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WAVE NUMBER DOMAIN REPRESENTATION OF TIRE VIBRATION

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ABSTRACT

The use of both non-parametric and parametric wave number decomposition techniques to characterize tire vibration is described here. When a tire was driven radially at a point on its treadband, measurements of the resulting radial treadband vibration were made around the treadband circumference by using a laser Doppler velocimeter. By performing a circumferential wave number decomposition of the space-frequency data, the propagation characteristics of the waveguide modes that contributed to the response of a tire could be visualized. However, to obtain quantitative estimates of the real and imaginary dispersion relations for each of these modes, an iterative Prony series approach has been used. By curve-fitting to the latter data, it is possible to obtain wave velocity and attenuation rate estimates on a mode-by-mode basis.

1 - INTRODUCTION

It is now well known that tire/road interaction noise can contribute significantly to passenger vehicle passby noise levels. As a result, there is interest in studying tire vibration particularly as it relates to sound radiation. In earlier work, non-parametric, i.e., FFT-based, circumferential wave number decomposition techniques were used to characterize tire vibration [1]. Those results suggested that the response of a typical passenger car tire is controlled by a small number of relatively slowly propagating flexural modes acting in combination with more quickly propagating modes associated with extension of the treadband. Each of these modes may be associated with a particular cross-sectional mode shape, and thus it was concluded that tires could be effectively modeled as waveguides. In the present work, emphasis has been placed on estimating quantitative information about the propagating tire modes: in particular their velocities and attenuation rates. To this end, the tire vibration has been represented as a sum of complex exponentials, each having a complex wave number. The latter have been identified by using an iterative Prony series procedure, whose implementation is described here. The complex dispersion relations that result from that procedure may then be used to estimate the phase and group velocities of each mode, along with their corresponding attenuation rates.

2 - EXPERIMENTAL PROCEDURE

Figure 1 shows the measurement set-up. A Firestone P215/70R14 M+S tire inflated to 40 psi was used here: it was placed horizontally on a tire balancer as shown. The tire was forced radially at the center of its treadband by using a mini-shaker (Brüel and Kjaer 4810). The force was applied through a 2 cm diameter aluminum disk glued to center of the treadband. The mini-shaker was held by a cantilever beam that was itself attached to a plate bolted to the wheel. A PCB 208A03 force transducer connected between a stinger and the aluminum disk was used to monitor the force input to the tire. A counter weight was placed on the wheel to balance the weight of the drive arrangement. The output of a random noise generator was passed through a Wavetek 852 bandpass filter and a QSC Model 1080 power amplifier before being delivered to the mini-shaker. The radial velocities of the treadband were measured at 206 equally-spaced locations around the circumference of the tire over the frequency range from 100 Hz to 1000 Hz by using a Polytec OFV-040 laser vibrometer and OFV 3000 controller. The measurement track was covered with reflective paint to ensure a good signal-to-noise ratio. The position of the laser was fixed during the measurements; the tire and drive arrangement were rotated to allow measurements to be made at different circumferential locations. The outputs of the force transducer and laser vibrometer were delivered to a Brüel and Kjaer 2032 signal analyzer after the former signal was amplified using a PCB 480D09 charge amplifier. A Matlab computer program was used to control the signal analyzer and to record all the necessary information.

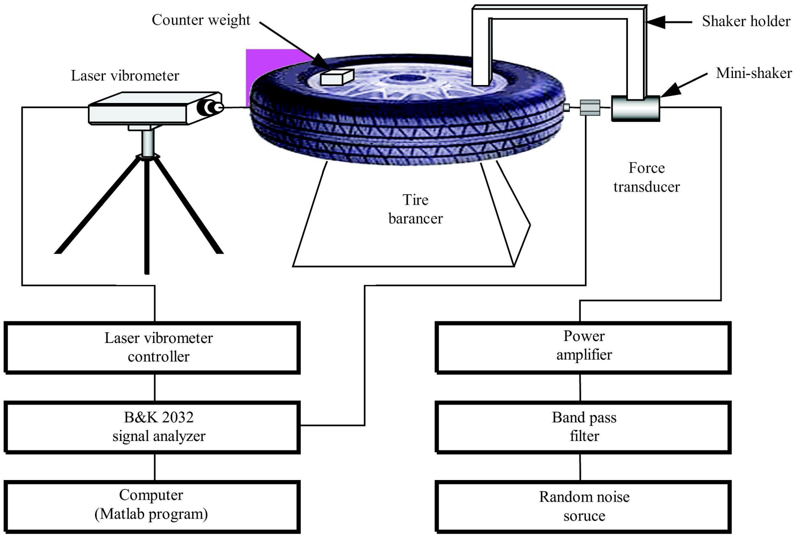


Figure 1: Experimental setup.

