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# A CELL MODEL FOR THE ACOUSTICAL PROPERTIES OF FIBROUS ABSORBENTS

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**ABSTRACT**
A new two-parameter cell model is developed to predict the acoustical properties of fibrous absorbents with fibres oriented normal to the sound propagation direction. The expressions for dynamic drag parameters are derived for monodisperse as well as for polydisperse arrays. A relationship between complex density and complex compressibility has been obtained. The model gives good agreement with acoustical data if measured values of permeability and porosity are used.

## 1 - INTRODUCTION

Tarnow [1], [2] has used a cell model to calculate acoustical characteristics of fibrous absorbents. The cell model approach has been used frequently to predict acoustical properties of concentrated suspensions. In our previous work [3], the cell model approach was used to predict the acoustical properties of a stack of solid spheres. The Kuwabara cell model [4] was modified to match the cell volume averaged fluid velocity in the direction of flow with the macroscopic flow velocity. To give accurate agreement between cell model predictions, data and numerical results, it was found necessary to use an adjustable cell radius. Here the same approach is used to model air filled fibrous materials.
If the fibre density and heat capacity significantly exceed those of air then the complex density and compressibility depend on the drag term $D(\omega)$ and heat transfer term $S_H(\omega)$ respectively:

$$\rho(\omega) = \rho_0 + \frac{D(\omega)}{-i\omega\phi},\ C(\omega) = \left(\frac{\partial\rho}{\partial p}\right)_T \frac{1}{\rho_0}\left(1 - \rho_0 \frac{\alpha^2}{\left(\frac{\partial\rho}{\partial p}\right)_T}\frac{T_0}{\rho_o c_p}\left(1 + \frac{S_H(\omega)}{-i\omega\phi\rho_0 c_p}\right)^{-1}\right) \quad (1)$$

where $\rho_0$, $T_0$, $\alpha$ and $c_p$ are fluid density, temperature, the volume thermal expansion coefficient and heat capacity at constant pressure, $\phi$ is the porosity.

## 2 - THE CELL MODEL EXPRESSION FOR THE COMPLEX DENSITY

In an oscillatory flow of incompressible viscous fluid with macroscopic fluid velocity $\vec{u_0}e^{-i\omega t}$ normal to a fixed cylindrical fibre, the fluid velocity field around this fibre can be represented by:

$$\vec{v} = \vec{u_o} - |\vec{u_0}|\, curl\vec{a}$$

where $\vec{a}$ is a vector potential.
As has been shown [1], the vector potential has only a $z$-component (in cylindrical coordinates), $\vec{a} = a(r)\sin\Theta\vec{e_z}$. The function $\xi = \partial_r a + \dfrac{a}{r}$ obeys the following equation

$$\partial_r\left[\left(\partial_{rr} + \frac{\partial_r}{r}\right)\xi - \frac{\omega\rho_0}{i\eta}\xi\right] = 0 \quad (2)$$

The boundary conditions for $\xi$ are derived from non-slip boundary conditions at the fibre surface and a condition of zero vorticity on the outer cell boundary:

$$\xi|_R = 2,\ \ \partial_r \xi|_b = 0 \tag{3}$$

Also boundary conditions for potential $a(r)$ are used:

$$\left.\frac{a}{r}\right|_R = 1,\ \ \partial_r a|_R = 1 \tag{4}$$

An additional boundary condition for the potential function $a(r)$ on the outer cell boundary,

$$\left.\frac{a}{r}\right|_b = \frac{R^2}{b^2} = \Psi \tag{5}$$

results from matching the macroscopic fluid velocity and cell volume averaged fluid velocity:

$$\langle u_x \rangle = \frac{1}{\pi\left(b^2 - R^2\right)} \int_0^{2\pi} \int_R^b \left(u_r \cos\Theta - u_\Theta \sin\Theta\right) r dr d\Theta = u_0 \left[1 - \frac{a|_b\, b - R^2}{b^2 - R^2}\right] = u_0$$

In our cell model for a stack of spheres, the parameter $\Psi$ was considered to be adjustable since it accounted for the effect of contacts between adjacent particles. In fibrous materials with high values of porosity, this effect should not be important. Consequently, in the following comparisons, it is assumed that $\Psi = 1 - \phi$. However, we do not exclude the possibility that it might be an adjustable parameter for more dense arrays of cylinders. For this reason, the parameter $\Psi$ appears explicitly in the analytical results.

