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EFFICIENT ACOUSTIC BEM CALCULATIONS ON AXIS-SYMMETRIC BODIES WITH NON-AXIS-SYMMETRIC FIELDS USING ELLIPTIC INTEGRALS AND FFT

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ABSTRACT
A method is shown whereby non-axis-symmetric acoustic fields from axis-symmetric geometries can be calculated very efficiently. The method is a hybrid of the method of Juhl [1], which uses elliptic integrals and the method of Kuijpers et al [2], which uses FFT and is much faster than both. This has been implemented in a Matlab© program and three test cases illustrate that the method works even at high frequencies. The Matlab© source code is freely available from **http://www.dat.dtu.dk/∼openbem/**.

1 - BACKGROUND AND THEORY

The boundary element method is a useful tool for predicting sound fields governed by Helmholtz equation. It can however be quite demanding with respect to computation time and computer memory size. If the geometry of the problem is axis-symmetric, one can take advantage of special axis-symmetric formulations (e.g. Juhl [3]), which only require the generator of the axis-symmetric geometry to be discretised. This requires much less memory and also saves computation time. Non-axis-symmetric fields are handled by expanding the acoustic variables into a Fourier series, e.g.

$$\phi\left(r,z,\theta\right)=\sum_{m}\phi_{m}\left(r,z\right)e^{im\theta} \tag{1}$$

where (r,z, θ) are the usual cylindrical co-ordinates, i is the imaginary unit and m is an integer. **Note that in this paper, expanding into a Fourier series has nothing to do with the time-frequency domain transformation!** It can be shown that if the surface impedance is axis-symmetric the Helmholtz integral equations decouple so that each term can be calculated independently of the others and finally the total solution can be obtained as the sum of the individual solutions.
Among other things, finding the solution requires evaluation of the following two integrals:

$$\begin{gathered}\int_{0}^{2\pi}\cos\left(m\theta\right)\frac{e^{ikR}}{R}d\theta\\ \int_{0}^{2\pi}\cos\left(m\theta\right)\frac{\partial}{\partial n}\left(\frac{e^{ikR}}{R}\right)d\theta\end{gathered} \tag{2}$$

where k is the wave number, R is the distance from the running integration point to the collocation point and n is the unit vector normal to the surface. It is the efficient and accurate evaluation of these integrals that are the topic of this paper.
Due to the analytical complexity of these integrals, numerical integration is popular and has been reported to give good results (e.g. Kuijpers et al [2]). In the same paper, the authors show that (by using FFT) the computation time required to evaluate the integrals for all m-values simultaneously is not much larger than the computation time required for just one value of m. Since one very often needs 10 or more m-values, this is a substantial saving. The price (not mentioned in the paper) however is that the boundary

element matrices must be stored for all m-values simultaneously, so the method effectively reduces the computation time by a factor of 10, but increases the memory required by a factor 10. Fortunately, the extra memory space required is easily stored on disk (if necessary) without any significant increase in computation time.

The method of Kuijpers et al [2] is simple, but the authors of the present paper have poor experience with it. This is because both integrals become singular when $R \rightarrow 0$, and numerical integration therefore requires closely spaced integration points near singularities in order to achieve acceptable accuracy. Ideally one would vary the spacing of integration points accordingly, but FFT requires evenly spaced integration points and therefore forces the use of many more points than are strictly necessary. The extra computation time due to the added number of points very easily exceeds the computation time saved by using FFT. Furthermore, the computation time required is very sensitive to practical implementation, but even for optimum implementation it is unlikely that the high frequency examples discussed later in this paper could have been practically calculated with the method of Kuijpers et al [2].

