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SENSITIVITY OF AMPLITUDE AND PHASE DIFFERENCES OF NOISE CANCELLATION SYSTEM IN A RECTANGULAR DUCT

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ABSTRACT
A series of preliminary studies using a full-scale model of straight rectangular duct have been conducted in laboratory. Due to inconsistent results found from these experiments, a computer simulation using FEM with 3-dimensional isoparametric elements and twenty nodal points was applied to investigate the influence of amplitude and phase differences on active noise cancellation effects. The working object was an air conditioning duct system with $2.0 \times 0.12 \times 0.12$ m^3 dimensions with anechoic end. The secondary source was located at the middle of the duct with acoustically hard walls or $\partial p/\partial n$. The results revealed that, (1) noise reduction of about 70 dB achieved with an amplitude difference between the noise and the anti noise waves not more than $\pm$ 0.025 dB, (2) a phase shift of not more than $\pm$ 0.25° deviation from 180° produced cancellation effect of about 23 dB.

1 - INTRODUCTION

Since introduced by Paul Leug in 1934, the simple principle of active noise cancellation has been widely discussed and investigated. However, theoretical advantages of this method compared with the passive noise reduction technique still require further research and improvement, especially in its practical or commercial applications [1].

The cancellation principle of Active Noise Control (ANC) technique is mainly based on two physical quantities resulted from superposition of primary and secondary waves, i.e. the amplitude and phase differences between noise and anti noise signals. Theoretically, the secondary source should produce anti noise wave with exactly the same amplitude and 180° phase shift in contrast to the noise wave at every position of interaction. Less cancellation may be achieved when amplitude difference between noise and anti noise waves is not zero, whereas, amplification effect may occur when phase shift between the two waves is not exactly 180°. These problems were found in previous laboratory research using a full-scale model [2], [3] using the early Lueg's model (see Figure 1). Difficulties in finding an exact position or distance ($d = \lambda/2$) between sensor microphone and secondary loudspeaker for achieving 180° phase shift has caused uncertainty in the cancellation results.

This investigation was intended to study the effect of amplitude and phase differences to achieve optimum cancellation using a computer simulation where unpredictable physical occurrences, such as, duct vibration, duct-end reflection, can easily be avoided. A FEM was applied to build up the computer simulation due to some reasons, such as, a complex geometrical domain can be represented by interrelated simple geometrical sub-domains (finite elements); an approximation function (or interpolation function) developed by a linear combination of polynomials, can be derived from each finite element.

2 - ANC IN A RECTANGULAR DUCT

Sound waves propagated in a rectangular duct can be assumed to form plane waves when frequency of sound waves is less than cut-off frequency of the rectangular duct, $f_c = \dfrac{c}{2d}$, where c is sound velocity and d is the largest dimension of the cross sectional duct area. This one-dimensional propagation was also specified to assure zero pressure when a single secondary source was driven.

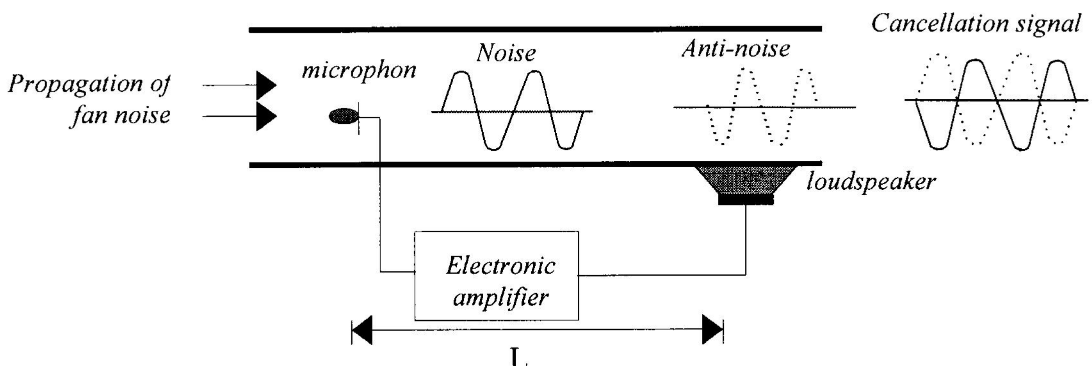

Figure 1: A diagram of simple active noise control.

