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THE MULTI-INPUT MULTI-OUTPUT THEOREM (MINT) FOR CAUSALITY PROBLEMS REDUCTION IN ACTIVE NOISE CONTROL

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ABSTRACT
This paper deals with the causality constrain in active noise control of interior noise, namely it studies the possibility of reducing this problem in cases where it does not have a fundamental nature connected to true delays in the system, but is more related to the enclosures reverberation time. The MINT theorem can be used to show that, for such cases, if the number of reference signals is greater than the number of noise sources and if the number of secondary signals is greater than the number of error microphones, the causality problems can be greatly reduced. However, in some cases, the problem can becomes badly conditioned and without use from a practical viewpoint. This is of special importance for inversion of the pre-reference path, because the reference signal can receive a large amount of ambient noise. The article describes some computer simulations of such systems in small enclosures to evaluate to what extent this problem appears in practice. In addition, a MC-LSL algorithm is presented to solve the problems associated with the high correlation of the reference signals of these systems.

1 - INTRODUCTION
One of the most important factors limiting noise reduction for active control in interior noise is the causality constrain. The reference microphone, error microphones and secondary sources should be placed so that the traveling delay from the noise source to the error microphone is greater than the sum of the delays from the noise source to the reference and from the secondary source to the error microphone. This would in fact solve the problems if only direct path sound transmission occurred. However, for a real enclosure, the transfer function between the several signals of interest are not pure delays, they are affected by the enclosure reverberation time, resulting in a non minimum-phase system and causality problems even for those cases. These problems have two sources. They can result from inverting the secondary path transfer function, and from inverting the pre-reference path transfer function, from the noise source to the error microphone. This paper addresses some issues related to the use of the multi-input/output inverse theorem, MINT, to try and minimize these problems. It's possible to show that, if the direct path delay constrains are met, and if the number of secondary sources are greater than the number of error microphones the secondary path can be inverted. The same is true for the pre-reference path if the number of reference microphones is greater than the number of noise sources.
This is only true, however, if the transfer functions of the paths to be inverted don't have common zeros. If the zeros are close to each other (in other words if the transfer functions are somewhat similar) then the inversion problem is badly conditioned. This results in an inverse with a very high gain, and the system becomes very sensitive to noise in the input signal. This is especially important when inverting the pre-reference path, due to measuring errors and interference from other acoustics sources.
Another problem associated with using the MINT theorem for pre-reference path inversion (the same is true for secondary path) is that since there are more reference signals than noise sources, the input signals will be highly correlated, resulting in poor performance of LMS based algorithms.
This paper tries to address these issues. Although the MINT theorem has been referred several times as possible solution to causality problems. It has been used mainly to determine the number of reference

signal or secondary sources used. Actual evaluations of the practical applicability of the theorem, taking into account the previous considerations, have not been made. An example with room's responses measured in a small office is used to show that the MINT theorem can, in fact, be used to greatly reduce causality problems at least in some cases. Second, a numerical stable multi-channel lattice algorithm, adapted to be used in ANC is presented which is shown through computer simulations to produce good results for the example proposed, here the MC-LMS algorithms performs badly. Finally, a study based on theoretical models for a small rectangular enclosure is presented which tries to determine how serious the bad conditioning problem is in several possible setups for the noise sources, reference microphones and secondary loudspeakers.

2 - THE MINT THEOREM

Acoustic paths are in general non mininum phase transfer functions, so in order to obtain a stable inverse one cannot simply invert its Z transform. Lets look at the problem using matrix algebra: given the output of an acoustic system, $y(n)$, and the impulse response of the system, w_i, one can write the following equation:

$$\mathbf{y}^{L\times 1} = \mathbf{W}^{L\times M}.\mathbf{x}^{M\times 1},\ M = L + N - 1 \tag{1}$$

