Forced oscillation mode relations of acoustical guitars

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Seven classical guitars were investigated in terms of their forced oscillation patterns driven by all 78 tones played on the instrument on all six strings up to the 12^{th} fret. The forced oscillation patterns are correlated to the first four eigenmodes of the instruments, the Helmholtz, the wood monopole mode (0,0), and the two dipole modes (1,0) and (0,1). Strong correlations between the eigenmodes are consistently found for all guitars, so no orthonormal relation is found between the eigenmodes. For the forced oscillation patterns, the mode shapes are much more consistent, while the phases change dramatically depending on the driving frequency. Also a strong dependency on the string driving point is found, which in some cases is clearly audible. For the same frequency, the forced oscillation patterns driven by different strings even at the eigenmodes of the guitars are also quite different, which is not consistent with the standard theory of forced oscillations. The string dependency is consistent within single strings.

1 Introduction

The acoustics of classical guitars has investigated the instrument in terms of its frequency response [1] [2] [4] [5]. The lower modes of the guitar body are found to have the Helmholtz resonance as its lowest frequency of around 100 Hz, followed by a monopole wood resonance at around 200 Hz, again followed by two dipole modes, the first splitting the top plate into left and right (1,0) around 250 Hz - 350 Hz, the higher splitting the top plate into top and bottom (0,1) between 350 Hz - 450 Hz. The mode frequencies are deviating strongly for the dipole and higher modes, while the Helmholtz is designed by most classical guitar builders to be around 100 Hz and therefore slightly below the open A-string tone. The eigenmodes have been visualized also using laser interferometry [3] [11]. Another methods of visualization is using microphone arrays and acoustic holography [6] or equivalent source methods [8]. These methods justify the associations of mode shapes to certain peaks in the resonance spectrum of the instruments. The guitar body is taken as a complex geometry, where top and back plates couple to the inclosed air, therefore all modes are combination modes.

In this standard view of the guitar, the guitar body is a resonator radiating the frequencies supplied by the strings as the generators. Therefore it is assumed that the impulse response of the guitar body is sufficient to characterize the instrument and the frequencies of the string drive the body modes more or less strong, according to the strength of the eigenmodes in the resonance spectrum. To investigate to which extend this view is correct, a sound method is to take a look at the forced oscillation patters of the guitar driven by the strings themselves. So rather considering the guitar acoustically in terms of driving its body by sinusodials, as done with the interferometry methods, or by knocking on the bridge, measuring its impulse response, in this paper the guitar is played normally, exciting the strings as a guitarist would do. The strings' frequencies then drive the guitar body which is forced to go with the string frequencies and radiates the sound into the surrounding air. Using these forced oscillation patterns in analysis has the advantage, that it contains all information about impedance, driving-point dependency, radiation characteristics and radiation strength, as well as it could give clues about the relation between the geometry of the guitar and its acoustics behaviour.

Using a microphone array technique, the characteristics of single guitars could successfully be associated with these radiation patterns [9]. Also a formula the calculate the Helmholtz for vihuelas with more than one soundhole was found by including the radiation patterns as a weighting function [8]. When forward-propagating these patterns into the surrounding air, complex radiation characteristics can be found, showing evanescent waves, beam-like radiation, or highly complex patterns [10].

The paper is part of a large project building a software for instrument builders to be able to construct guitars in the computer and listen to their sounds before actually building the instruments themselves. Within this project, 32 guitars have been extensively measured. This paper considers a small amount of seven of these guitars to approach the problem and give first insights into the global picture. So e.g. the eigenmodes mentioned above and discussed in the paper extensively did not appear with most of the guitars investigated. Indeed, they were only present with about one third of the guitar sample.

2 Method

Seven guitars were placed in an unechoic chamber in front of a microphone array consisting of 121 microphones recording simultaneously with a sample frequency of 48 kHz each for two seconds. Each tone of the instrument was plucked and its sound was recorded. All notes up to the 12^{th} string were played on all six strings, resulting in a total of 78 recordings of each guitar. From these recordings the forced oscillation patterns were calculated. Additionally, the eigenmodes of the guitar were measured by knocking on the guitar bridge. In all cases the system was considered linear in such a manner as a different driving strength of single frequencies is not expected to change the forced oscillation patterns on the guitar, or of its eigenvalues.