3 - PRONY SERIES PROCEDURE

In the present work it was assumed that the radial tire vibration could be represented as the sum of a relatively small number of circumferentially propagating waves, each having a complex wave number [1]. When a complex wave number, $k_\theta = \beta - i\alpha$, is used to represent a spatially attenuated wave propagating around a tire, its imaginary part accounts for the spatial attenuation. It is appropriate to identify spatially attenuated waves of this type by using a series of complex exponential functions whose exponents are themselves complex. This exponential series identification method, the Prony method, is very well known [2-5]. When using the conventional implementation of the Prony method, a relatively large number of exponential terms (i.e., a large model order) must normally be used to model roots related to measurement noise. Since the bias of the wave number estimate resulting from measurement noise decreases as the model order is increased, the Prony series model order should normally be as large as possible. As a result, many of the series components identified in this way are not related to propagating wave components. In order to limit the model order to the number of propagating waves, an iterative Prony method proposed by Therrien and Velasco [6] was used here. This procedure can be used to find a Prony series representation of the spatial data that minimizes an estimation error norm, and which is therefore optimal in a least squares sense. Therefore, with relatively small bias error, it can be used to identify wave propagation parameters while limiting the model order to the number of wave propagating around the tire. However, since the error norm is not itself a quadratic function, the iterative method does not always locate the global minimum when the parameter starting points are chosen arbitrarily. To resolve this concern, it is necessary to select starting parameter values close to their optimal values to guarantee an optimal solution regardless of the starting point. In the present case, parameter starting values were selected from amongst the solutions of the conventional Prony method. Note that when a dominant wave component is removed from the exponential series representation of a function, the residual error should be large. Thus, noise roots that do not contribute substantially to the error norm when excluded from the Prony series can be separated from the true system roots whose exclusion causes a large increase in residual noise. This selection method may still fail when the magnitudes of the noise roots are comparable with those of the true roots. To resolve this concern, the small number of noise roots were removed at one time under user control, and then the iterative Prony procedure was applied based on the selected roots. Both the root selection and the optimization procedure were applied repeatedly until the desired identification was obtained.

4 - COMPARISON WITH WAVE NUMBER TRANSFORM TECHNIQUES

Figure 2 shows the measured normalized radial velocities and the magnitude of the corresponding spatial Fourier transform, both on a decibel scale [1].

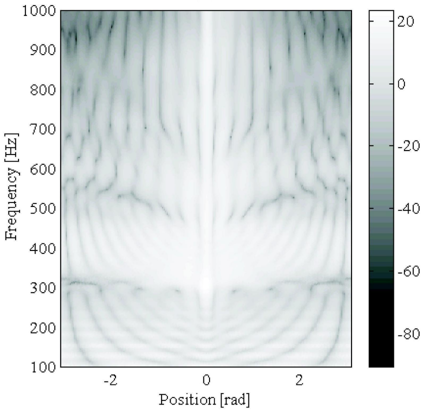


Figure 2(a): Treadband velocity.

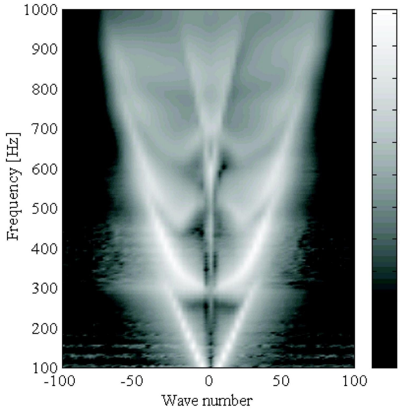


Figure 2(b): Result of wave number transform.

Figure 3 shows the results at three individual frequencies: it is clear that energy dissipation causes the magnitude of the treadband velocity to attenuate exponentially as waves propagate along the tire from the drive point. When the wave number transform technique is applied to the measured data, only the real part of the dispersion relation is obtained. That information allows the wave speed to be inferred, but not the corresponding attenuation characteristics. Further, because of the exponentially decaying characteristic of each wave component, each of the latter has a finite bandwidth in the wave number domain at a specific frequency, thus sometimes making it difficult to identify peak locations in the frequency-wave number domain.