The input buffer length (M) must equal to the output buffer length (L) plus the impulse response filter length (N) minus one. This means that the $\mathbf{W}$ matrix is not square, and the system, in general, cannot be exactly inverted. That changes in a multi-channel system with O_c output channels and I_c input channels:

$$\begin{gathered}\mathbf{y}^{(O_c.L)\times 1} = \mathbf{W}^{(O_c.L)\times(I_c.M)}.\mathbf{x}^{(I_c.M)\times 1},\ M = L + N - 1 \text{ and } O_c.L = I_c.M \\ \iff O_c.L = I_c.\left(L + N - 1\right)\end{gathered} \tag{2}$$

If there is one more output than input channels then, to the matrix to become square, one simply must have $L = I_c.\left(N - 1\right)$ and, in general, the system as an exact inverse. This is not true, however, if the paths of the transfer function have common zeros, which corresponds to the case where the matrix is not invertible.

With, one more output than input channels, the inverse filters could become quite long. Much shorter filters are obtained for the case where the number of outputs is twice the inputs. For that case $L{=}N{-}1$ only.

It should be emphasized that the exact inverse only exists if the paths do not have common zeros. More, if the zeros of the several paths are close together then the inverse is bad conditioned. As mentioned before, one of the goals of this paper is determine to what extent these are serious constrain to the applicability of these techniques in real world active noise control systems.

3 - ENCLOSURE MODELLING

Accurate modelling of an enclosure is a difficult task. In order to maintain the problem tractable, all the work was done with a simple rectangular enclosure with locally reacting halls.

A mode base approach is used, as similar to the one used by Elliot, since this is accurate at low frequencies where active noise control is effective. These formulas assume a small surface admittance so that the coupling between the several modes is small. Following the work of Elliot the sound pressure at a given point in an $L_1 \times L_2 \times L_3$ enclosure is given by:

$$\begin{aligned}p_\omega\left(\mathbf{x}\right) &= \sum_{n=0}^{\infty} \frac{\omega\rho_0 c_0^2.\psi_n\left(\mathbf{x}\right)}{V\left[\left[2\zeta_n\omega_n\omega + j\left(\omega^2 - \omega_n^2\right)\right)\right.} \int_V \psi_n\left(\mathbf{y}\right).q_{vol}\left(\mathbf{y}\right)dV, \\ k_n &= \pi.\sqrt{\left(n_1/L_1\right)^2 + \left(n_2/L_2\right)^2 + \left(n_3/L_3\right)^2/c}\end{aligned} \tag{3}$$

$$\begin{aligned}&\psi_n\left(\mathbf{x}\right) = \sqrt{\varepsilon_{n1}.\varepsilon_{n2}.\varepsilon_{n3}}.\cos\frac{n_1\pi x_1}{L_1}\cos\frac{n_2\pi x_2}{L_2}\text{c os}\frac{n_3\pi x_3}{L_3}\ ; \\ &\text{if } n_i = 0, \varepsilon_{ni} = 2,\ \text{else } \varepsilon_{ni} = 1\end{aligned} \tag{4}$$

The viscous damping ratio, ζ_n, results from the linear dependence of the halls absorption coefficients with frequency. A volume velocity sound source, $q_{vol}\left(\mathbf{y}\right)$, is assumed somewhere in the enclosure. The sound velocity and the steady state sound pressure are given by c_0 and ρ_0 respectively.

4 - MC-LSL ALGORITHM

Multi-channel LSL Algorithm using a priori estimation errors with error feedback

- $\eta^{Ic\times 1}$, $\psi^{Ic\times 1}$ - Forward and backward a priori prediction errors
- $\Gamma_f^{Ic\times Ic}$, $\Gamma_b^{Ic\times Ic}$ - Forward and backward reflection coefficients
- $Fi^{Ic\times Ic}$, $Bi^{Ic\times Ic}$ - Sum of weighted forward and backward a priori prediction errors
- $k^{Oc\times Ic}$ - Regression coefficients
- $\alpha^{Oc\times 1}$ - A priori estimation error
- $\gamma^{1\times 1}$ - Conversion factor
- Ic, Oc, M - Input channels, output channels and filter order