For calculating the forced oscillation patterns from the plucked tones, for each tone all 121 recordings were Fourier analysed, resulting in 121 complex spectra. For each tone the fundamental frequency and the first 20 partial frequencies were calculated using a Wavelet algorithm. As Wavelets are able to zoom into the sound with arbitrarily chosen frequencies, it was possible to calculate the precise frequency of each partial with a precision of 10^{-3} Hz. This precision is not audible, still it ensures very precise phase calculations which are needed when back-propagating the sound onto the guitar top plate. So from each tone for 20 partials, 121 complex amplitudes at the different microphone positions were calculated.

From these complex amplitudes, distributed in space, using a Minimum Energy Method (MEM) [7] the sound-field was back-propagated onto the top plate of the guitar resulting in the forced oscillation patterns.

Due to lack of space in this proceeding paper and as we want to focus on interesting results below, the MEM method is given here only very briefly (for details see [9]

[10]). It is a multipole method, assuming as many poles are microphones are present. The pressure p_m^j at the j-th microphone is assumed to be the sum of the pressures p_g^i at the N radiating points i weighted with a radiation matrix R_0^{ij} connecting the radiation and microphone points like

$$p_m^j = \sum_{i=1}^{N} p_g^i \, R_0^{ij} \tag{1}$$

with radiation matrix

$$R_0^{ij} = \frac{1}{\Gamma_0^{ij}} e^{\imath k r^{ij}} , \tag{2}$$

for a single frequency with wave vector k, distances between radiation and measurement points r^{ij} and a function representing the amplitude drop from radiation to measurement point Γ_0^{ij}

$$\Gamma_0^{ij}(\alpha) = r^{ij}(1 + \alpha(1 - \beta^{ij})) . \tag{3}$$

This is an ill-posed problem, where only small measurement noise of any kind will lead to dramatically wrong solutions immediately. To stabilize this, Γ depends on α, a parameter changing the shape of the monopoles more or less into cardioid shapes. When defining

$$\beta^{ij} = \mid \frac{\mathbf{r}^{ij}}{\| \mathbf{r}^{ij} \|} \cdot \mathbf{n}^i \mid , \tag{4}$$

then, as $\Gamma(r) = 1/r$ for normal radiation direction and $\Gamma(r) <= 1/r$ for all off-normal directions, α shapes the monopole. For $\alpha = 0$ it is a normal monopole, for $\alpha > 0$ the monopole more and more becomes a beam assuming focussed radiation into the normal direction. It can be shown that with such a system, the real surface vibration can be calculated by finding the α for which the reconstruction energy is minimal. When increasing α only slightly above the optimum value, the ill-posed problem in presence of noise is avoided very robust by changing the results only very slightly.

All measurements shown below have been performed with a microphone array of 128 mics in an anechoic chamber at the Institute of Musicology in Hamburg. Each mic was recorded with a sampling frequency of 48 kHz simultaneously to assure a high spatial and temporal resolution capable to analyze musical transients as well. All recordings were done in the near-field to cover evanescent waves.

As all microphone-array methods of back-propagation can only work if all source points are considered, the field of back-propagation was also taking the air around the top plate into consideration, as the radiation from the back plate and sides are scattered around the instrument [6]. The same method was applied for the eigenmodes, taking the sounds from knocking on the top plate and finding the peaks in the spectrum. These peaks were associated with the modes by visualization of their eigenmode shapes.

3 Results

Among the many results derivable from the data, we concentrate on the correlations between the first four eigenmodes and the forced oscillation modes. These eigenmodes are the Helmholtz (HH), the top plate monopole mode (0,0), and two top plate dipole modes of left/right split (1,0) and top/down split (0,1). The HH between the seven guitars used here ranged from 99 Hz to 108 Hz, as expected for normal classical guitars. The monopole mode ranged from 180 Hz to 223 Hz with a mean of 212 Hz. The (1,0) mode had frequencies between 252 Hz and 299 Hz, still there were two guitars around 300 Hz and five guitars around 250/260 Hz. The (0,1) mode ranged from 347 Hz to 440 Hz and so was very wide spread, with many possible values in between.

