inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 7.0

LOCALISATION OF SOURCE - APPLICATION ON A CELLO

C. Langrenne, A. Garcia

Laboratoire d'Acoustique du CNAM, 292 rue Saint Martin, 75141, Paris Cedex 3, France

Email: langrenn@cnam.fr

Keywords:
VIBROACOUSTIC, HELMHOLTZ FORMULATIONS, HOLOGRAPHY, LOCALISATION

ABSTRACT
An isoparametric element formulation is used for implementing the Helmholtz integral formulation associated with arbitrary shaped three dimensional bodies [1]. The inverse problem is solved, *i.e.*, the normal velocity distribution is determined on the surface of the instrument. Because of difficulties to solve the inverse problem (this is an ill-posed problem, *i.e.*, the solution is very sensitive to the errors of measurement), a regularisation method is used to find the most nearest solution of the real one [2]. The algorithm is presented on a cello which is a good example of sound source: one way, his shaped is complex, but it's possible to use accelerometers on his surface and, other way, the holes provide a good test to valid our algorithm.

1 - INTRODUCTION

For an industrial case, it is very important to know the velocity distribution on the structure of a sound source and localise high amplitude of displacements. Then, it is possible to made a treatment to reduce them and, with the velocity, the field around the machine can be calculated anywhere.
Some experimental techniques get directly the velocity on the structure (accelerometer, laser vibrometer). But, when the surface of the structure is complex − non planar, holes, membrane − no contact techniques must be employed.
Intensimetry grew up in the 80's. It is useful to localise high radiation on a machine. But, his application is delicate in low frequency and in near field [3].
Near-field Acoustic Holography (NAH) is used when the shaped of the source is simple − planar, cylindrical − because the treatment in the wave number domain is very fast [4].
To use this technique at arbitrary shaped three dimensional bodies, the Helmholtz formulations need to be solve to know the propagator operator between measurement pressure and structure velocity. An isoparameter element formulation is used for implementing the acoustic direct problem. The structure is discretised on four node quadrangles which allow non-planar mesh grid. This method requires only relatively few elements to achieve an acceptable accuracy in comparison of the collocation method [2].
Once the inverse problem is solved, the acoustic field can be calculated anywhere (near-field, far-field) around the machine. The reactive intensity can be used to localise the high radiation on the structure, while the active intensity show the radiated far-field by the sound source.

2 - INVERSE PROBLEM RESOLUTION: HELMHOLTZ FORMULATIONS

The starting point for the solution method is the classical Helmholtz integral which may be written in the form:

$$\frac{\Omega}{4\pi} p\left(\underline{r}\right) = \oint_{\Gamma} \left[p\left(\underline{s}\right) \frac{\partial G}{\partial n}\left(\underline{r}-\underline{s}\right) - i\rho_0 ckv\left(\underline{s}\right) G\left(\underline{r}-\underline{s}\right)\right] dS \qquad (1)$$

where $p\left(\underline{r}\right)$ is the pressure satisfying the Helmholtz equation $\left(\Delta + k^2\right) p\left(\underline{r}\right) = 0$ in the region V_1 (see fig. 1). The Green function G is:

$$G(r) = \frac{e^{ikr}}{4\pi r} + \frac{e^{ikr'}}{4\pi r'} \tag{2}$$

in where $r = |\underline{r} - \underline{s}|$ is the distance between any point $\underline{s}$ on Γ and $\underline{r}$ may be in V_1, V_2, or on Γ and r' is the distance between the mirror image $\underline{s}$' of $\underline{s}$ and $\underline{r}$. The coefficient Ω has the value 0 for $\underline{r}$ in V_2, 1 for $\underline{r}$ in V_1, and the value 1/2 for $\underline{r}$ on Γ provided there is an unique tangent to Γ at such $\underline{r}$. For $\underline{r}$ at an edge or corner of Γ, Ω has a different value which is a function of the local geometry of Γ at $\underline{r}$.