In general, three-dimensional wave equation can be written as follows,

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{1}$$

In order to simplify the calculation procedure, sound pressure p of the above equation can be modified into the following complex form,

$$p = P\cos(\omega t - \phi) = \mathrm{Re}\left[pe^{-j\omega t}\right] \tag{2}$$

Substitution of equation (2) to (1) yields the following Helmholtz relationship,

$$\nabla^2 p + k^2 p = 0 \tag{3}$$

where $k = \omega/c$ is wave number. This equation enable computation of wave equation using separation of space and time variables as indicated by the following differential equation,

$$D_x\frac{\partial^2\phi}{\partial x^2} + D_y\frac{\partial^2\phi}{\partial y^2} + D_z\frac{\partial^2\phi}{\partial z^2} - G\phi + Q = 0 \tag{4}$$

where, $D_x = D_y = D_z = 1$, $G = -k^2 = -\frac{\omega^2}{c^2}$, $Q = 0$ and $\phi = p$. In this study, solution of equation (4) is calculated using a numerical method (FEM).

3 - NUMERICAL SOLUTION OF SOUND-WAVE EQUATION USING FEM

A numerical technique provides solution values at each discrete point specified for a set of independent variables. Transformation of these independent variables will then give new values for the next discrete points. Therefore, a numerical analyses method can be used to solve physical phenomena of plane wave propagation in a rectangular duct [4]. There are several numerical methods available for different solution purposes, however, in this study, weighted residual using Galerkin method as the weighted function was applied.

Three-dimensional element was used to illustrate a model of volume of static fluid. Thus each element was assumed to have n nodal points and dependent variable ϕ as shown in the following equation,

$$\phi = \sum_{i=1}^{n} N_i\Phi_i \tag{5}$$

where N_i is shape function and F_i is nodal value. In matrix form, equation (5) can be written as follows,

$$\phi = [N]\{\Phi\} \tag{6}$$

with

$$[N] = [N_1, N_2, \ldots, N_n] \text{ and } [\Phi] = \begin{Bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_n \end{Bmatrix}$$

By using this shape function $[N]$ as weighted function, equation (4) can be derived to form the following integral residual equation for one element,

$$\left\{R^{(e)}\right\} = -\int_V [N]^T \left(D_x \frac{\partial^2 \phi}{\partial x^2} + D_y \frac{\partial^2 \phi}{\partial y^2} + D_z \frac{\partial^2 \phi}{\partial z^2} - G\phi + Q\right) dV = 0 \quad (7)$$

Combined $\left\{R^{(e)}\right\}$ of all elements will then provide an equation from which the numerical solution will be calculated. However, $\phi(x, y, z)$ is a discontinue function, therefore, the above equation should be determined in its first derivation, as follows,

$$\{R(e)\} = \{I(e)\} + \{k(e)\}\{\Phi(e)\} - \{f(e)\} \quad (8)$$

where $I^{(e)}$, $k^{(e)}$ and $f^{(e)}$ are interelement requirement, element of stiffness matrix and element of force vector, respectively. Based on physical characteristics of sound propagation in a rectangular duct, a concept of isoparametric with the same degree of approximation between the transformation coordinates and the variables is then considered for deriving the numerical solution.

4 - SIMULATION MODEL AND RESULTS

The working object was a rectangular duct with dimensions of 2 × 0.12 × 0.12 m^3, which has a cut-off frequency 1300 Hz. The primary and the secondary sources were positioned at $x = 0$ and $x = 1$ m from the left end, respectively. Dimension of the secondary loudspeaker of 0.12 × 0.12 m^2 was chosen so that it was smaller than the wavelength of anti-noise signal. Various ANC systems were then investigated and started from the simplest feedback model [5]. Problems found from the first model were then improved in the later models, by implementing additional components and rearranged to achieve optimum cancellation effects. The preliminary investigation elicited a phase difference of only 174.35° instead of 180° and cancellation effects of only 20 − 25 dB from noise amplitude of 70 dB. In addition, upstream propagation from the secondary source was also identified. These problems similar to that were found in the previous research using a full-scale model.

A Dipole-Chelsea model illustrated in Fig. 2 was applied to overcome the problems. The results indicated that a very limited space for positioning the microphone was found. This showed that an optimum cancellation due to amplitude and phase differences between the primary and secondary signals might only be achieved at view positions of microphone between dipole-sources. This sensitivity of amplitude and phase differences can be indicated in Fig. 4. It also pointed out that the secondary sources produced 153.97° phase shift at duct-end areas therefore it should be delayed 224.45° to achieve 0.22 dB amplitude interference or 69.78 dB cancellation (see Fig. 3).