Some years earlier, Soenarko [4] and Juhl [1] simultaneously suggested a different approach: The first integral in Equation (2) can be split up as follows:

$$\int_0^{2\pi} \cos(m\theta) \frac{e^{ikR(\theta)}}{R(\theta)} d\theta = \int_0^{2\pi} \cos(m\theta) \frac{1}{R(\theta)} d\theta + \int_0^{2\pi} \cos(m\theta) \frac{e^{ikR(\theta)} - 1}{R(\theta)} d\theta \qquad (3)$$

The last term will be referred to as the oscillating part because the integrand is non-singular and oscillating w.r.t. $R(\theta)$. It is easily integrated numerically. The other term on the right hand side will be referred to as the singular part because the integrand has a singularity at $R(\theta) = 0$ and is non-oscillating w.r.t $R(\theta)$. This part can be integrated analytically using elliptic integrals. A similar method is used for the second integral in Equation (2).

Although not mentioned in References [1] and [4], the method of Kuijpers et al [2] is very easily (without the drawbacks mentioned above) applied to the oscillating part of the integral resulting in large reductions in computation time. Furthermore, the analytical integration for the singular part is recursive in m, allowing similar savings in computation time if all the m-values are handled simultaneously.

Unfortunately, when implemented computationally, the analytical integration of the singular part is very sensitive to round off errors and the straight forward implementation easily breaks down for double precision arithmetic when m>6. Practical implementation is therefore slightly more complicated than the description in Reference [1]: More examination reveals that it is only when the integrand in the singular part is far from being singular that the analytical integration breaks down for m>6. When the integrand is very close to being singular, analytical evaluation works up to m=23 with double precision arithmetic. The solution has therefore been to use numerical integration when the integrand is far from singular and only use the analytical integration when the integrand is close to being singular. When m>23 however, this too breaks down. The solution method chosen for this case was an empirical formula that was developed to fit for m>20, but it must be stressed that for many practical applications, m<24 suffices. Due to space considerations, details of this method have been omitted here, but can be seen in the Matlab© implementation which is freely available from **http://www.dat.dtu.dk/~openbem/**.

2 - TEST CASES

Three test cases are used. All are scattering of a plane wave by a sphere at ka=40, where a is the radius of the sphere. In all cases the surface is locally reacting with a constant nominal admittance (i.e. normalised wrt. $1/\rho c$); the three values of nominal admittance used are 0, 1 and i. The solution for each test case is obtained in two ways: firstly by calculating the problem as a completely axis-symmetric problem (m=0) and secondly by treating the problem as an axis-symmetric geometry with a non-axis-symmetric incoming field (i.e. the incoming plane wave travels in a direction perpendicular to the axis of rotational symmetry of the geometry). In all cases, 80 quadratic elements are used and 25 randomly placed CHIEF points.

Figures 1, 3 and 5 show the modulus and real and imaginary parts of the solution on the surface of the respective spheres. There is clearly very good agreement between the two calculation methods. This was also found to be the case at lower frequencies. Figures 2, 4 and 6 show the modulus of the Fourier coefficients (see Equation (1)). Naturally these also show very good agreement, although a small error is apparent at certain m-values. This error is most apparent when nominal acoustic admittance is i, which also proved to be the test case that was most sensitive with respect to the above mentioned empirical formula.

3 - CONCLUSIONS

A method for calculating non-axis-symmetric sound fields from axis-symmetric bodies has been presented.