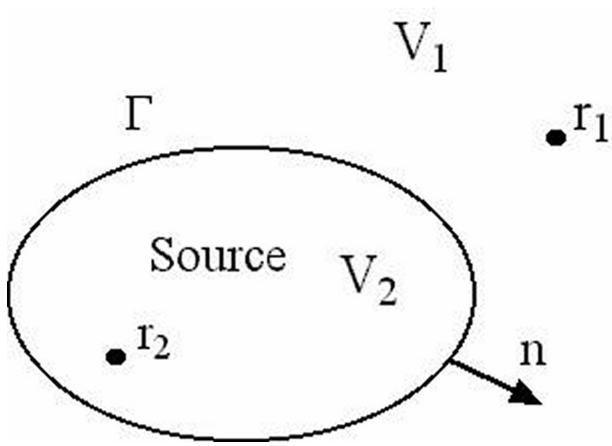

Figure 1: Configuration of the problem.

By the use of an isoparametric element formulation, the problem is discretised and the two following matrix formulations can be obtain [2]:

$$\underline{p_s} = \underline{\underline{S}}.\underline{v_s} \tag{3}$$

$$\underline{p} = \underline{\underline{F}}.\underline{v_s} \tag{4}$$

where $\underline{\underline{S}}$ relies the velocity to the pressure on the structure Γ and $\underline{\underline{F}}$, the velocity on Γ to the pressure in V_1.

The inverse problem that one tries to solve is part of class of ill-posed problems. The evanescent waves that are exponentially decreasing in the radial axis to the source, become inferior to the dynamic range of the measurement system. Small variations on the data induce divergence on the solution.

A Truncated Singular Value Decomposition (TSVD) method is used to get a solution close to the better one. While using the singular value decomposition $(\lambda_i, \underline{u}_i, \underline{v}_i)$ of the $\underline{\underline{F}}$ matrix, we obtain:

$$v_{s,\nu} = \sum_{i=1}^{\nu} \frac{1}{\lambda_i} \langle \underline{p} \, | \underline{u}_i | \, \underline{v}_i \rangle \tag{5}$$

where the first term correspond to the re-amplification of the $\underline{\ v_i}$ modes.

The principle of cross-validation is used to calculate this optimal regularised parameter [5].

3 - INTENSITY

Then, as the velocity is known on the structure, with the help of the formulation (3), the pressure on Γ can be calculated and the complex acoustic intensity is:

$$\underline{\Pi}(\underline{s}) = \frac{1}{2} p_s(\underline{s})\, v_s^*(\underline{s}) \tag{6}$$

where * means the complex conjugate of the velocity value.

The active intensity is written as:

$$\underline{I} = real\left(\underline{\Pi}(\underline{s})\right) \tag{7}$$

and the reactive intensity as:

$$\underline{J} = -imag\left(\underline{\Pi}(\underline{s})\right) \tag{8}$$

where the minus sign is arbitrary taken to give an outgoing direction from the structure.

4 - EXPERIMENTAL RESULTS

The 299 pressure measurements were made on a cylindrical mesh around the cello. This cello is discretised with 224 points. The following figures presents the results at 90 Hz which correspond to the resonant frequency of the holes of the cello. The cello is excited by a shaker placed on his bridge.

On figure 2, the accelerometers cannot be use on the holes, and an uniform vibration can be seen on the structure of the cello.