The drag term, $D$, is calculated as a sum of viscous forces acting on individual fibres:

$$D = \frac{FN}{u_0} = \frac{1-\phi}{\pi R u_0} \int_0^{2\pi} \left(-\ p|_R \cos\Theta - \sigma_{r\Theta} \sin\Theta\right) d\Theta = -\left(1-\phi\right)\eta \left[\left.\frac{\partial_r \xi}{r}\right|_R - \partial_{rr}\xi|_R + \frac{\omega\rho_0}{2i\eta}\right]$$

where $p$ is the pressure and $\sigma_{r\Theta}$ is the component of the stress tensor.

Solving equation (2) with boundary conditions (3)-(5) we get:

$$D\left(\omega\right) = -i\omega\rho_0 \left(1-\phi\right) \frac{\left(2\Psi - 4\right) Z_1\left(kR\right) + \left(1-\Psi\right) Z_0\left(kR\right)}{2\Psi Z_1\left(kR\right) + \left(1-\Psi\right) Z_0\left(kR\right)\left(kR\right)} \tag{6}$$

where

$$Z_{0,1}\left(kR\right) = i \left[J_{0,1}\left(kR\right) Y_1\left(\frac{kR}{\sqrt{\Psi}}\right) - Y_{0,1}\left(kR\right) J_1\left(\frac{kR}{\sqrt{\Psi}}\right)\right],\ \ k = \sqrt{\frac{i\omega\rho_0}{\eta}}$$

$J_{0,1}$ and $Y_{0,1}$ are Bessel functions.

## 3 - CELL MODEL RESULTS FOR DYNAMIC DRAG PARAMETERS

The drag parameters i.e. dc permeability $k_0$, characteristic viscous dimension $\Lambda$ and tortuosity $\alpha$ can be calculated from low- and high- frequency limits of the complex density [5]:

$$\lim_{\omega\to 0} \frac{\rho\left(\omega\right)}{\rho_0} = \frac{\phi}{k_0} \frac{i\eta}{\omega\rho_0}, \qquad \lim_{\omega\to\infty} \frac{\rho\left(\omega\right)}{\rho_0} = \alpha_\infty + \left(\frac{i\eta}{\omega\rho_0}\right)^{1/2} \frac{2\alpha_\infty}{\Lambda}$$

The low frequency limit of complex density gives:

$$k_0 = \frac{\phi^3\left(-3 + 4\Psi - \Psi^2 - 2\ln\Psi\right)}{16\left(1-\phi\right)\left(1-\Psi\right)} R^2 \tag{7}$$

This expression coincides with that obtained by Tarnow [1] (note that $k_{Tarnow} = k_o/\phi$). Using Hankel's asymptotic expansions for Bessel functions of large arguments, it is possible to extract expressions for tortuosity and characteristic viscous dimension from the high frequency limit of complex density. Hence

$$\alpha_\infty = \frac{1}{\phi},\ \ \Lambda = \frac{1-\Psi}{2\left(1-\phi\right)} R \tag{8}$$

As shown in Fig. 1, the agreement between numerical results for a regular array [6] and our cell model predictions for dc permeability is good over a wide range of porosity. However, as has been shown [1], the predicted values of permeability are significantly lower than measured values in glass wool.

The new cell model results for drag parameters can be extended to materials with non-identical fibres. If a log-normal distribution function $p(r)$ for fibre radius is adopted, the drag parameters of the polydisperse array $k_0^p$, $\Lambda^p$ and $\alpha_\infty^p$ are connected with corresponding characteristics of the monodisperse array in the following way:

$$k_0^p = \frac{\int_0^\infty r^2 p\,(r)\,dr}{\int_0^\infty \frac{1}{k_0} r^2 p\,(r)\,dr} = k_0 e^{d^2}, \qquad \Lambda^p = \frac{\int_0^\infty r^2 p\,(r)\,dr}{\int_0^\infty \frac{r^2}{\Lambda} p\,(r)\,dr} = \Lambda e^{-d^2}, \qquad \alpha_\infty^p = \alpha_\infty \tag{9}$$

where $\sigma$ is standard deviation.
The dc permeability in a polydisperse fibrous material is higher and characteristic viscous dimension is lower than for monodisperse materials. The value of tortuosity remains the same.