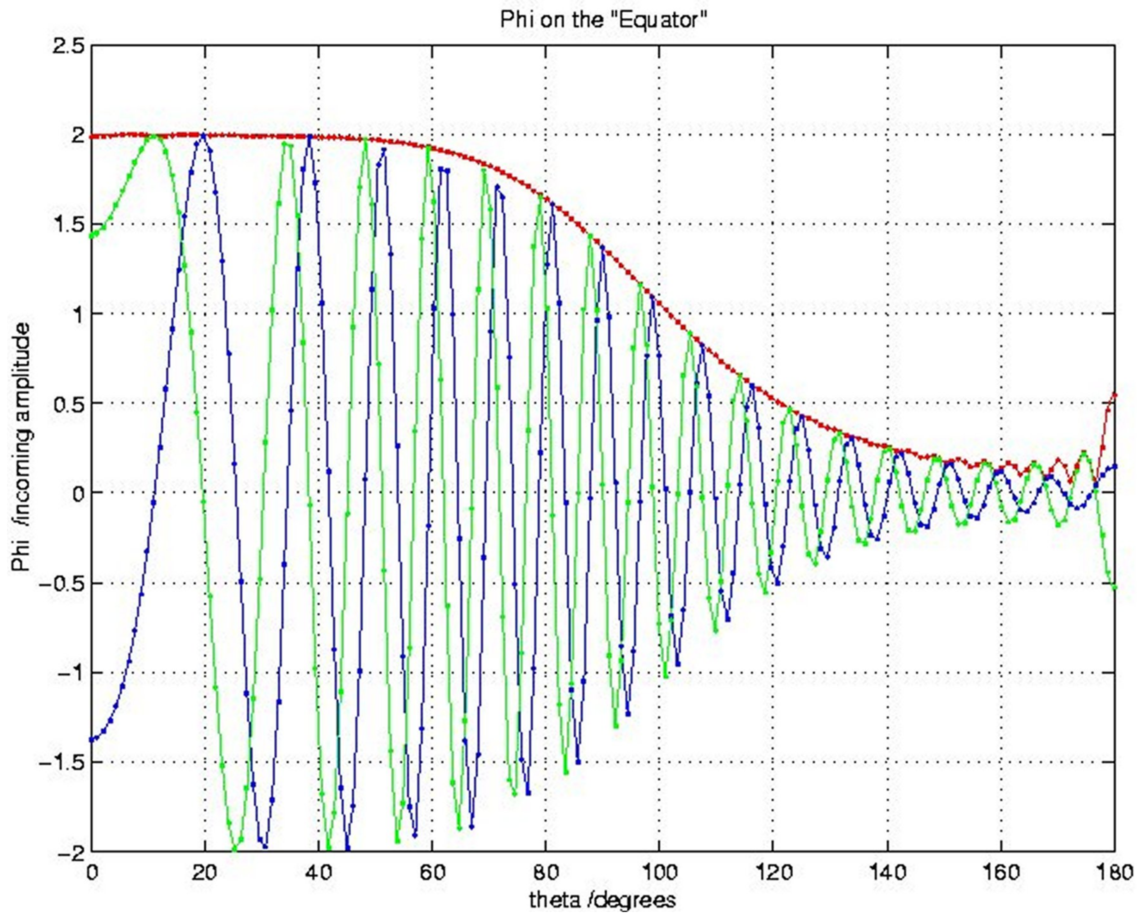

Figure 1: Solution on surface of sphere with nominal admittance 0; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.

The method is a refinement of the method previously published by Juhl [1], using a technique from Kuijpers et al [2] as well as some new ideas to improve performance at high frequencies and high Fourier order. The resulting method is superior with respect to both computation time and practical accuracy. This is illustrated with some examples, where it is found that purely imaginary admittances can be significantly more difficult to handle than other boundary conditions. Never the less, the improved method performs very well also in this case.

ACKNOWLEDGEMENTS
The freely available code OpenBEM was written by a team consisting of the two authors and Vicente Cutanda. This code formed a basis for this work.