3.1 Correlations between eigenmodes

First the correlations between the eigenmodes are considered. In Tab. 1 these correlations between the first four eigenmodes are shown, correlating the absolute values (phase correlations discussed below). The values are mean and standard deviation for seven guitars. Strong correlations appear between the Helmholtz (HH) and modes (0,0) as well as (0,1). The reason is mainly the strong radiation from the soundhole present also with the (0,0) and (0,1) modes. Also, the HH mode has some monopole radiation from the top plate, as the energy in the HH is supplied from the string via bridge and top plate to the inclosed air. Therefore, the top plate must vibrate a bit also with the HH mode. The dipole mode (1,0), which is the left/right split on the top plate correlates least with the HH, as expected. Indeed, within the low frequency range there is a monopole and a dipole radiation pattern. Still both correlate strongly, too, the (1,0) dipole mode correlates with the (0,0) monopole with 0.57. As the (1,0) mode is not radiating via the soundhole too much, this correlation is caused by asymmetries of both modes. Also a strong correlation between the two dipole modes (1,0) and (0,1), the up/down split of the top plate with a correlation of 0.57 is present, too. All combinations show low standard deviations, indeed the values do not change considerable according to the top plate geometry. Also the frequencies of the modes do not influence the correlations considerably. Although all guitars have the HH frequency around 100 Hz as usual, the dipole frequency may be between 250 Hz to 300 Hz, as discussed above.

First the correlations between the absolute values were discussed, as the phase correlations differ considerably. When looking at the modes visually, the HH, the monopole and the dipole patterns are clearly present. This is represented by the absolute values. Still the picture changes when the phases are considered. In Tab. 2 the correlations between the modes, only using the phases are shown, again mean and standard deviations for the seven guitars. The mean correlations are very low all through. Still the standard deviations are very high. E.g. the HH / (0,0) monopole correlation is 0.12, while its SD is 0.302, nearly three times higher. Another example is the (0,0) / (0,1) correlation of 0.09 mean with a SD of 0.17.

So although the modes look like regular monopole and dipole modes when examined visually or using the absolute values, still their phase relations may be very complex and deviating strongly between guitars.

	HH	(0,0)	(1,0)	(0,1)
HH	1 (0)	0.82 (0.062)	0.22 (0.082)	0.60 (0.091)
(0,0)		1 (0)	0.56 (0.048)	0.77 (0.083)
(1,0)			1 (0)	0.57 (0.050)
(0,1)				1(0)

Table 1: Mean and (standard deviations) for mode correlation of the **absolute values** between guitar eigenmodes for seven classical guitars.

	HH	(0,0)	(1,0)	(0,1)
HH	1 (0)	0.12 (0.302)	0.06 (0.074)	-0.02 (0.100)
(0,0)		1 (0)	0.10 (0.118)	0.05 (0.170)
(1,0)			1 (0)	0.09 (0.114)
(0,1)				1(0)

Table 2: Mean and (standard deviations) for mode correlation of the **phases** between guitar eigenmodes for seven classical guitars.

3.2 Correlations between eigenmodes and forced oscillation patterns

The same picture as with the correlations of the eigenmodes appear when examining the correlations between the eigenmodes and the forced oscillation patterns. In Fig. 1 the correlations between the (1,0) dipole mode and the forced oscillation patterns of all 78 notes played on the guitar up to the 12^{th} string using the first partial of the tones respectively are plotted sorted by frequency. Again, the correlations between the absolute values only (blue) and the phases only (red) are displayed. As the eigenvalues may be in any phase relation to the forced oscillations, for each forced oscillation pattern separately, the eigenmode shape is circled around 2 π and the highest correlation is used in the plot. This is done for four guitars, the two upper ones have the dipole around 250 Hz, while the two lower ones have the dipole around 300 Hz. The black vertical lines in the figures indicate the position of the eigenmodes, from left to right: HH, (0,0), (1,0), (0,1).