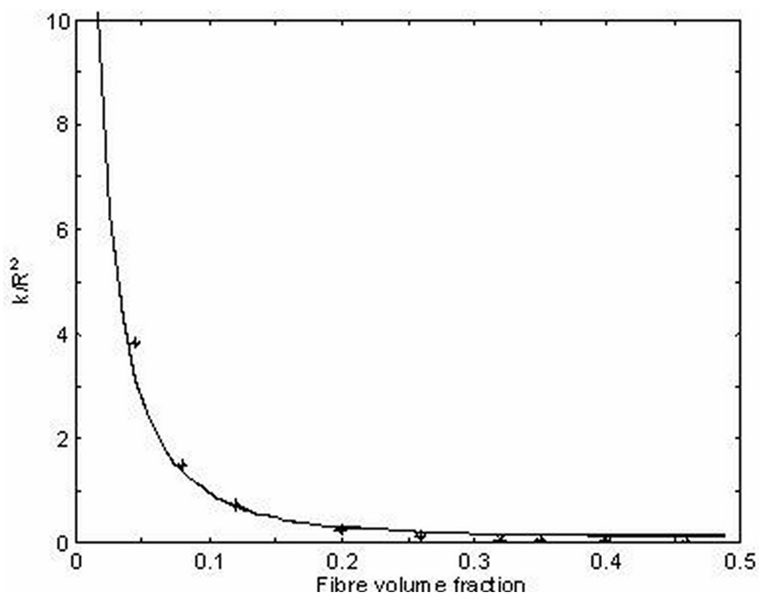


**Figure 1:** Comparisons between cell model predictions for normalised dc permeability and the numerical results for regular array [6].

## 4 - THE RELATIONSHIP BETWEEN THE COMPLEX DENSITY AND THE COMPLEX COMPRESSIBILITY IN A FIBROUS MATERIAL

Now we consider a heat transfer from a fibre subject to oscillating temperature $T_1 e^{-i\omega t}$ in the surrounding fluid. The amplitude of the dimensionless temperature distribution $T(r)$ in the fluid can be found from the equation of heat transfer

$$\Delta_r T - \frac{\omega N_{pr} \rho_0}{i\eta} T = 0 \tag{10}$$

In the 'mirror approximation' [7], the boundary conditions on the fibre surface and cell boundary are:

$$T|_R = 1, \;\; \partial_r T|_b = 0 \tag{11}$$

An expression for the heat transfer term is [8]:

$$S_H = -2\,(1-\phi)\,k \frac{\left.\frac{\partial_r T}{r}\right|_R}{1 + \frac{2i\eta}{\omega N_{pr}\rho_0} \frac{\Psi}{1-\Psi} \left.\frac{\partial_r T}{r}\right|_R}$$

From the symmetry of problems (10)-(11) and (2)-(3) it is possible to find a relationship between function $\xi$ and dimensionless temperature. It gives the relationship between drag term $D\,(\omega)$ and heat transfer term $S_H(\omega)$, which leads to the following relationship between complex density and complex compressibility:

$$C(\omega) = \left(\frac{\partial\rho}{\partial p}\right)_T \frac{1}{\rho_0}\left[1 - \frac{\alpha^2 T_0}{c_p\left(\frac{\partial\rho}{\partial p}\right)_T F(\omega N_{pr})}\right],\ F(\omega) = 1 + \frac{1-\Psi}{2}\left[\frac{\rho(\omega)}{\rho_0} - \frac{1}{\phi}\right] \tag{12}$$

For materials with low density, the cell model requires knowledge of two parameters. As the model underestimates permeability for real materials, we suggest rearrangement of the terms in the derived expressions, to use the measured values of permeability and porosity [9]. In Fig. 2, the cell model predictions for real and imaginary parts of wave number, using the measured porosity and flow resistivity, are compared with data [10]. The agreement is quite good over the whole frequency range.
From (12), the following specific relationship between the viscous and thermal parameters of the empirical model developed by Johnson, Allard, Lafarge *et al* [5], [11], [12] can be derived:

$$k_0^{'} = \frac{2}{(1-\Psi)}k_0,\ \Lambda' = \frac{2}{(1-\Psi)\,\alpha_\infty}\Lambda \tag{13}$$

where $\Lambda'$ and $k'_0$ are the characteristic thermal dimension and dc thermal permeability respectively. The relationships between these parameters (i.e. $k_0^{'} = 2k_0, \Lambda' = 2\Lambda$), derived elsewhere [11], do not include tortuosity (it was supposed to be equal to 1). However for high porosity materials they give results similar to (13).

