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CURVE SQUEAL OF RAILBOUND VEHICLES (PART 1): FREQUENCY DOMAIN CALCULATION MODEL

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ABSTRACT
A frequency domain model is developed to describe squeal noise generated by the unstable stick-slip phenomenon in the creepage between wheel and rail. The model is based on a combination of existing models for contact mechanics, contact dynamics, wheel dynamics and rail dynamics. The friction coefficient is linearized to allow frequency domain calculations. This enables fast calculations and enhances insight into frequency dependent phenomena relating to squeal. Examples of such phenomena are the influence of the wheel dynamics and the lateral contact position on squeal noise. The model is validated and can be used as a design tool for measures against squeal noise.

1 - INTRODUCTION

Curve squeal is the intense tonal noise that can occur when a railway vehicle traverses a curve or switch. During the curve passage, some wheels rub with the flange against the rail. Other wheels perform lateral creepage because the wheel movement does not align with the rolling direction. The creepage can show unstable stick-slip behaviour, causing the wheel to oscillate and radiate loud annoying noise. This paper describes a frequency domain model to predict the dominant squeal noise frequency. The calculations of the dynamic force and vibration amplitudes and higher harmonics are performed in the time domain. Part 1 of this paper describes the squeal noise model, part 2 [4] and 3 [5] describe the validation of the model.

2 - CONTACT MECHANICS

During a curve passage, the wheel movement direction and the rolling direction do not align, causing lateral creepage. Lateral creepage s is defined as the ratio of lateral creep velocity v_{sy} over rolling velocity V_0. For small rolling angles, the stationary lateral creepage s_0 and rolling angle [rads] are approximately equal. The lateral creep causes lateral contact forces which are equal to the product of vertical contact force and friction coefficient. Fingberg [1] gives equations for the stationary friction coefficient μ_0 for the non-rolling contact:

$$\mu_0 = \frac{\tau_R \tau_W}{\tau_R + \tau_W} \frac{\pi ab}{F_0} \tag{1}$$

- τ_R, τ_W: shear strength wheel and rail material (e.g. 4.10^8 - 6.10^8 [N/m^2]),
- a, b: semi-axis length of Hertz contact ellipse in the rolling and lateral direction [m],

and for the creepage dependent friction coefficient $\mu(s)$ for the rolling contact (see figure 1):

$$\mu(s) = \begin{cases} \mu_0 \left\{ s' - \frac{1}{3}s'^2 + \frac{1}{27}s'^3 \right\} \left\{ 1 - 0.5e^{-\frac{0.138}{|sV_0|}} - 0.5e^{-\frac{6.9}{|sV_0|}} \right\} & \text{for } s' \leq 3 \\ \mu_0 \left\{ 1 - 0.5e^{-\frac{0.138}{|sV_0|}} - 0.5e^{-\frac{6.9}{|sV_0|}} \right\} & \text{for } s' > 3 \end{cases} \tag{2}$$