When taking the first guitar, the Admira Cordoba on the top of the plot for the absolute values, the forced oscillation patterns correlation with the dipole eigenmode clearly peak at the position of the dipole. Although this correlation is increasing up to the (1,0) mode frequency, it is not considerably decreasing to higher frequencies. This means that even higher forced patterns have a strong correlation with this dipole. Also the increase starts quite early between the HH and the (0,0) mode. Therefore, the theoretical expectation of the forced patterns having a single resonance at the dipole is not appearing here. It may be present for the lower frequency, it fails for higher ones. The zig-zag behaviour for higher frequencies is discussed below. This peak at (1,0) is present with all guitars, also the increase from lower frequencies to the (1,0) peak. Still the two lower guitars, Ramirez and Hense, have a second peak around

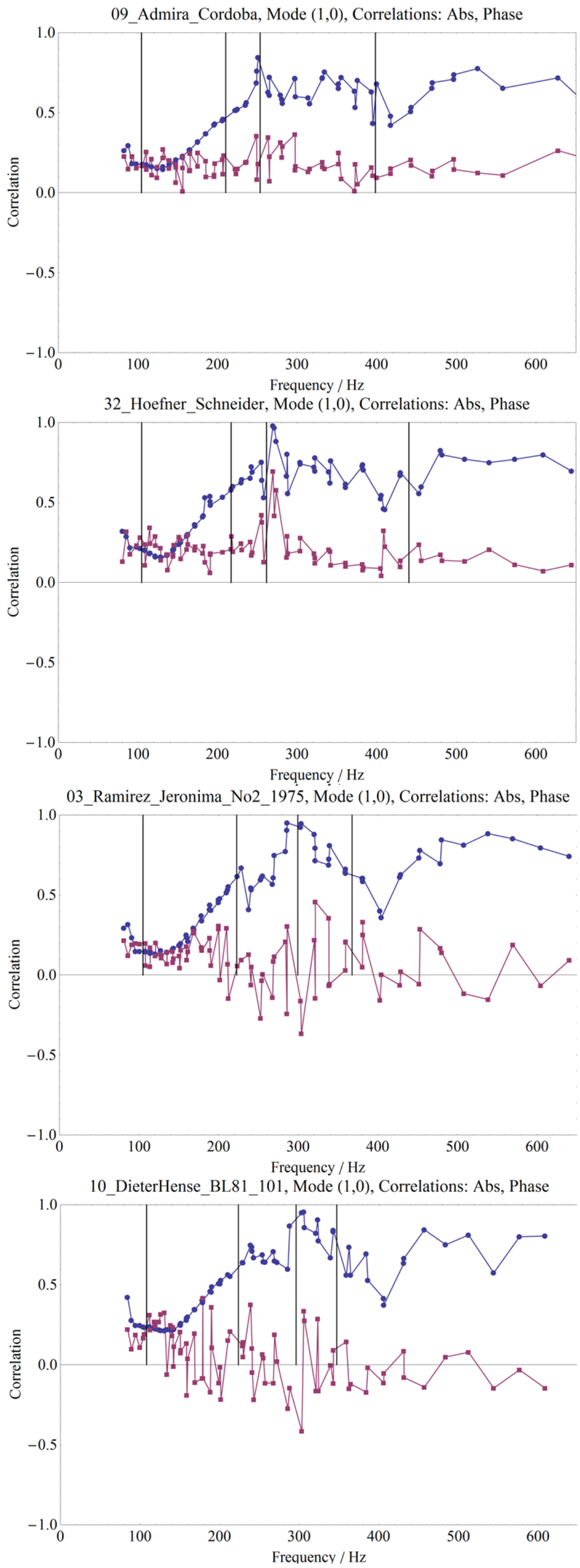

Figure 1: Correlations between eigenmode (1,0) and forced oscillation patterns for four guitars for all notes played on all six strings up to the 12^{12} fret, only first partial of each tone used. The plot shows the correlations between the absolute values only (blue) and the phases only (red). The black vertical lines indicate the position of the eigenmodes, from left to right: HH, (0,0), (1,0), (0,1).

