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A ONE DIMENSIONAL WAVE MODEL FOR AN AUTOMOTIVE TYRE

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ABSTRACT
A one dimensional wave equation of an infinite flattened tyre belt is generated. The belt vibration is controlled by bending, tension, shear and the sidewall stiffness. The dispersion relations for two waves in the belt are calculated and used to find both the input impedance and attenuation on a tyre belt of infinite extent. Tension and the sidewall controls the deformation and stiffness below 100 Hz. Waves propagate around the belt above this frequency. The wave speeds due to bending and shear were predicted and measured. The model presented here should be valid for the prediction of tyre response above about 400 Hz when for a car tyre the modal behaviour is observed to cease. In this high frequency region the tyre at the input appears to be of infinite extent.

1 - INTRODUCTION
The intention here is to present a wave model of the tyre in which an equation of motion is satisfied by a set of waves. The model is therefore not so restricted by: high frequencies, heavy damping or frequency dependent material properties as previous rigid ring or modal models [1,2]. The tyre belt is represented as a tensioned Timoshenko beam upon an arbitrary sidewall impedance which means that the upper frequency is limited by the resonances across the belt depth. In practical terms this limit would be the first shear resonance of the tread blocks.. The tyre is flattened like a snake skin and is of infinite extent in the direction of the belt. This approach is thought to be most appropriate at high frequencies when the damping inhibits modal behaviour and the sound radiation is most significant from the vibration local to the contact patch. The result is only valid above the ring frequency of the tyre.

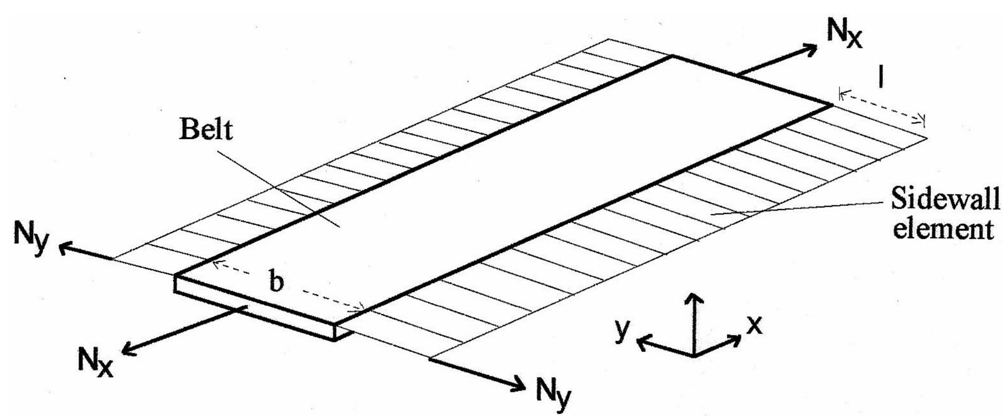

Figure 1: Belt and sidewall model.

2 - THE TYRE MODEL
The tyre dynamics are described by a one dimensional wave equation. The belt is modelled as a Timoshenko beam to accommodate bending, shear and the rotary inertia effects that are significant at high

frequencies. The equations of equilibrium also include the axial tension caused by the air pressure and the in built prestress. The sidewall is represented as a line impedance in the direction of the tyre circumference, indicated in Figure 1 as the y direction. The controlling parameters of belt bending stiffness, mass, tension, shear stiffness, rotary inertia and sidewall impedance are represented in non dimensional form. The dispersion curves relating wavenumbers to the frequency are obtained from the solution to the wave equation. These wavenumbers are substituted back into the equations of equilibrium to give the relative amplitudes of wavetypes and the transfer functions around the belt.
Consider a section of an infinite Timoshenko beam under tension along the x-direction shown in Figure 2.

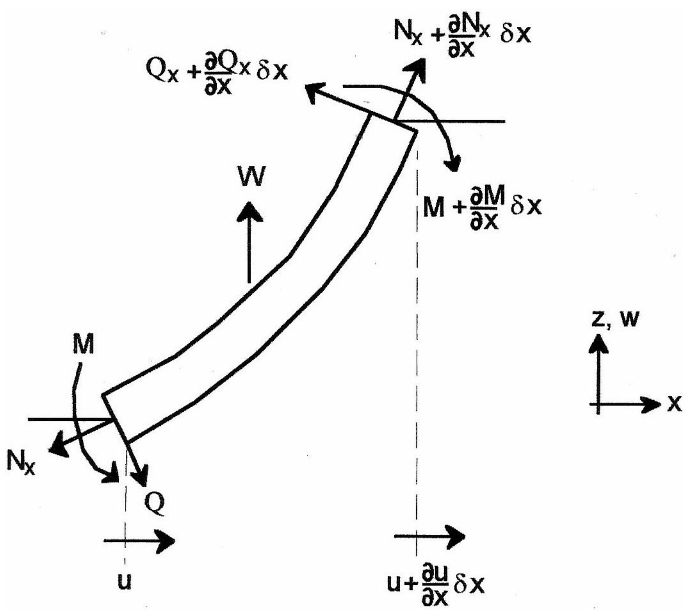

Figure 2: Forces on a section of the belt element in the x-direction.