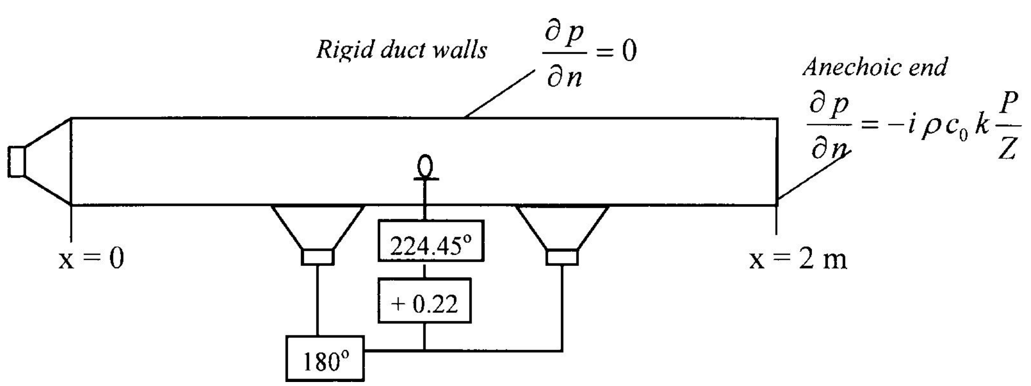

Figure 2: The final simulation diagram using Dipole-Chelsea model.

5 - CONCLUSIONS

FEM with 20 nodal points is sufficiently accurate to represent physical phenomena of an ANC system using Dipole-Chelsea model. Satisfactory results also found for an ANC simulation using LMS Filter (recent study, not reported here). The study indicates that amplitude and phase differences become two sensitive factors, which influence the cancellation results. The study indicates that amplitude differences should be not more than ± 0.025 dB and ± 0.25° shift should be maintained from 180° phase differences between the primary and the secondary sources.

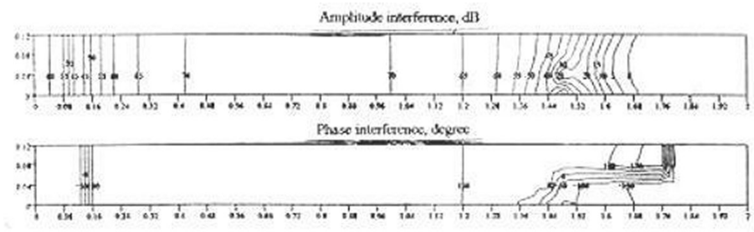

Figure 3: Distribution of amplitude and phase interference along the duct length at 100 Hz.

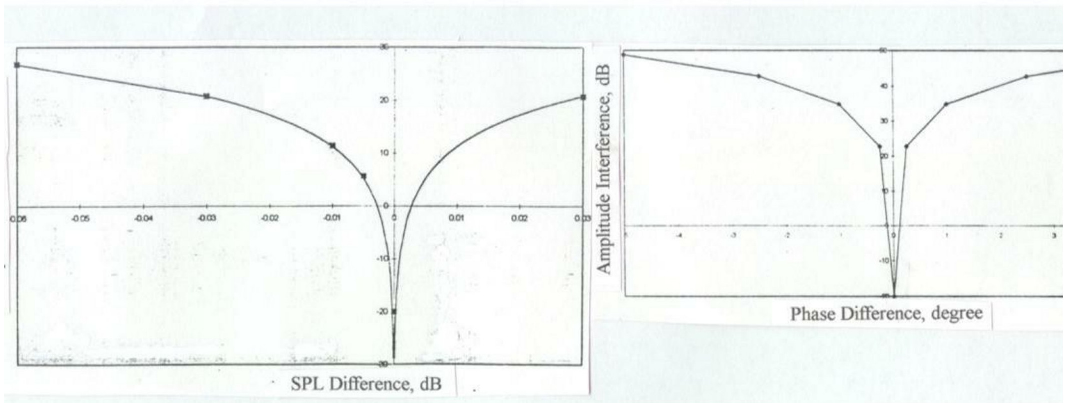

Figure 4: Sensitivity of amplitude and phase differences versus optimum interference level between primary and secondary sources.

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