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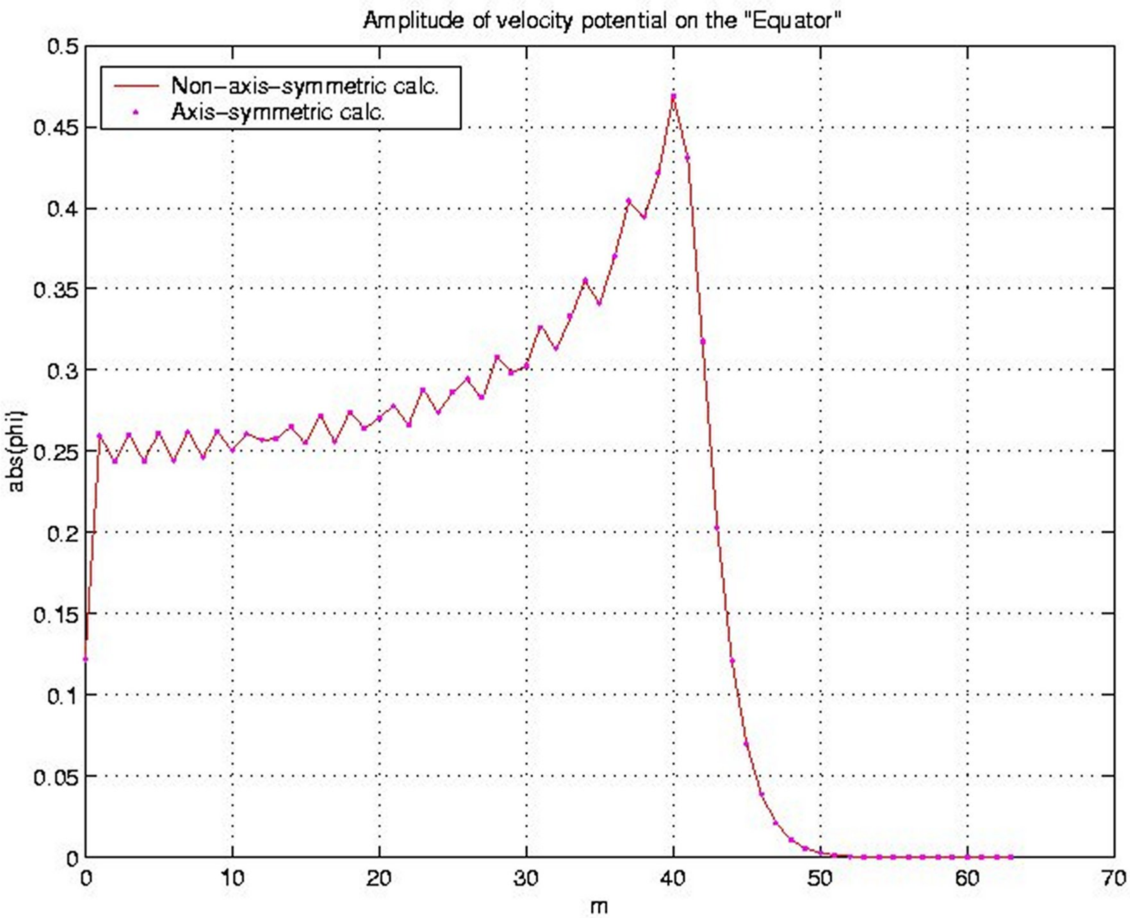

Figure 2: Harmonics of solution from Figure 1; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.