The tension per width in the beam in the axial direction is denoted by N_x. M and Q are the bending moments and shear forces per unit width acting on the belt section. N_x can be written as

$$N_x = \overline{\sigma}_x h \tag{1}$$

where h is the beam thickness and $\overline{\sigma}_x$ is the mean axial stress across the section. The bending moment M can be found by integrating the axial stress σ_x weighted by z the distance from the section neutral axis, the belt of thickness h is assumed to be symmetric about the neutral axis:

$$M = \int_{-h/2}^{h/2} \sigma_z z dz = -B_x \frac{\partial^2 w}{\partial x^2} \tag{2}$$

where B_x is the bending stiffness/width. The out of plane displacement w is given by the equation of vertical equilibrium

$$\frac{\partial Q}{\partial x} + N_x \frac{\partial^2 w}{\partial x^2} = \mu_x \ddot{w} + \frac{Z_s \dot{w}}{b} \tag{3}$$

where μ_x is the mass/area of the belt. Rotational equilibrium of the bending moments and shear stresses yield

$$Q - \frac{\partial M}{\partial x} = \rho l \ddot{\beta} \tag{4}$$

where ρl_x is the rotary inertia/unit width and β_x is the rotation angle due to bending. The total rotation of the belt element is the sum of the shear angle α_x plus the rotation angle β_x i.e.,

$$\frac{\partial w}{\partial x} = \alpha + \beta \tag{5}$$

The bending moment M is related to the bending angle via equation (2):

$$M = -B_x \frac{\partial \beta}{\partial x} \tag{6}$$

Similarly the shear force Q is related to the shear angle by

$$Q = S_x \alpha_x \tag{7}$$

where S_x is the shear stiffness of the belt/width. For a shear modulus G, S_x=Gh. If the above equations (1-7) are combined and a harmonic solution of the form $e^{i\omega t}$ is applied the fourth order differential equation of motion is obtained:

$$-\frac{\partial^4 w}{\partial x^4}(1+\Phi) + \frac{\partial^2 w}{\partial x^2}\left(\chi - (1+\Phi)k_{cx}^2 - k_{sx}^2\right) + \left(k_{bx}^4 - k_{sx}^2 k_{cx}^2 - \frac{i\omega Z_s}{bB_x}\right) w = 0 \tag{8}$$

Further simplification of equation (8) is achieved using the following normalised parameters to describe the tension contribution:

$$\Phi = \frac{N_x}{S_x} \; X = \frac{N_x}{B_x}$$

and also the wavenumbers for the four possible propagating waves. The associated deformation patterns are given in Figure 3. For the tension wave the wavenumber k_{tx} is defined in equation 9. This is the wave mechanism of a guitar string in which the restoring force for lateral displacements arises from the tension.

$$k_{tx}^2 = \frac{\omega^2 \mu_x}{N_x},\; k_{bx}^4 = \frac{\omega^2 \mu_x}{B_x},\; k_{sx}^2 = \frac{\omega^2 \mu_x}{S_x},\; k_{cx}^2 = \frac{\omega^2 \rho I_x}{B_x} \tag{9}$$

The wavenumber k_{bx} for bending waves is defined in equation 9, the associated deformation pattern is seen in Figure 3a. The wavenumber k_{sx} and deformation pattern of shear waves is shown in equation 9 and Figure 3b respectively.

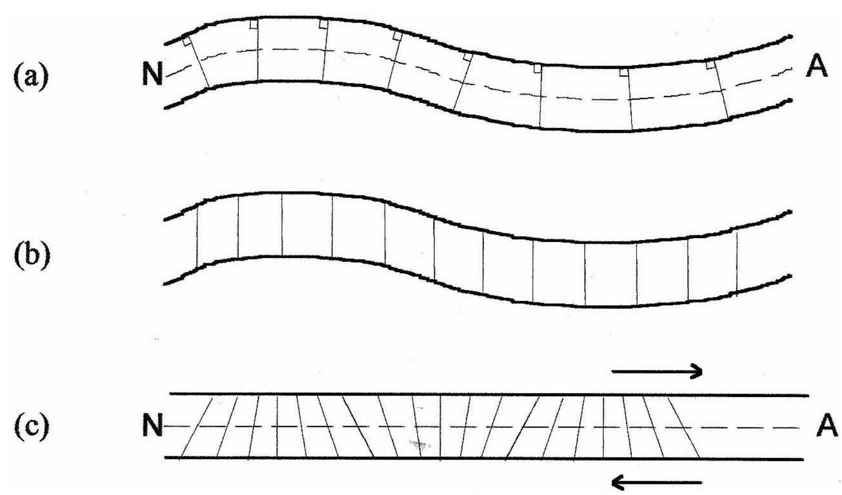

Figure 3: (a) bending waves, (b) shear waves, (c) rotational waves.

