and select exactly one answer to the question 'Is the wolf note worse or better?' from the five possible: much better, a little bit better, pretty much the same, slightly worse, much worse. The players were asked to respond as quickly as possible. If they were unsure which case is easier to play, or needed to recalibrate their sense of ease of playing with the bare cello condition, the experimenter moved the clip for comparison. Participants repeated this assessment process after each change. To ensure the reliability of results of this perceptual experiment, there are three trials each for bare cello and each clip, thus twelve trials all in all, carried out in a randomised sequence.

For the second set of psychoacoustic experiments, another three test subjects were asked to provide detailed comments on their perceptions of the tested cello. During the experiment, the clip 4 was kept clamping the bridge tightly. Changes were made by rotating the adjustable mass in the clip to one of ten angles from 80º to 225º on each trial. Before the formal test, the players were allowed to calibrate their responses by being exposed to the easiest case (180º) and hardest case (225º) according to the results of acoustical measurements, so as to give a preferred rating scale. The scale of rating for every player was different because it was chosen according to personal preference. The task for the test subjects was the same as the previous test: to play the wolf note and rate how hard to play the wolf note was, before and after each change. The only difference is that they were asked to give an absolute rating of the severity of the wolf, rather than a relative judgement compared to the previous state. To ensure the reliability of results of this perceptual experiment, each case was tested twice so that there were a total of twenty trials, in randomized order. The consistency of their rating was found to be good, as one would have hoped. The correlation between the players' judgments and acoustical measurements will be discussed in the next section.

6 Results

The results from the first set of playing tests revealed something slightly unexpected: clip 1 and clip 2 made very little perceived difference to the wolf note compared to the bare cello, but clip 3 gave a strong effect of improved ease of playing. This conclusion is compatible with the acoustical measurements of minimum bow force, shown in Fig. 8. Based on the measurements of input admittance of the bare cello without damping the C string, the wolf note peak at about 173 Hz in the minimum bow force plot has average amplitude of 11 N. The height of the wolf note peak in the minimum bow force plot was affected very little by clips 1 and 2, but significantly reduced by clip 3.

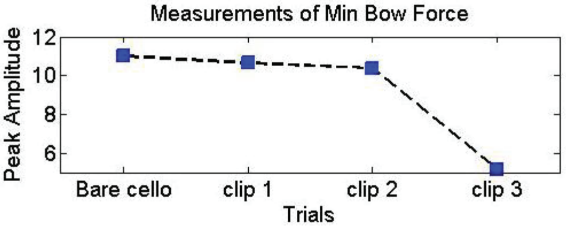


Figure 8: Predicted peak value of minimum bow force for the bare cello and the three clips of experiment 1.

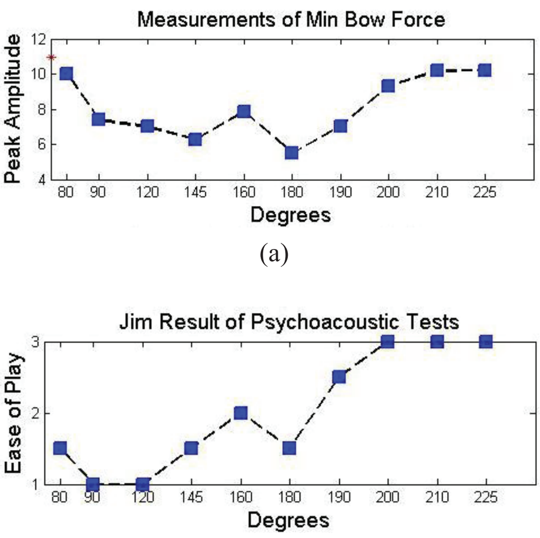
These results gave the motivation for designing the second set of acoustic and playing experiments, with the clip 4. It was obvious that, in order to obtain useful data to test more clearly whether the minimum bow force measurement has predictive power for playability judgments, more controllable gradations in the peak height variation were needed. The adjustable peg depicted in Figure 6 provides this: preliminary experiments were used to refine the design of clip 4 so that the peak height in the minimum bow force plot could be adjusted over a useful range by changing the angle of the movable piece. The amplitudes of the wolf note peaks with ten different values of this angle α are shown in Figure 9 (a).