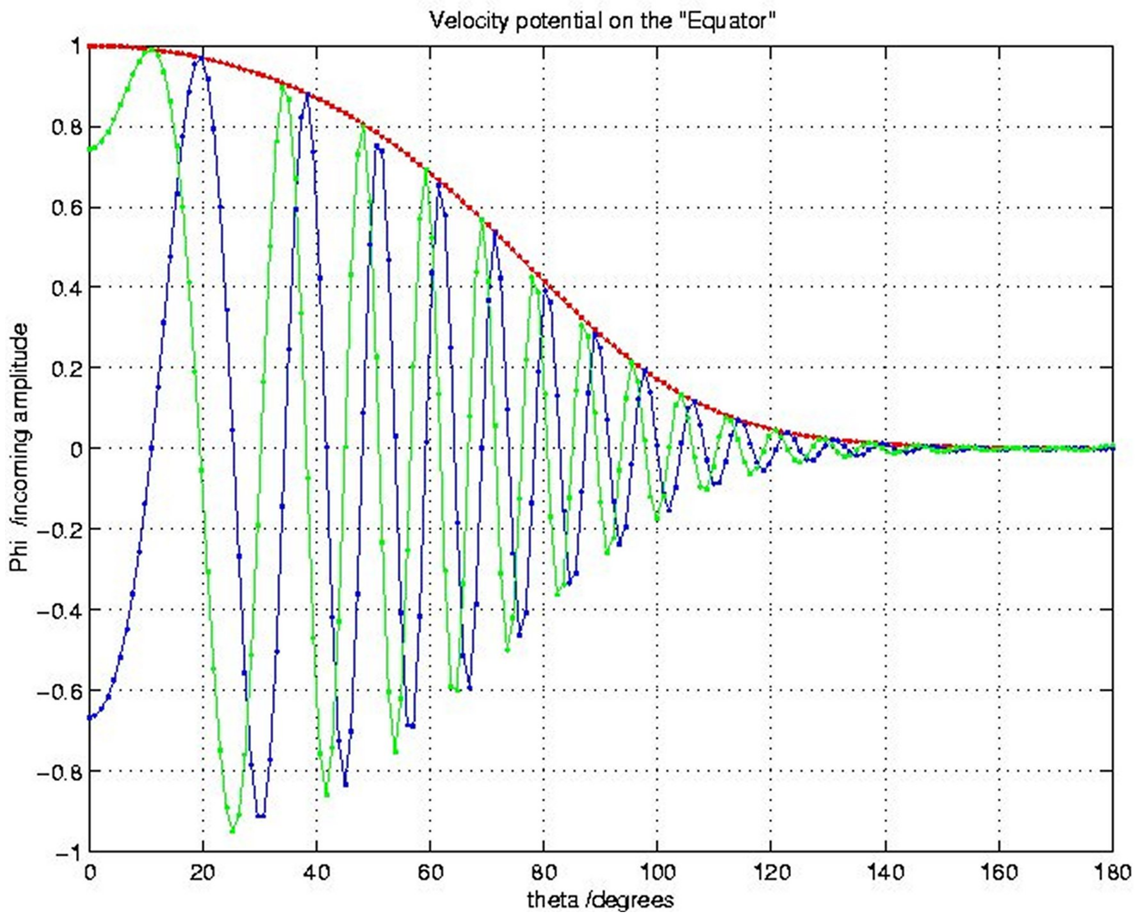

Figure 3: Solution on surface of sphere with nominal admittance 1; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.