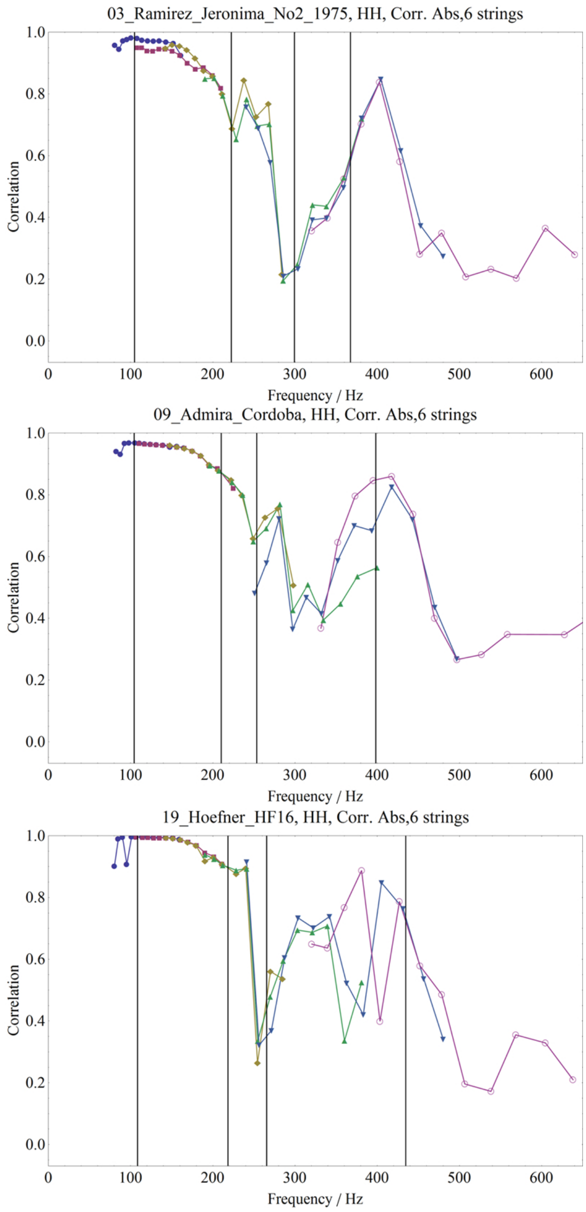

Figure 2: Correlations between the Helmholtz eigenmode and all forced oscillations of played notes, first partial separated by strings, so each line has 13 points, according to 13 notes played from the open string to the 12^{12} fret. The black vertical lines indicate the position of the eigenmodes, from left to right: HH, (0,0), (1,0), (0,1).