Figures 9 (b), (c), and (d) give the mean ratings over this same range of angles by three tested players. It is very reassuring to see that measured minimum bow force has reasonably good consistency with subjective judgments from players, at least in terms of some qualitative features of the plots.

The psychoacoustic results were also processed with acoustical results to give values of the correlation coefficient r_{xy} . The calculation of r_{xy} is made as if we have two series of n measurements of X and Y. X denotes the measurements of wolf note peaks, written as x_i , and Y indicates the ratings from players written as y_i where $i = 1, 2, \ldots, n$. Then the sample correlation coefficient r_{xy} is defined as

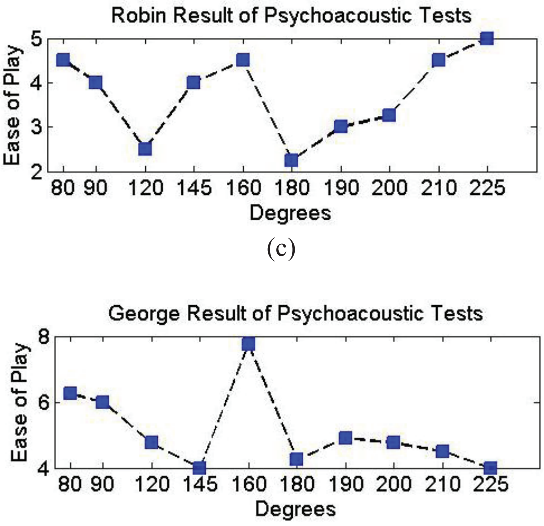
$$r_{xy} = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}} \qquad (2)$$

where $\bar{x}$ and $\bar{y}$ are the mean values of X and Y. The correlation coefficients r_{xy} for the first two participants are 0.6 and 0.6. A lower correlation of 0.3 are obtained for the third one as shown in Figure 9 (d). The lower value in this case arises from the fact that this player took a particular disliking to the behaviour at angle 160°, more so than the other players, and this distorted the numerical pattern of his gradings of the other angles.



(a)

(b)



(c)

(d)

Figure 9: Comparison between perceptual judgments and acoustical measurements

7 Conclusions and discussions

The work described in this paper is a preliminary investigation of playability problems associated with minimum bow force and wolf notes, and the results are very encouraging since they reveal a pattern of behaviour which is at least qualitatively in agreement with playing experience. The variations of wolf note peaks in minimum bow force plots generally agreed with the changes of perceptions on the wolf note, providing a direct link between the acoustical parameter, constructional changes and perceptual preferences. The results might inform efforts to improve the quality of the cello in the future.

Nevertheless, one tested subject in the second set of playing test, who is an experienced luthier and player, tended to show relatively poor performance in correlations. This highlights the fact that playability is a subjective judgement, likely to vary with personal factors such as age, gender, training, playing style and so on. Also, players might ask themselves different questions and assign different priorities while judging the ease of playing of the tested wolf note.

There are many possible directions for future work along the lines presented here. It is very interesting to note the detailed comments made by the test subjects on other difficulties of playing apart from the wolf note on the C string. Terms like ‘more even’ and ‘shifted a little bit’ were used by players to describe variations in the wolf note during playing. These might suggest some subtle changes in the minimum bow force or other acoustical parameters. Also, it would surely be possible to obtain quantitative judgments of preference in respect of other playability issues. Of special interest might be the term ‘range of tone’ used frequently by string players. Moreover, notice from Fig. 1 that the intersection point of the maximum and minimum bow force lines will move with a change of the minimum force limit. At least within the assumptions underlying the Schelleng diagram, this means that there will be variations in how close the bowing point can approach to the bridge. What kind of playability issues might be associated with this variation is still an open research question.

All the preceding results were obtained using only one cello. It is a possibility that this particular instrument has a significant impact on players' preference. In order to test the reliability of these results, acoustic experiments and playing tests should be carried out with other cellos, and perhaps other bowed instruments such as the bass viol.

Finally, it would be useful to investigate the physical mechanism by which that the moment of inertia of the clip added to the bridge can influence the wolf note. Presumably the nodal lines of the mode causing the wolf note are being shifted, in such a way as to alter the strength of coupling to the bowed string. To examine this possibility would require further testing to investigate the motion of the bridge and the associated vibration behaviour of the cello body. Such testing would be a good target for future research, to bring the results closer to the detailed concerns of instrument makers.

Acknowledgments

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