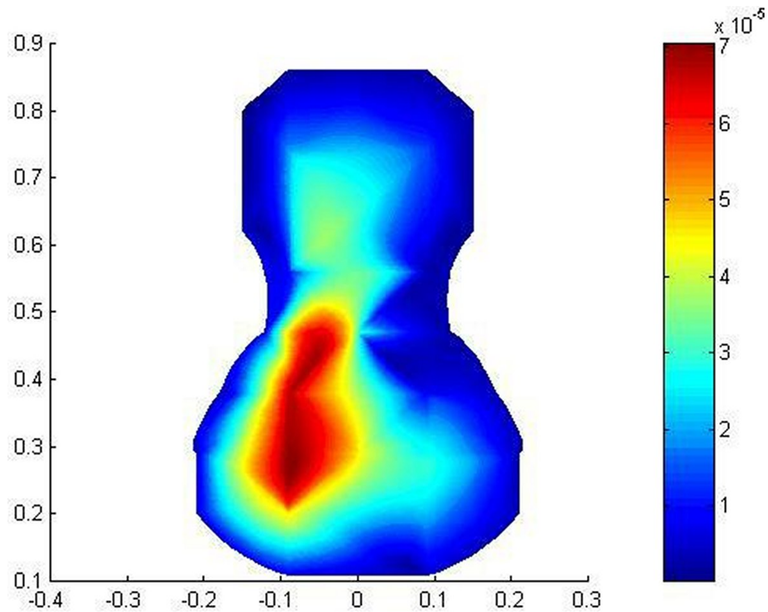

Figure 2: Velocity get from accelerometers (90Hz).

The figure 3 shows the result of the back propagation. The holes of the cello can be easily localised as in the figure 5 which represents the reactive intensity.

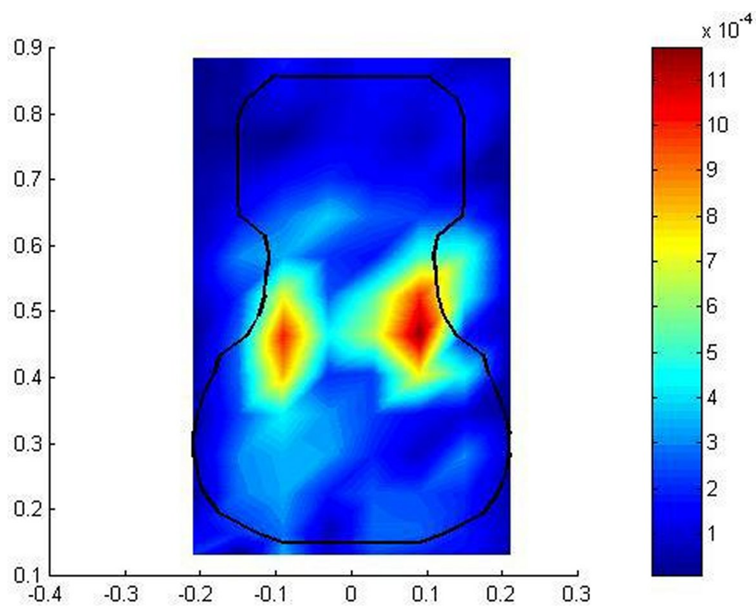

Figure 3: Velocity get from back propagation (90 Hz).

The figure 4 shows the active intensity. The holes cannot be localised, and in far-field, the radiated field seems radiated from a zone between the holes of the cello. But, this is the normal intensity on the structure. If vectors (2D or 3D) are used or stream lines, the active intensity localises only one hole, this one which have the greatest amplitude (figure not presented here).