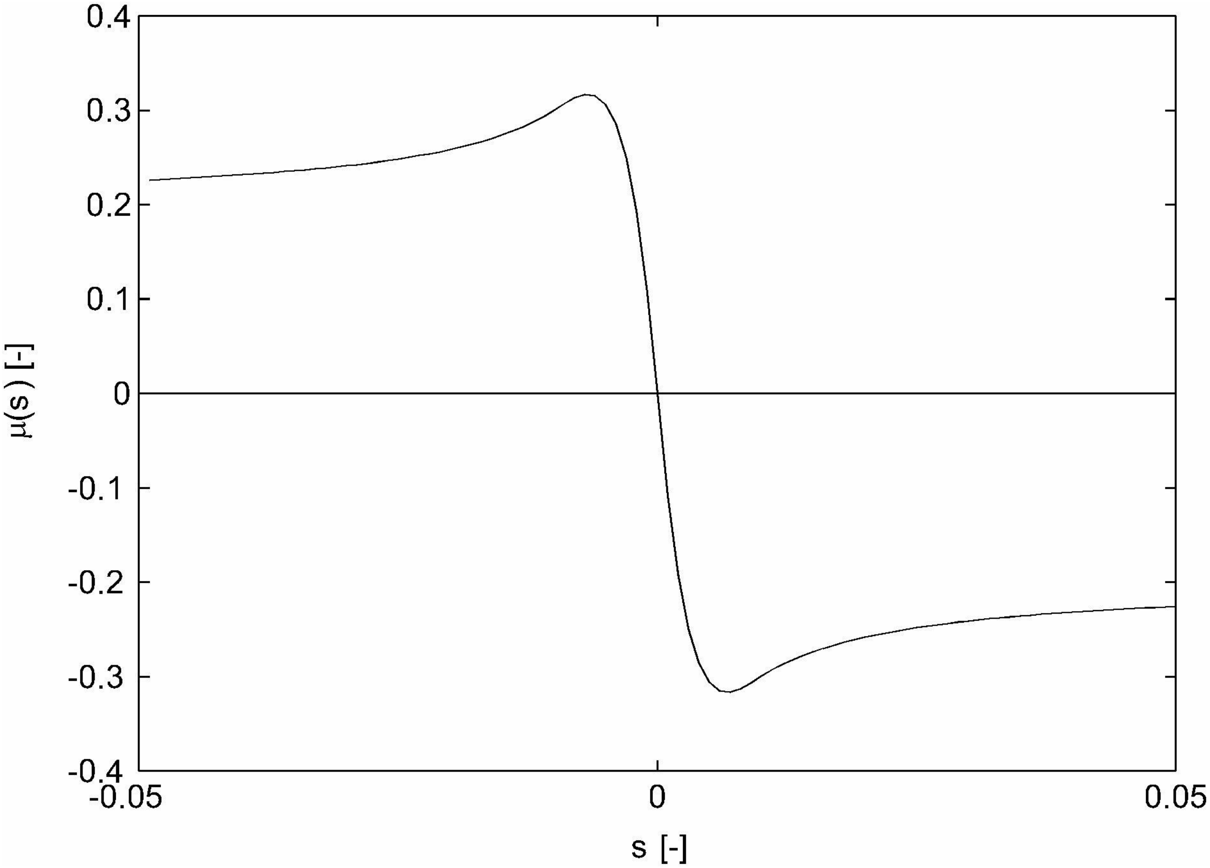

Figure 1: Example of friction coefficient for rolling contact (V_0 = 20 m/s).

with $s' = \dfrac{sGabC_{22}}{\mu_0 F_0}$, G = Young modulus of steel, C_{22} = Kalkar constant ($C_{22} \approx 2.39 + 1.36\,(a/b) - 0.025\,(a/b)^2$).

For squealing wheels, the vertical contact force $F_0 + f_x\,(t)$, the lateral contact force $F_0\mu\,(s_0) + f_y\,(t)$, and the lateral slip $s_0 + v_{sy}\,(t)\,/V_0$, consist of a constant and a time variant part. Their relation is:

$$F_y = (F_0 + F_x)\,\mu\left(\alpha + \frac{v_{sy}}{V_0}\right) - F_0\mu\,(\alpha) \approx F_x\mu\,(\alpha) + v_{sy}\frac{F_0}{V_0}\frac{\partial\mu\,(\alpha)}{\partial v_{sy}} \tag{3}$$

If the situation of starting squeal is considered, in which the vibration amplitude is still very small, the non-linear friction coefficient $\mu\,(s)$ can be linearized and equation (3) becomes:

$$F_y = (F_0 + F_x)\,\mu\left(\alpha + \frac{v_{sy}}{V_0}\right) - F_0\mu\,(\alpha) \approx F_x\mu\,(\alpha) + v_{sy}\frac{F_0}{V_0}\frac{\partial\mu\,(\alpha)}{\partial v_{sy}} \tag{4}$$

Transformation of the time variant part from the time domain to the frequency domain gives:

$$F_y\,(\omega) = F_x\,(\omega)\,\mu\,(s_0) + \frac{F_0}{V_0}\frac{\partial\mu\,(s_0)}{\partial s}V_{sy}\,(\omega) \tag{5}$$

This equation gives the lateral force as a result of vertical force and creepage. In turn, the vertical force and the creep velocity are a function of the lateral force as described in the next section.

3 - WHEEL AND RAIL DYNAMICS

The TWINS software package [2] is used the to calculate the wheel, rail and contact spring mobilities:

- Y_{Wxx}, Y_{Wyy}, Y_{Wyx}: vertical, lateral and cross-mobility for the wheel in the contact point,
- Y_{Rxx}, Y_{Ryy}, Y_{Ryx}: vertical, lateral and cross-mobility for the rail in the contact point,
- Y_{Cx}, Y_{Cy}: vertical and lateral mobility of the contact spring.