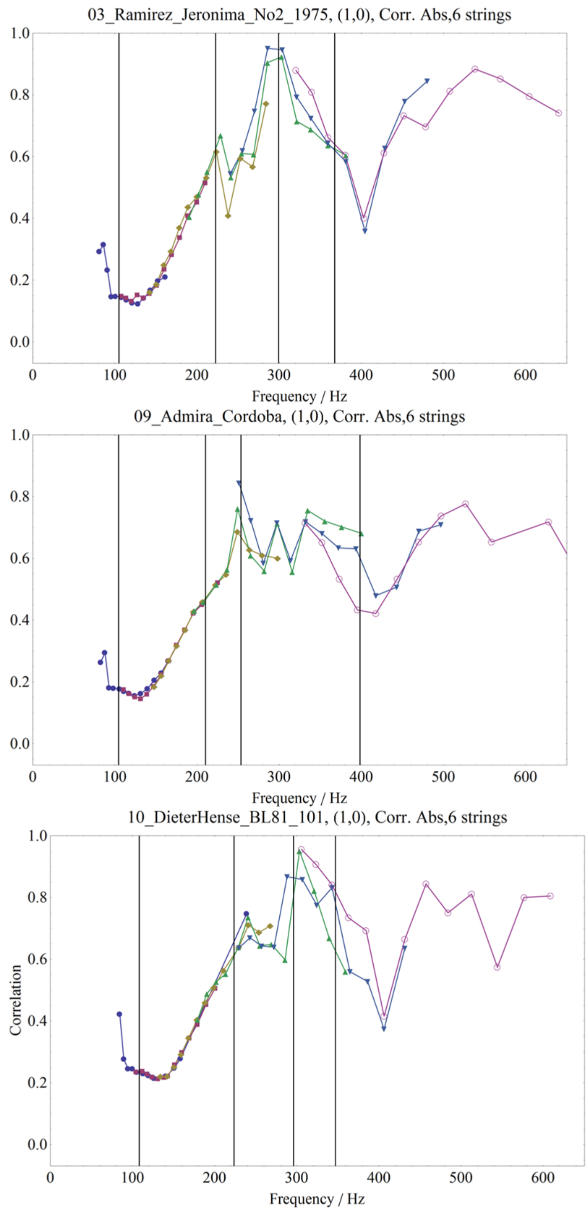

Figure 3: Correlations between the (1,0) dipole eigenmode and all forced oscillations of played notes, first partial separated by strings, so each line has 13 points, according to 13 notes played from the open string to the 12^{12} fret. The black vertical lines indicate the position of the eigenmodes, from left to right: HH, (0,0), (1,0), (0,1).

the (0,0) mode, again showing a correlation between these modes. As discussed above, this may be caused by slightly asymmetrical modes shapes, which are present with nearly all guitars. That this peak appear with the two guitars of a higher (1,0) mode may be caused by the higher frequency of the (0,0) mode with these guitars compared to the the two upper ones with a (1,0) eigenmode frequency around 250 Hz.

Now considering the phases of the forced oscillation patterns, the correlations are again mostly much lower and show very different patterns. Basically, the guitars with (1,0) mode eigenfrequency around 250 Hz have a much smoother phase correlations compared to the ones where the (1,0) frequency is around 300 Hz. The reason for this behaviour is not known. The Hoefner guitar is the only showing a clear peak of the phases at (1,0), too. Still it is interesting to see that this peak is a bit higher than the actual dipole eigenmode. Indeed, both correlations, with the absolute and the phases are considerably low right before the eigenvalue, where the forced oscillation pattern changes tremendously.

3.3 Dependency on driving point

In Fig. 1 above the (1,0) resonance frequency the correlations of the absolute values often show a zig-zag pattern. To examine this closer, the correlations are separated by the six strings. As all notes on the strings up to the 12^{th} fret are present, in Fig. 3 each single line consists of 13 frequency points. Here only the correlations of the absolute values are shown for the sake of clarity. Again three different guitars are taken as examples. Examining the correlations around the (1,0) peak, the different strings show consistent correlation changes within themselves. These correlations often differ considerably between the strings. The audibility of this differences are discussed below. So e.g. the Ramirez guitar in Fig. 3 shows differences above the (1,0) resonance. The mangenta line of the high e-string has higher correlations than the blue line of the b-string, which is again higher in correlation compared to the green line of the g-string. With the Admira guitar in the middle figure this also is very evident for the correlations around the (0,1) mode frequency. Also the Hense guitar shows strong differences between the string correlations above (1,0) consistent within the single strings. It is also interesting to see that the correlations between the strings are perfectly the same for lower frequencies around 100 Hz.

To show another example of the dependency of strings upon the driving point, as well as consistency within strings, the correlations of the forced patterns with the Helmholtz (HH) are shown in Fig. 2. Here it is interesting to see that the Ramirez guitar has a difference even around the HH resonance between the lowest E-string (blue) and the A-string (red). This does not appear with the other two guitars. On the other side, the Ramirez has much higher string consistency all through the frequency range compared to the Hoefner HF 16 guitar at the bottom, where driving point differences are very strong for frequencies above the (1,0) resonance.

Note that the correlations shown in Fig. 3 and Fig. 2 are taken from the absolute values. According to the theory of forced vibrations [12] the shape of a forced oscillation pattern does not change with the driving point, only its maximum amplitude. This is not met with the results from the guitars investigated here and therefore the standard theory of forced oscillations need to be revised, which is beyond the scope of this paper.

3.4 Radiation strength of same frequency on different strings

To estimate if the differences in radiation patterns of the top plate for one common frequency played on different strings are audible to a listener, as an example the note c was used with its fundamental partial at about 250 Hz with the 03 Cordoba guitar. This note can be played on the 10^{th} fret of the D-string, the 5^{th} fret of the g-string, or the 1^{st} fret of the b-string. The Cordoba guitar was used as its dipole eigenfrequency mode (1,0) is at 253.4 Hz and so very close to the forcing frequencies. Again, for these three notes only the first partial was used, so three radiation patterns are compared. The radiation patterns of these strings differ, as can be seen in Fig. 3 for the 09 Admira_Cordoba guitar (middle plot) at the dipole mode (1,0) frequency of about 250 Hz (third vertical line from left).