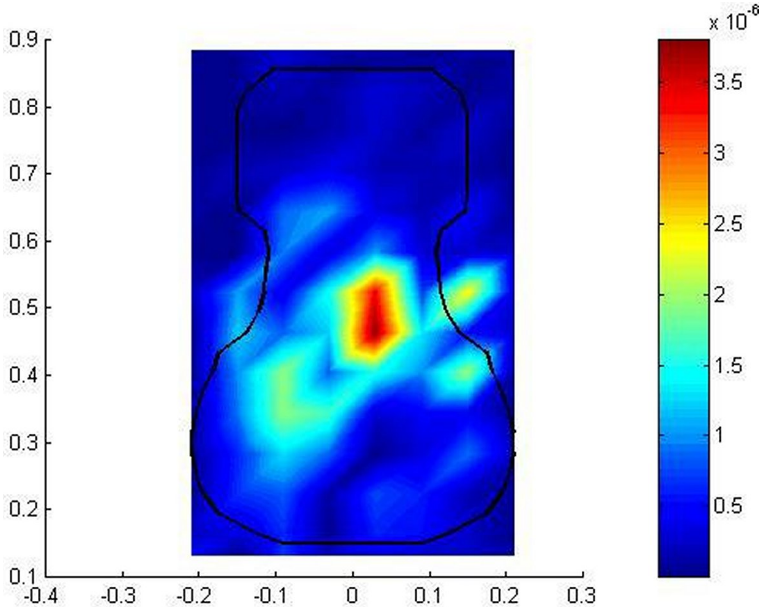

Figure 4: Active intensity (90 Hz).

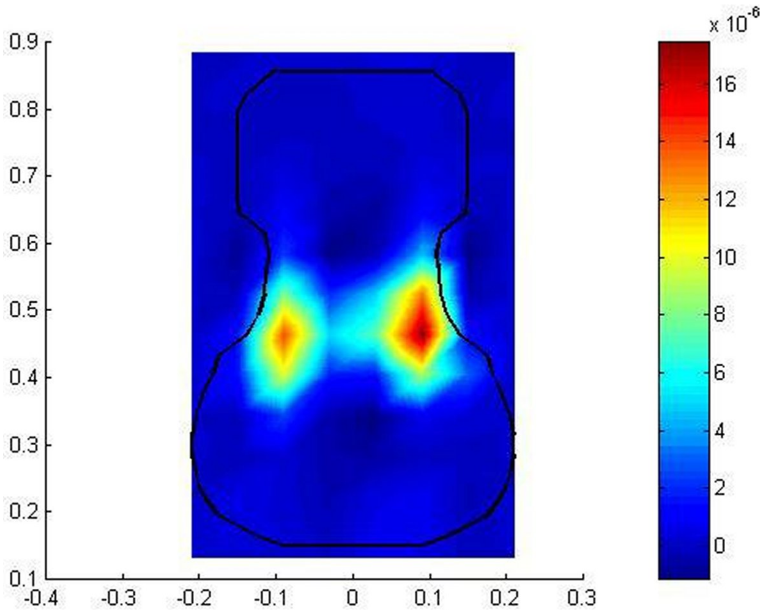

Figure 5: Reactive intensity (90 Hz).

5 - CONCLUSION

The result shows that a no-contact technique to determinate velocity on the structure is a good solution to localised high level of vibration. The vibro-acoustic problem can be solved to know the radiated field around a sound source by the use of the intensity representation. Particularly, the reactive intensity is a good way to localise the surface with high velocity. The active intensity is less accurate and show only the predominant sound source which is responsible of the radiated field on the far field. It's important to note that, in measurements, the normal value of the intensity is insufficient to localise properly the sound source on a structure.

More results will be presented at different frequencies with vectors and stream lines of intensity.

ACKNOWLEDGEMENTS
We thank A. Chaigne, ENST (Paris), for disposal of the cello.

REFERENCES

1. **M. Bonnet**, *Equations intégrales et éléments de frontières*, CNRS ed. / ed. Eyrolles, 1995
2. **C. Langrenne and A. Garcia**, Integral formulations for the vibroacoustic characterisation of a cello, In *Inter-Noise 99*, 1999
3. **F. Fahy**, *Sound Intensity*, Elsevier Applied Science, 1989
4. **J. D. Maynard**, *Encyclopedia of Acoustics*, ed. M.I. Crocker, 1997
5. **C. Langrenne and A. Garcia**, Solving of the inverse problem through a Tikhonov regularisation method for an acoustic source with arbitrarily shaped surfaces, In *Inter-Noise 97*, 1997