The squeal noise appears at wheel resonance frequencies. At these frequencies the following relations hold: $Y_{Wyx} \gg Y_{Ryx}$, $Y_{Wyy} \gg Y_{Ryy}$ and $Y_{Wyy} \gg Y_{Cy}$. As a consequence, the rail vibrations can be neglected compared to the wheel vibrations. Using these simplified relations to describe velocity over the friction element, see figure 2, yields:

$$F_x \left(Y_{Wxx} + Y_{Rxx} + Y_{Cx} \right) + F_y \left(Y_{Wxy} + Y_{Rxy} \right) = 0 \tag{6}$$

$$\left(F_x Y_{Wyx} + F_y Y_{Wyy} \right) + \left(F_y Y_{Cy} \right) = 0 \tag{7}$$

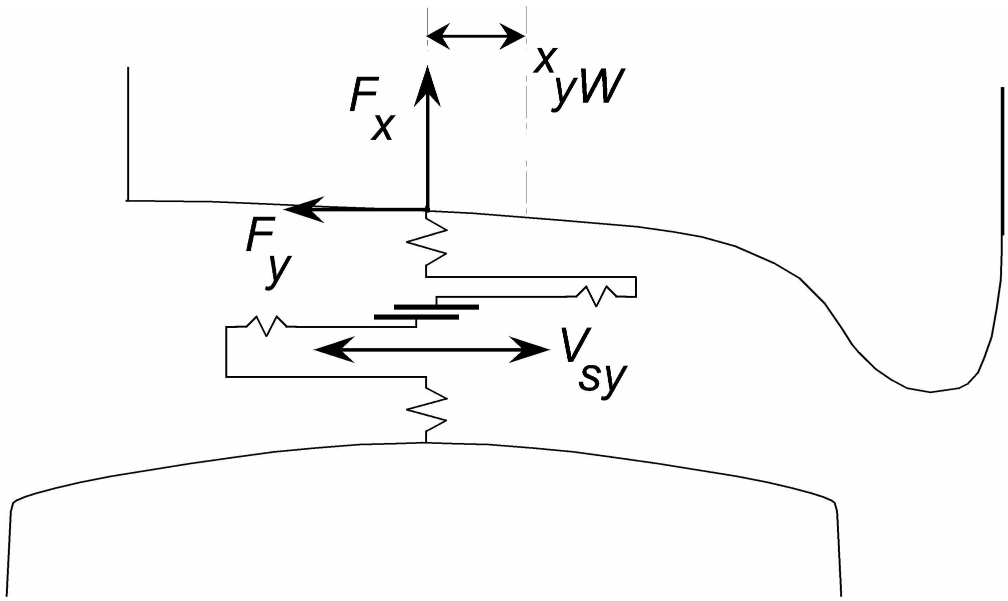

Figure 2: Modelling of wheel and rail dynamics, contact springs and friction elements.

The mobilities Y_{Wxx} and Y_{Wyx} depend on the lateral offset of contact position on the wheel, x_{yW}:

$$Y_{Wyx} = Y_{Wyx0} + x_{yW} Y_{Wyz0} \tag{8}$$

$$Y_{Wxx} = Y_{Wxx0} + 2x_{yW} Y_{Wxz0} + x_{yW}^2 Y_{Wzz0} \tag{9}$$

Y_{Wyx0}, Y_{Wyz0}, Y_{Wxx0}, Y_{Wxz0}, Y_{Wzz0}: cross-mobilities between vertical and lateral forces and moment about the z-axis in the nominal lateral contact position on the wheel thread.
Substitution of equations (6-9) gives the transfer functions for $F_x(\omega)/F_y(\omega)$ and $V_{xy}(\omega)/F_y(\omega)$. Figures 3 and 4 show the influence of the contact position on the transfer functions V_{xy}/F_y and F_x/F_y. These transfer functions determine the loop gain $H(\omega)$ and the dominant squeal noise frequency (see sections 4 and 5).

4 - NYQUIST CRITERIUM FOR STABILITY

Combination of the equations for contact mechanics and wheel dynamics gives a loop gain for the lateral contact force.

$$F_y = (F_0 + F_x)\, \mu \left(\alpha + \frac{v_{sy}}{V_0} \right) - F_0 \mu(\alpha) \approx F_x \mu(\alpha) + v_{sy} \frac{F_0}{V_0} \frac{\partial \mu(\alpha)}{\partial v_{sy}} \tag{10}$$

The instability of the system can be determined by the Nyquist criterion, which states that the system is unstable (the wheel squeals) for frequencies where the Nyquist contour $H(\omega)$ passes the real axis at the right side of 1 (corresponding to a loop gain $H(\omega)$ larger than 1 and a phase shift of 0°). The Nyquist contour shows that squeal noise can occur at the frequencies of the axial wheel modes.