Radiation strength of guitars is important under several conditions, either the guitar is recorded in the near-field at at distance of about 10 cm in front of the top plate (at several possible positions), or recorded at a larger distance of about 1 m to allow a stronger amount of room reverberation to be on the recording. On the other side, a close audience at about 3 m and a distant one at about 10 m are considered. For all four cases, the pressures of all three modes are calculated on a plane in those distances. The plane width and hight were altered, for the .1 m and 1 m case a 1 m x 1 m plane was used and for the 3 m and 10 m distance a 3 m x 3 m plane was calculated with a grid of 11 x 11 positions in all cases. From these pressure values at the four planes the differences in pressure between the three modes were calculated in dB.

Tab. 3 show the maximum, minimum, and mean pressure differences for the three combinations. The highest value of 12.8 dB is at a distance of 1 m between the d- and the g-string. Here, the mean is 6.1 dB, which is clearly audible. On the other hand, there may be no differences found at all, like for the 3 m distance between all combinations, although a mean is present. As expected, the 10 m distance shows least differences. Surprisingly, the .1 m distance has less differences than the 1 m case, so the near-field patterns of the three radiation patterns are more similar than at the medium distance of 1 m. Overall we find a strong dependency of radiation strength between distance and mode shapes with strong variations. Many differences are audible, even at larger distances. This is a strong indication, that the driving point of forced oscillations at the same frequency may result in strong loudness differences in radiation.

3.5 Conclusions

Many unexpected results appear when examining not only the eigenvalues of guitars but also the forced oscillation patterns.

- The eigenmodes of classical guitars are not orthonormal one to another.
- The basic eigenmode shapes are consistent between guitars.

Relation (dB) max (min) [mean]	d-string (249.0 Hz) vs. g-string (249.4 Hz)	d-string (249.0 Hz) vs. b-string (251.0 Hz)	g-string (249.4 Hz) vs. b-string (251.0 Hz)
close micing, 10 cm	6.5 (1.6) [3.0]	6.8 (0.0) [4.8]	4.3 (0.0) [1.9]
distant micing, 1 m	12.8 (1.8) [6.1]	10.9 (0.0) [3.9]	4.0 (0.7) [2.4]
close audience, 3 m	8.1 (0.0) [2.0]	5.8 (0.0) [2.3]	5.6 (0.0) [2.1]
distant audience, 10 m	2.7 (0.2) [1.4]	3.8 (0.8) [2.2]	4.1 (2.9) [3.6]

Table 3: Comparison of radiation strength in dB, **maximum (minimum) [mean]**, between three forced oscillation patterns at nearly the same frequency around 250 Hz of the note c at three different strings and four distances from the guitar. 121 pressures are calculated at a plane at distances .1 m, 1 m, 3 m, and 10 m in front of the top plate with plane width and hight 1x1 m, 1x1 m, 3x3 m, and 3x3 m respectively. Maximum, minimum and mean are calculated from the 121 values on the plane.

- The phase relations between the eigenmodes are very different between guitars.
- The correlations between the eigenmodes and the forced oscillation patterns peak at the eigenmodes frequencies but have large correlations in other frequency regions, too.
- The forced oscillation patterns differ depending on the driving point (strings).
- The radiation strength of forced patterns of different string driving points for the same frequency may be up to 10 dB or more, even in larger distances.
- The different strings show consistent correlation behaviour within themselves.

Therefore the standard resonance theory cannot be applied to classical guitars. Because of the strong mode correlations mode coupling may be expected within the guitar body. Also the differences in the forced oscillation patterns between strings of the same frequency call for a revision of the forced oscillation theory, which expects the amplitudes of the patterns to differ, but not their basic shape. Overall, the guitar body need to be considered as a much more active part of tone production and not only as a passive resonator.

Acknowledgments

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