Figure 3c displays the deformation pattern for a wave which is described here as 'rotational'; and can be seen to be a degenerate bending wave, as there is stretching and compression above and below the neutral axis without the out of plane displacement. The wavenumber k_{cx} is a function of bending stiffness B_x and second moment of area I_x. The mass of the beam section does not appear directly which indicates that there is no section translation in this wave as with longitudinal, shear, and bending waves. This wave has the possibility of propagating within the tyre contact patch where out of plane motion is constrained, and so it may be associated with dynamic phenomena such as tyre squeal.
Substituting the solution $w = We^{-ikx}$ for equation 8 gives the wavenumber polynomial:

$$k^4(1+\Phi) + k^2\left(\chi - (1+\Phi)k_{cx}^2 - k_{sx}^2\right) + \left(k_{bx}^4 - k_{sx}^2 k_{cx}^2 - \frac{i\omega Z_s}{bB_x}\right) = 0 \tag{10}$$

This is a quadratic in k^2, the solution obtained using a MATLAB program therefore, produces two wavenumber pairs at each frequency. Each wavenumber has a real and imaginary part describing the direction, wavelength and attenuation. The real wavenumbers of a particular belt [3] is shown in Figure 5.

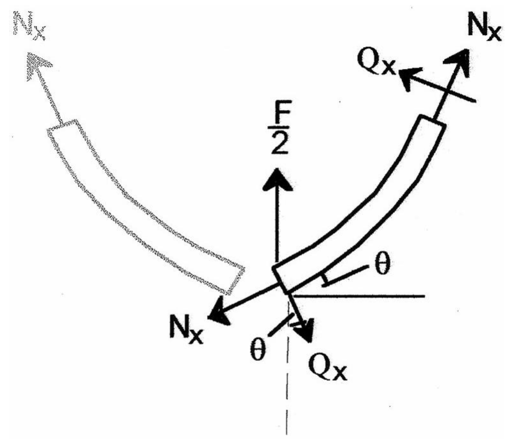

Figure 4: Forces on belt element.

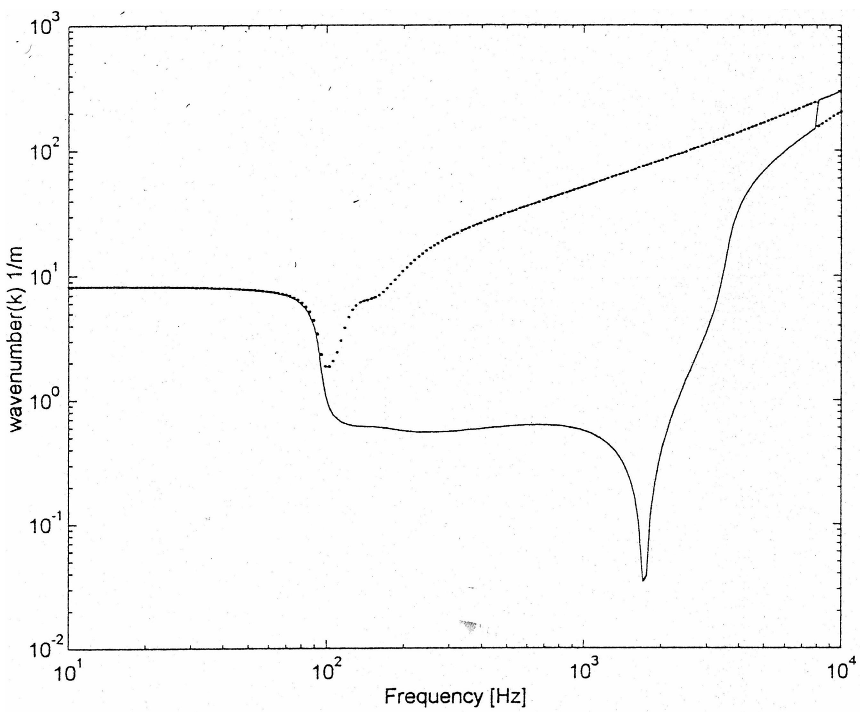

Figure 5: Dispersion curve for a tyre belt on a sidewall, real wavenumbers.