Initialization	Iterations (m=0... $M-1$)
$\eta_0(n) = u(n)$ $\psi_0(n) = u(n)$ $\alpha_0(n) = d(n)$ $Bi_m(0) = \delta.I$ $Fi_m(0) = \delta.I$ $\gamma_0(0) = 0$ $\Gamma_{f,m}(n) = 0$ $\Gamma_{b,m}(n) = 0$ $k_m(0) = 0$	$\eta_m(n) = \Gamma_{f,m}(n-1).\psi_{m-1}(n-1) + \eta_{m-1}(n);$ $\psi_m(n) = \Gamma_{b,m}(n-1).\eta_{m-1}(n) + \psi_{m-1}(n-1)$ $Bi_m(n) = \frac{1}{\lambda}\left(Bi_m(n-1) - \frac{Bi_m(n-1).\psi_{m-1}(n).\psi^*_{m-1}(n).Bi_m(n-1).\gamma_{m-1}(n)}{\lambda + \psi^*_{m-1}(n).Bi_m(n-1).\psi_{m-1}(n).\gamma_{m-1}(n)}\right)$ $Fi_m(n) = \frac{1}{\lambda}\left(Fi_m(n-1) - \frac{Fi_m(n-1).\eta_{m-1}(n).\eta^*_{m-1}(n).Fi_m(n-1).\gamma_{m-1}(n-1)}{\lambda + \eta^*_{m-1}(n).Fi_m(n-1).\eta_{m-1}(n).\gamma_{m-1}(n-1)}\right)$ $\Gamma_{f,m}(n) = \Gamma_{f,m}(n-1) - \eta_m(n).\psi^*_{m-1}(n-1).\mathrm{B}i_m(n-1).\gamma_{m-1}(n-1)$ $\Gamma_{b,m}(n) = \Gamma_{b,m}(n-1) - \psi_m(n).\eta^*_{m-1}(n).Fi_m(n).\gamma_{m-1}(n-1)$ $\gamma_m(n) = \gamma_{m-1}(n) - \psi^*_{m-1}(n).\mathrm{B}i_m(n).\psi_{m-1}(n).(\gamma_{m-1}(n).\gamma_{m-1}(n))$ $k_m(n) = \alpha_m(n).\psi^*_{m-1}(n).\mathrm{B}i_m(n).\gamma_{m-1}(n) + k_m(n-1);$ $\alpha_m(n) = \alpha_{m-1}(n) - k_m(n-1).\psi_{m-1}(n)$

Table 1.

The use of several reference sensors creates problems to the convergence of the LMS algorithm, due to the high correlation between the several signals. The use of constant filters to de-correlate the signals is proposed, with good results. However, in real world systems an adaptive technique may be more desirable. This paper proposes the use of a version of the MC-LSL algorithm. The proposed algorithm has a computational complexity of the order of $Ic^2 \times M$, and did not present numerical problems during the simulations. The algorithm is a generalization of the "single channel LSL algorithm using a priori estimation errors with error feedback" described by Haykin, and is obtained throw algebraic manipulations of the LSL algorithm.

5 - SIMULATION RESULTS

As said before, this paper focuses in pre-reference path inversion. The goal is to estimate the driving noise signal using the signal from one or two reference microphones. This is however a bit over demanding, compared to what is required in practice. A more reasonable goal is to estimate a signal from another microphone, $d(n)$, given the reference signals. This might seam similar to the multichannel adaptive cancellation problem, but has a major difference: $d(n)$ cannot be delayed, and the system must be causal.

5.1 - Office room

Figures 1 and 2 represent simulations obtained using impulse responses from a real office room with about 1 s reverberation time. They are intended to illustrate the practical usefulness of the MINT in real world situations, and the gain obtained with the MC-LSL algorithm. A band limited white noise signal, representing the driving signal of the noise source, was feed throw three filters, representing real transfer functions from a loudspeaker to three reference microphones in the office. Additional uncorrelated noise signals were added to the input of the filters simulating ambient noise. This limited the maximum theoretical noise reduction achieved to about 30 dB.