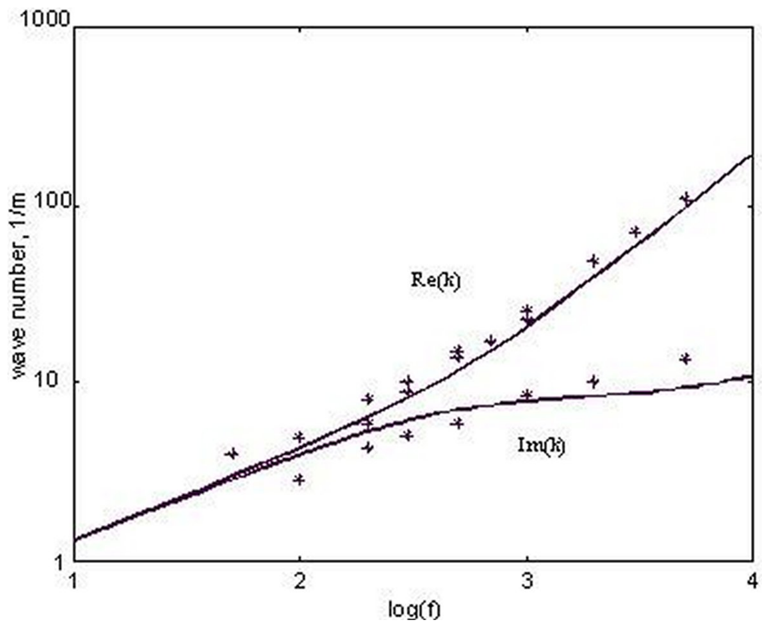


**Figure 2:** Comparison between cell model predictions (solid lines) and data [10] for frequency dependence of wave number. Data are for glass wool with density 16 kg/m$^3$ and measured value flow resistivity $\sigma = \eta/k_0$=5500 Pa.s/m$^2$; cell model predictions use the measured porosity and dc permeability $k_0$.

## 5 - CONCLUSION

The modified Kuwabara cell model approach [3] has been used to derive expressions for dynamic drag parameters of mono- and polydisperse fibrous materials with fibres oriented normally to sound propagation direction. A new relationship between the complex density and compressibility has been derived. The cell model leads to explicit relationships between viscous and thermal parameters for a fibrous material. However, the model underestimates the permeability of a fibrous material in comparison with available data. On the other hand, when measured values of permeability and porosity are used, the new cell model gives good agreement with acoustical data. For these reasons, we suggest the use of the measured values of permeability and porosity when making predictions with the cell model.

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**REFERENCES**

1. **V.Tarnow**, Calculation of the dynamic flow resistivity of fiber materials, *J. Acoust. Soc. Am.*, Vol. 102, pp. 1680-1688, 1997

2. **V.Tarnow**, Compressibility of an air in fibrous materials, *J. Acoust. Soc. Am.*, Vol. 99, pp. 3010-3017, 1996

3. **Umnova, K.Attenborough, K.M.Li**, Cell model calculations of dynamic drag parameters in packings of spheres, *J. Acoust. Soc. Am. (accepted for publication)*, 2000

4. **S.Kuwabara**, The forces experienced by randomly distributed parallel circular cylinders or spheres in a viscous flow at small Reynolds numbers, *J. Phys. Soc. Japan*, Vol. 14, pp. 527-532, 1959

5. **D.L.Johnson, J.Koplik, R.Dashen**, Theory of dynamic permeability and tortuosity in fluid-saturated porous media, *J. Fluid. Mech.*, Vol. 176, pp. 379-402, 1987

6. **D.S.Clague, B.D.Kandhai, R.Zhang, R.M.A.Sloot**, Hydraulic permeability of (un)bounded fibrous media using the lattice Boltzman method, *Phys. Rev. E*, Vol. 61, pp. 616-625, 2000

7. **Y.Hemar, N.Herrmann, P.Lemarechal, R.Hocquart, F.Lequeux**, Effective medium model for ultrasonic attenuation due to the thermoelastic effect in concentrated emulsions, *J. Phys. 2 France*, Vol. 7, pp. 637-647, 1997

8. **N.A.Gumerov, A.I.Ivandaev, R.I.Nigmatulin**, Sound waves in monodisperse gas - particle or vapor - droplet mixtures, *J. Fluid. Mech.*, Vol. 193, pp. 53-74, 1988

9. **R.Kirby, A.Cummings**, Predictions of the bulk properties of fibrous materials at low frequencies, *Appl. Acoust.*, Vol. 56, pp. 101-125, 1999

10. **V.Tarnow**, Measurement of sound propagation in glass wool, *J. Acoust. Soc. Am.*, Vol. 97, pp. 2272-2281, 1995

11. **J.F.Allard, Y.Champoux**, New empirical equations for sound propagation in rigid frame fibrous materials, *J. Acoust. Soc. Am.*, Vol. 91, pp. 3346-3353, 1992

12. **D.Lafarge, P.Lemariner, J.F.Allard, V.Tarnow**, Dynamic compressibility of air in porous structures at audible frequencies, *J. Acoust. Soc. Am.*, Vol. 102, pp. 1995-2006, 1997