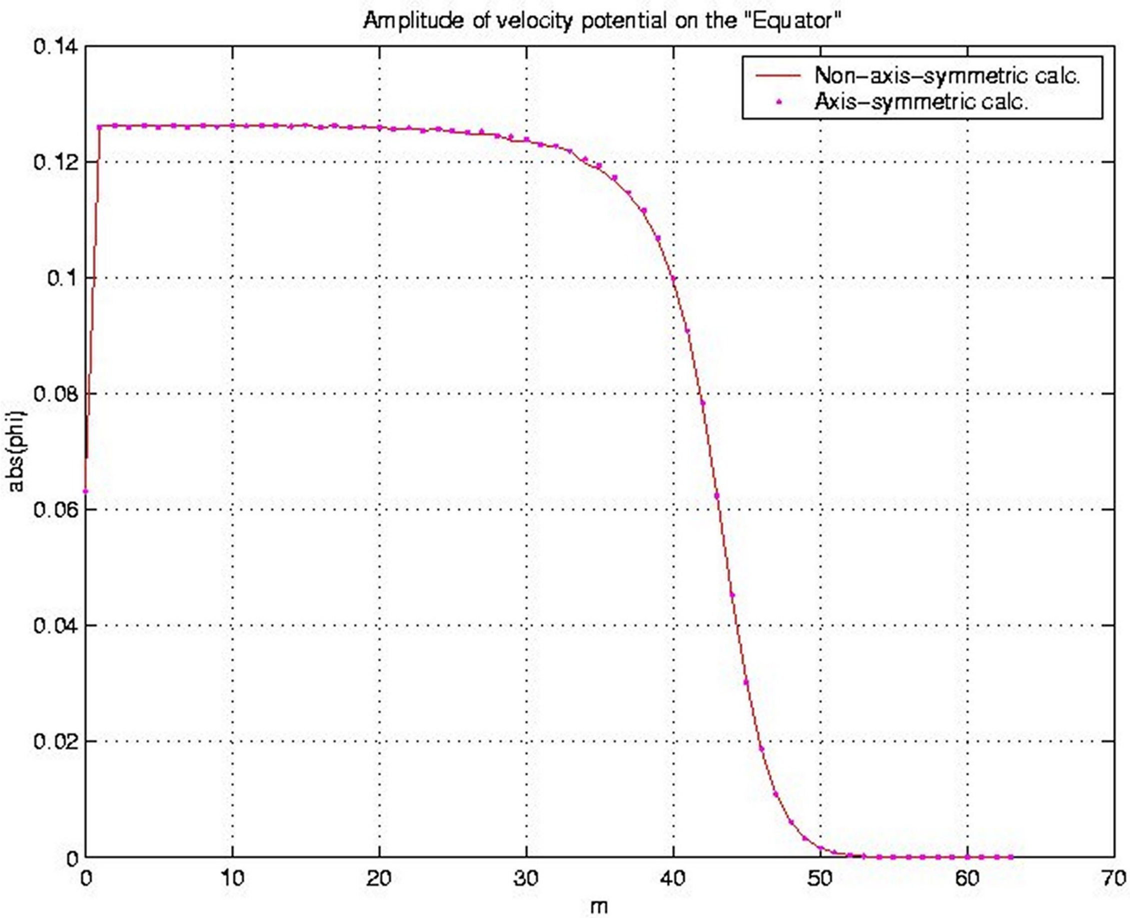

Figure 4: Harmonics of solution from Figure 3; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.

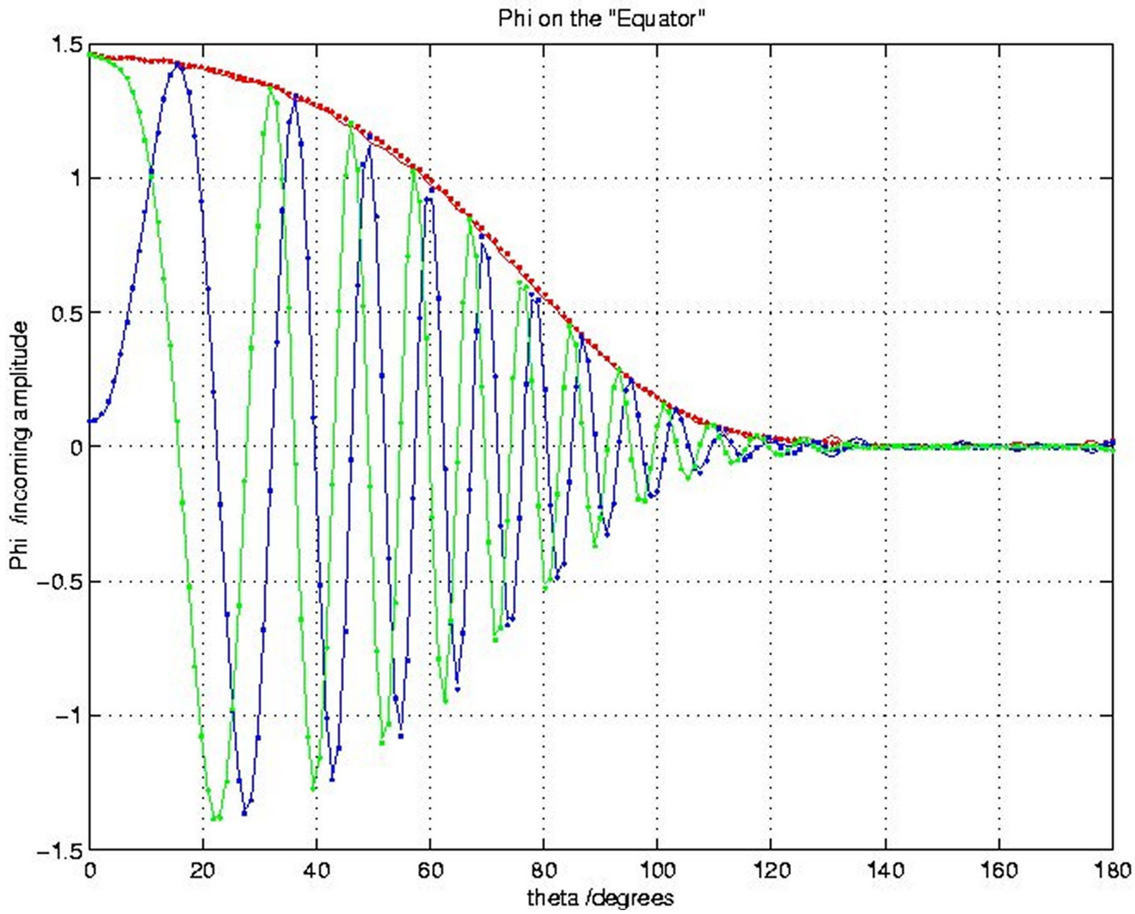

Figure 5: Solution on surface of sphere with nominal admittance i; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.