Figure 1 compares the multichannel LMS and LSL algorithm. It clearly shows that the MC-LMS algorithm performs badly while the MC-LSL algorithm achieves nearly optimal results.
Figure 2 compares the estimation error achieved with one and with two reference signals. With one reference signal, the causality problems are evident; the estimation error is much higher than the optimal 30dB. With two references, and the MC-LSL algorithm, once again, the results are nearly optimal.

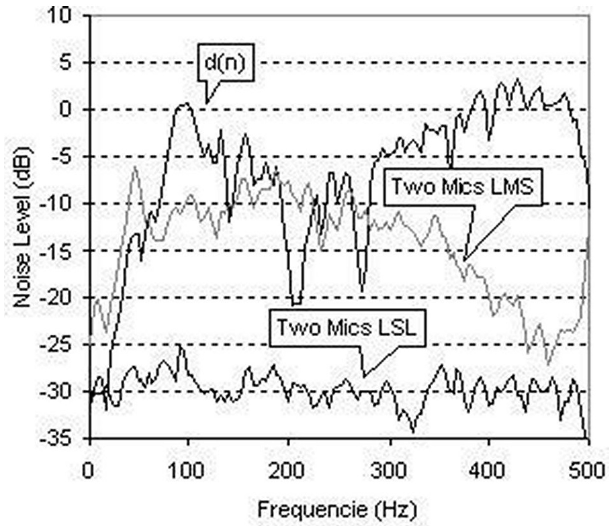

Figure 1: Comparison of the estimation error with the MC-LMS algorithm and the MC-LSL algorithm, for impulse responses of a real office room.

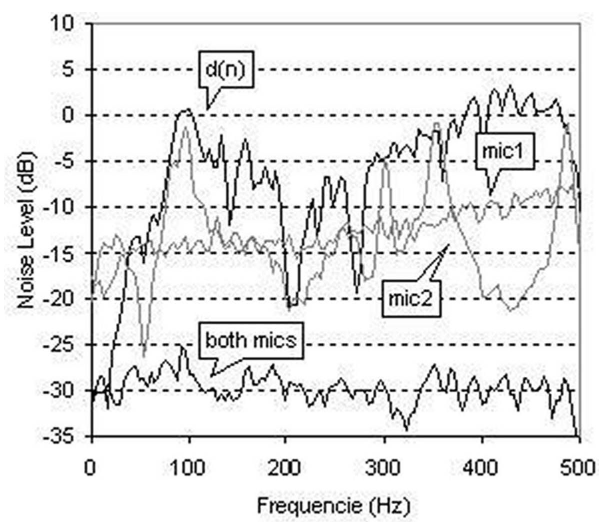

Figure 2: Comparison of the estimation error with one and two reference microphones with the MC-LSL algorithm, for impulse responses of a real office room.

5.2 - Enclosure simulation

This section presents some results from computer simulations of a small rectangular enclosure. It had 1.55 ×1.40 ×1.15 m, about the same volume as a small family car. The viscous damping ratio was set so that the reverberation time has about 0.5 s. Also, in order to kept the reverberation time finite, a minimum value for the damping constant was set, corresponding to the value at 100 Hz, in fact violating the linear relation to frequency.
The simulation refers to an enclosure exited by one noise source, and fill with ambient noise. Some effort was done to make the ambient noise evenly distributed, so that the noise canceling patterns were not related the ambient noise sources. With this in mind, the ambient noise was modelled by an infinity number of noise sources throw the enclosure. All the noise sources were driven by band-limited white

noise at 500 Hz. The placement of the microphones and the primary noise source has always done at about 0.6 m high and at the position marked in the figures.
Figure 3 shows how the noise reduction at the error microphone as it is moved across the room. It show a constant high level of 25 dB, close to the optimum, in most of the area, and a strong reduction as soon as it gets closer to the noise source than the reference microphones. This seams to indicate that the MINT algorithm can be applied in practice. When the causality problems are not fundamental, the use of the two references allowed the system to perform almost optimally. When the error microphone is closer than any of the references to the noise source not even the use of two references can help.