3 - BELT INPUT MOBILITY

If the power input to a tyre belt is required it is necessary to find the input mobility at the input at x=0. This is obtained by applying a unit normal force/width F and calculating the associated velocity. The two wavenumbers are found at each frequency then are substituted back into the equations of equilibrium to give the relative wave amplitudes. The absolute amplitudes depend upon the boundary conditions at $x = 0$. For the case considered here a line force/width $Fe^{i\omega t}$ is applied, and there is a zero slope condition from the symmetry.

By taking a symmetrical section half the normal force on the beam produces a shear force/width, Q_x and a tensile force/width N_x as seen in Figure 4. The angle θ shown in Figure 4 is composed of two parts described in equation (5); the shear angle α_x and the rotation angle β_x. Equation (5) can be written and expanded as

$$\alpha_x = \sum_{p=1,2} \alpha_{px} e^{-ik_p x}, \; \beta_x = \sum_{p=1,2} \beta_{px} e^{-ik_p x} \tag{11}$$

The subscript p denotes a particular wave branch of the dispersion curves corresponding to an axial wavenumber k_p. The shear force causes shear and bending therefore from equations (4), (5) (6) and the boundary conditions yields the out of plane velocity $\dot{w}$ at any point on an infinite belt:

$$\dot{w} = \frac{\omega F}{2\left(S_x + N_x\right)\left(k_1^2 - k_2^2\right)} \left\{ \frac{1}{k_1} \left[\left(k_1^2 - k_{cx}^2\right) + \frac{S_x}{B_x} \right] - \frac{1}{k_2} \left[\left(k_2^2 - k_{cx}^2\right) + \frac{S_x}{B_x} \right] \right\} \tag{12}$$

The input mobility modulus, found by setting $x = 0$, using the wavenumbers of Figure 5 is plotted in Figure 6. Below 100 Hz the sidewall stiffness dominates and there is only deformation local to the contact. At about 100 Hz there is the rigid body belt resonance of the belt upon the sidewall. Above this frequency there is the propagation around the belt of a tension-bending wave until 2 KHz. This wave converts to a shear wave and the rotational wave cuts on around this region. There is very heavy attenuation which would, on a finite tyre, prevent the formation of standing waves above 500 Hz. The tyre behaves as if it is of infinite extent above this frequency.

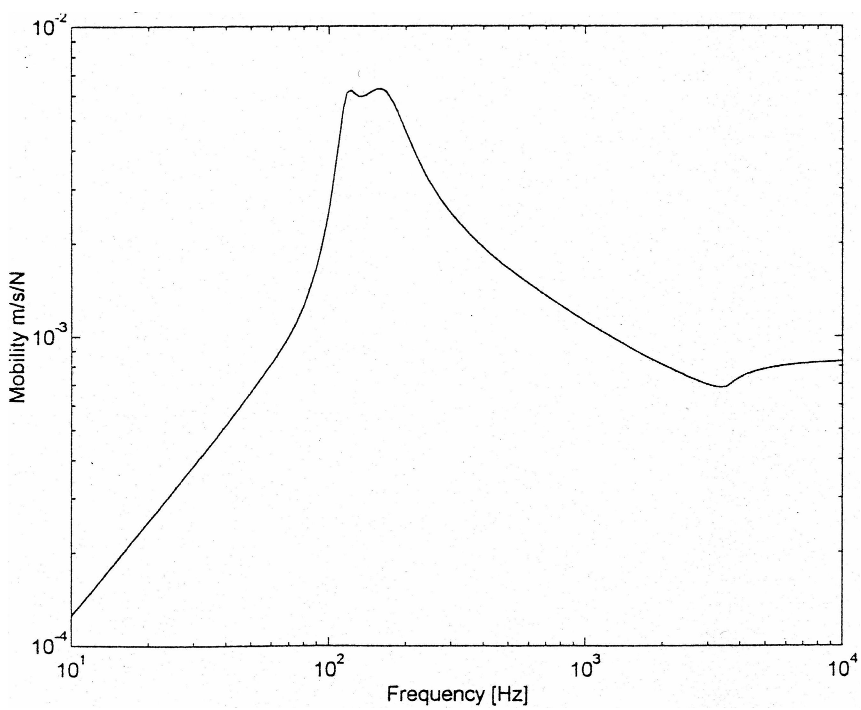

Figure 6: Input mobility modulus for an infinite tyre belt on a sidewall.

4 - CONCLUSIONS

A wave model for a tyre belt was made for one dimensional waves on a flat belt. The model included bending, tension, shear and rotary inertia. Sidewalls can be modelled with some detail, but here the are treated only as a simple stiffness. Resonant behaviour of the belt does not seem significant above 500 Hz when the sound radiation begins to increase, suggesting that this infinite model could be adequate for radiation calculations.

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