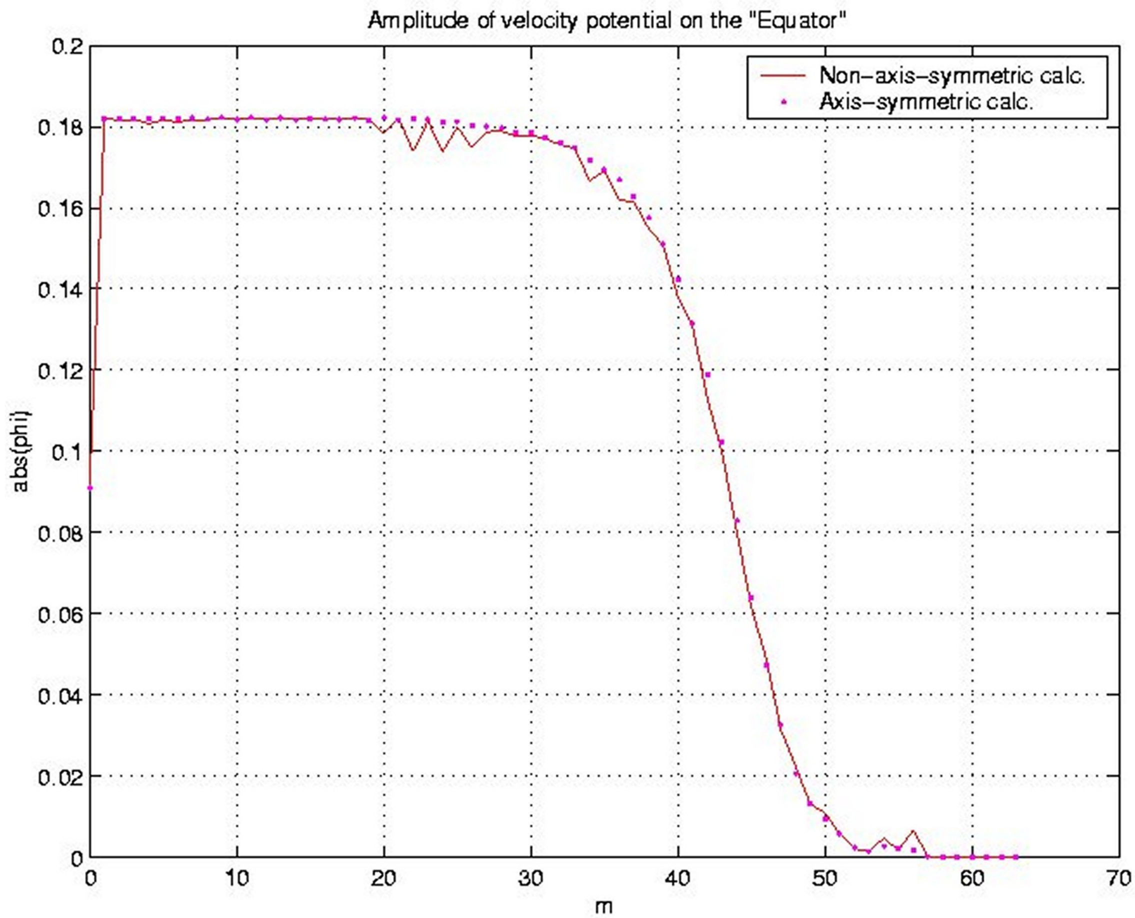

Figure 6: Harmonics of solution from Figure 5; ___ non-axis symmetric solution, ♦♦♦ axis-symmetric solution.