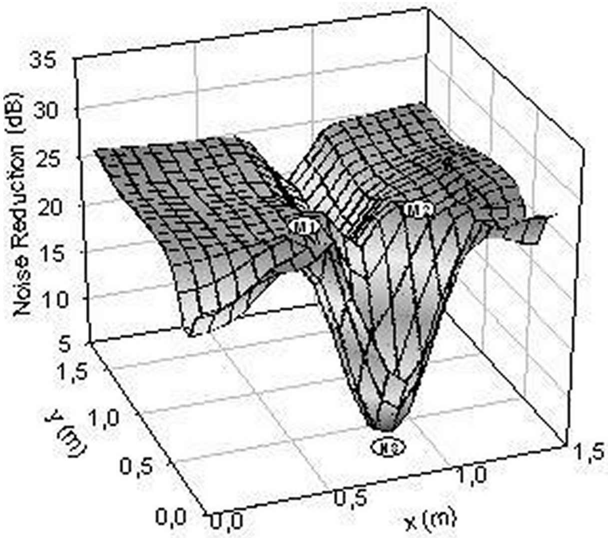

Figure 3: Noise reduction at the error microphone as it moves along the enclosure; with two reference, M1 and M2.

In figure 4, the variation of the noise reduction at the error microphone, with one reference which is moved across the enclosure, is shown. When the reference is close to the noise source, and the major signal contribution is from direct path transitions, the noise reductions are high. However, as it gets further away, the noise reduction drops, and only rises close to the error microphone or its images. When another reference microphone is added, the change is dramatic. A new figure would be show if it were not so simple. With a new reference, at the point of M2 (figure 3), the noise reduction changes to an almost constant 25 dB for any point in the room that M1 is placed. This clearly shows the advantage of multiple references.

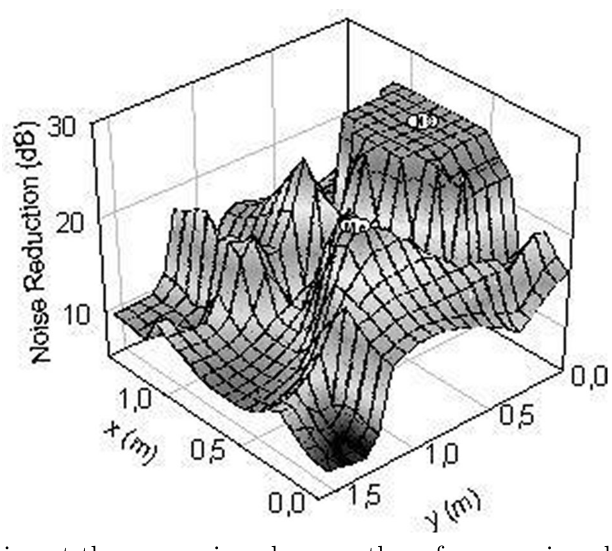

Figure 4: Noise reduction at the error microphone as the reference microphone is moved along the enclosure.

It should be said that, these results represent only one specific set of experiment, and that the results were not always this good. Namely, there seams to be some positions for the error microphone and reference microphones, where the two references don't help much. However, these cases appear sporadically, and the results are typical samples from the more common cases.

6 - CONCLUSION

This paper dealt with the practical applicability of the MINT algorithm, for inversion of the pre-reference path. Namely, with the problems off high correlation of the reference signals, which can degrade the performance of the LMS algorithm, and the sensitivity to measurement and ambient noise. The results were very good. First, the MC-LSL algorithm has shown great promise in solving correlation problems; second, given real world and simulated environments the measurement and ambient noise did not seam to constrain seriously the applicability of the MINT algorithm. In conclusion, the use of several references seams to be a good way to minimize the problems of inverting the pre-reference path.

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