5 - DOMINANT FREQUENCY, AMPLITUDE AND HIGHER HARMONICS

Heckl [3] shows that squeal noise occurs only at the resonance frequency of the mode with the largest growth factor. The growth factor is proportional to the ratio of the loop gain over the modal mass of

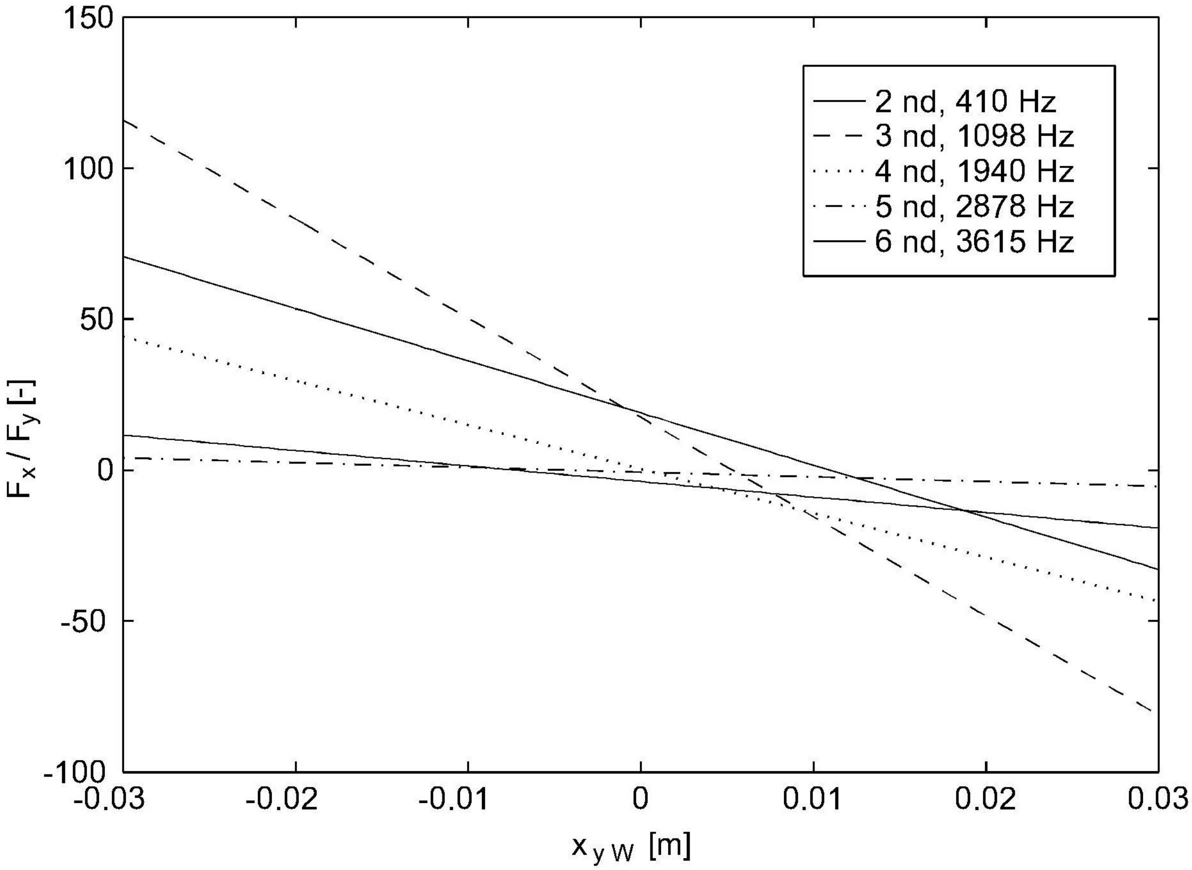

Figure 3: Transfer function ReF_x/F_y for axial modes dependent of contact position x_{yW}.

the corresponding mode. The modal masses of the axial modes are approximately equal. Hence, the frequency of the mode with the largest loop gain becomes the dominant squeal noise frequency ω_1.
The non-linear relationship between friction coefficient $\mu(s)$ and lateral creepage s causes higher harmonics in the force and vibration spectra. The lateral vibration of the wheel will therefore consist of the dominant frequency and higher harmonics, $k\omega_1$ with k=1, 2, 3,...
The amplitude of the squeal oscillations is calculated in the time domain. The relationship v_{sy}/F_y is sampled only for the frequencies $k\omega_1$ and transformed to the time domain. The relation $F_x(\omega_1)/F_y(\omega_1)$ is only used for the dominant frequency ω_1. This sampling reduced calculation time and is only allowed after determination of the dominant frequency. As a consequence the spectrum of the squeal noise consists of the dominant frequency and higher harmonics of the dominant frequency. Several parameters as lateral contact position on the wheel thread determine which of the wheel resonant frequencies becomes predominant.