These various concerns have been resolved by fitting a Prony series to the spatial data as explained in Section 3. The results of the Prony series identification at three frequencies are shown together with the measured data in Fig. 3. It is apparent that the measured data is well represented by the Prony series. The real and imaginary wave numbers obtained from the Prony series identification at each frequency are plotted separately in Fig. 4. The brightness of the result at each frequency represents the magnitude of the complex amplitude associated with that frequency-wave number combination: thus the dominant mode at any frequency may be identified. Each continuous curve in these figures is associated with a particular cross-sectional mode shape of the treadband [1]. The modes that behave like waveguide flexural waves cut on below 700 Hz and are associated with the membrane and flexural stiffness of the tire carcass. A fast wave cuts on in the vicinity of 600 Hz: this wave is believed to be related to the extensional stiffness of the treadband and to represent primarily in-plane motion. The most interesting feature in Fig. 4 is the behavior near the cut-on frequency of the higher modes. For example, near the cut-on frequency of the second mode (approximately 300 Hz), the first mode begins to be significantly attenuated and the second mode "cuts on" when it has a large imaginary wave number: i.e., the second mode is initially nearly evanescent. As the frequency increases, the imaginary wave number of the first mode increases and that of the second mode decrease and the dominant mode shifts from the first to the second mode.

5 - PHASE VELOCITIES AND GROUP VELOCITIES

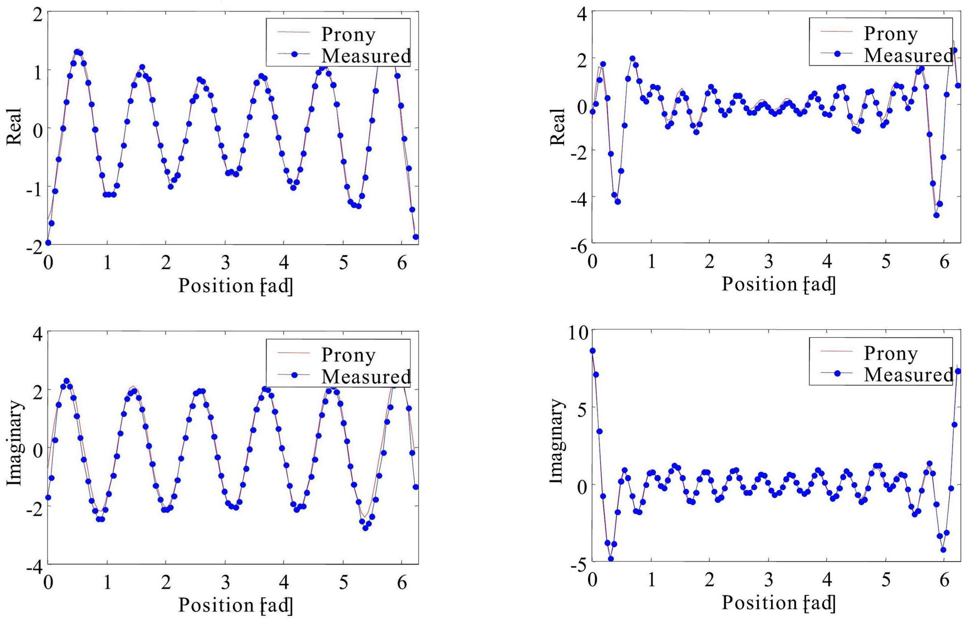
By fitting the dispersion curves of the first and second modes with polynomials, analytical expressions for the dispersion characteristics of these modes were obtained. By using these polynomial expressions, the wave speeds and spatial attenuations per wavelength then can be calculated: the former from the dispersion curves in the real wave number domain and the latter from those in the imaginary domain. Both results are shown in Fig. 5. As expected, the phase velocity of each mode approaches an asymptotic limit from above as the frequency increases, while the group speed increases towards the same limit from below. It can also be seen that the attenuation per wavelength approaches a maximum near the modal cut on frequency.

6 - CONCLUSION

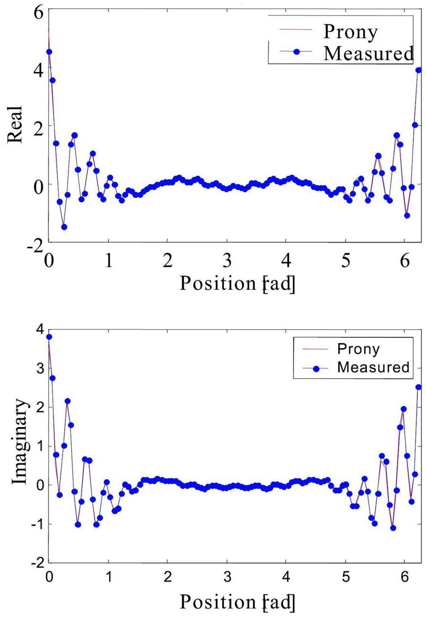
Here tire treadband vibration has been measured and studied by using wave decomposition techniques at frequencies below 1000 Hz. Whereas the spatial Fourier transform yielded only real and banded wave number estimates, the Prony series procedure described here could be used to decompose the measured data into a small number of exponential terms (including an exponential decay factor). It is assumed that each of these terms corresponds to a particular waveguide mode traveling circumferentially around the tire. The Prony series results can then be used to estimate the velocity and spatial rate of attenuation of each mode that contributes significantly to the tire vibration.