6 - CONCLUSIONS

Based on existing models for contact mechanics, contact dynamics, wheel dynamics and rail dynamics, a frequency domain model is developed. An essential step in the transformation from time to frequency domain is the linearization of the friction coefficient, which what is allowed for small vibration amplitude. The frequency domain model is used to determine whether or not squeal noise occurs and at which frequency. The model provides a short calculation time and an insight in related frequency dependent phenomena. The calculation of the dynamic force and vibration amplitudes and higher harmonics are performed in the time domain. The model predicts squeal noise for given wheel, rail, wheel load, lateral contact position on wheel thread, rolling velocity and rolling angle. The model is validated and used as a design tool for measures against squeal noise.
Wheel dynamics play a significant role in squeal noise. Squeal occurs at one of the resonant frequencies of axial wheel modes. The lateral contact position on the wheel tyre influences which resonant frequency becomes dominant.

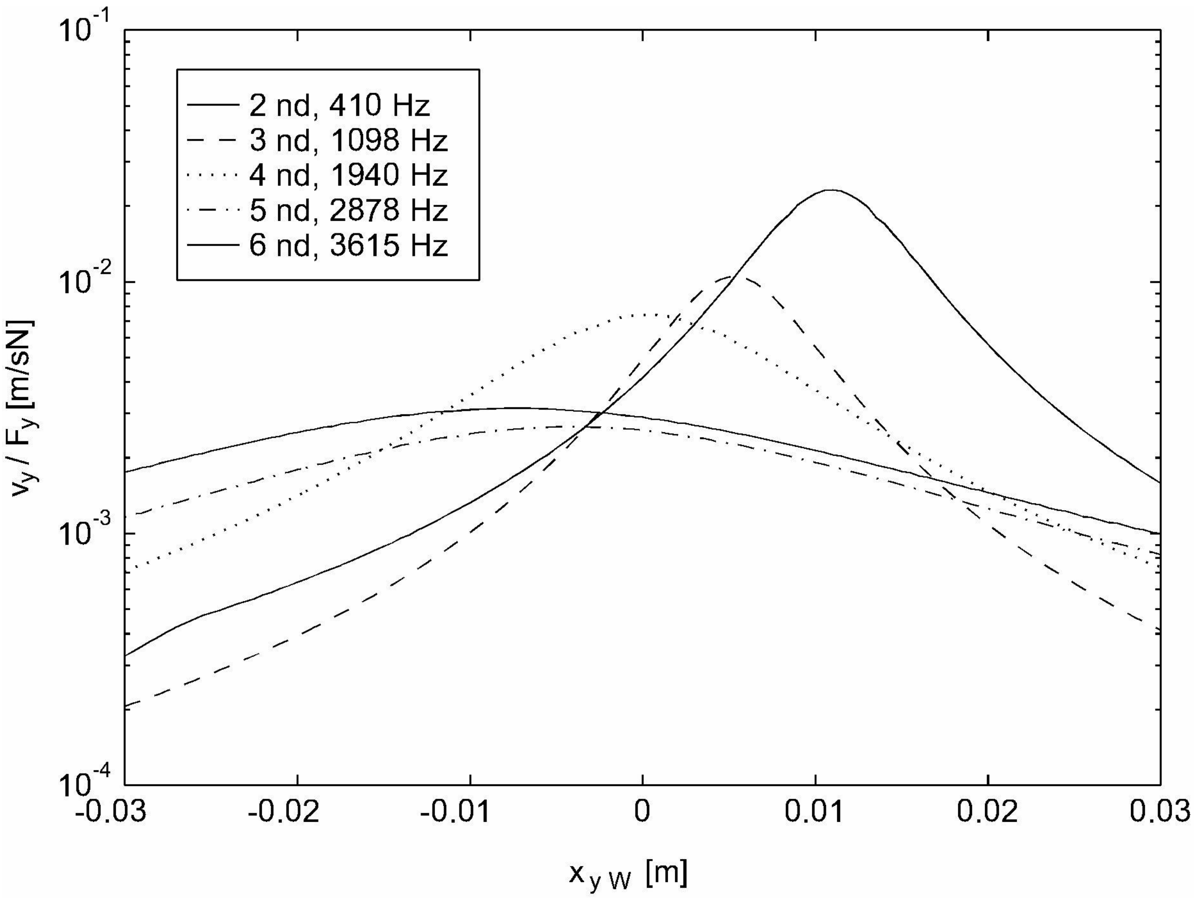

Figure 4: Transfer function $\mathrm{Re}\, V_{sy}/F_y$ for axial modes dependent of contact position x_{yW}.

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