REFERENCES

1. **J.S. Bolton, H.J. Song, Y.K. Kim, and Y.J. Kang**, The wave number decomposition approach to the analysis of tire vibration, In *Proceedings of NOISE-CON 98*, pp. 97-102, 1998
2. **F.B. Hildebrand**, *Introduction to Numerical Analysis*, 1956
3. **P. Davies and J.K. Hammond**, A comparison of Fourier and parametric methods for structural system identification, *Trans. ASME*, Vol. 106, pp. 40-48, 1984
4. **S. Braun and Y.M. Ram**, Determination of structural modes via the Prony model: System order and noise induced poles, *J. Acoust. Soc. Am.*, Vol. 81, pp. 1447-1459, 1987
5. **K. Grosh and E.G. Williams**, Complex wave-number decomposition of structural vibrations, *J. Acoust. Soc. Am.*, Vol. 93, pp. 836-848, 1993
6. **C.W. Therrien and C.H. Velasco**, An iterative Prony method for ARMA signal modeling, *IEEE Trans. Signal Proc.*, Vol. 43, pp. 358-361, 1995

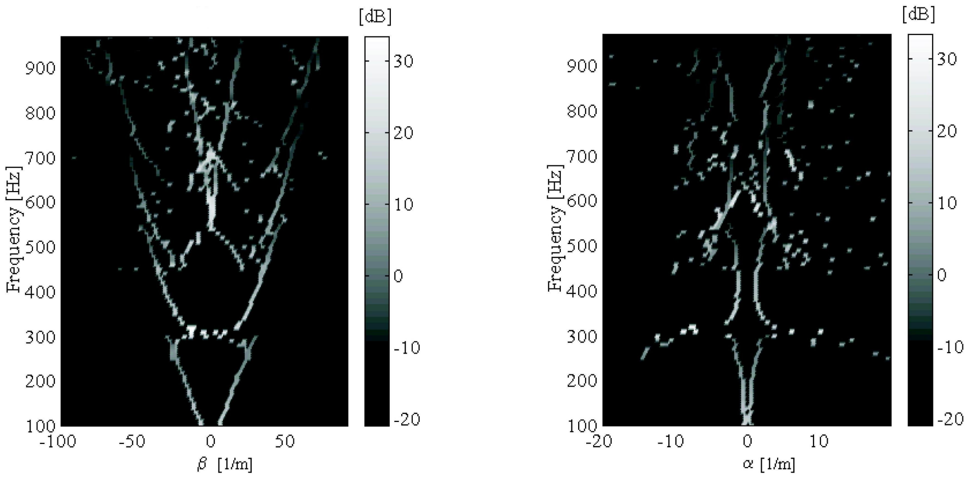


(a): 200 Hz. **(b):** 500 Hz.



(c): 800 Hz.

Figure 3: Prony series fits to the experimental data.



(a): Real wave number. **(b):** Imaginary wave number.

Figure 4: Prony series results.

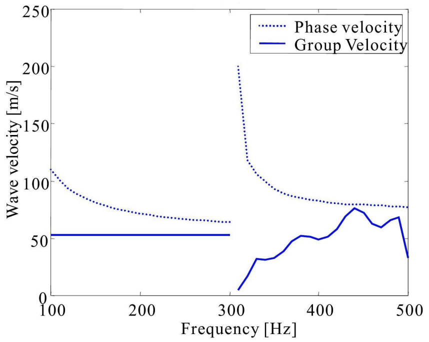


Figure 5(a): Phase and group velocities.

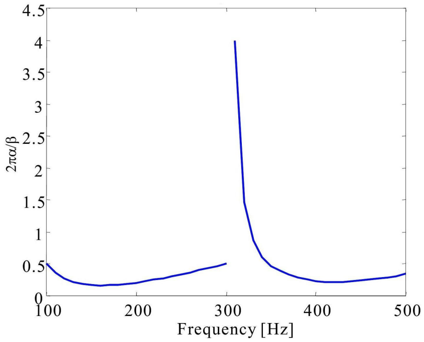


Figure 5(b): Spatial attenuation per wavelength.