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WAVEFIELD SUPPRESSION AND ITS APPLICATION TO MID-FREQUENCY STRUCTURAL-ACOUSTICS

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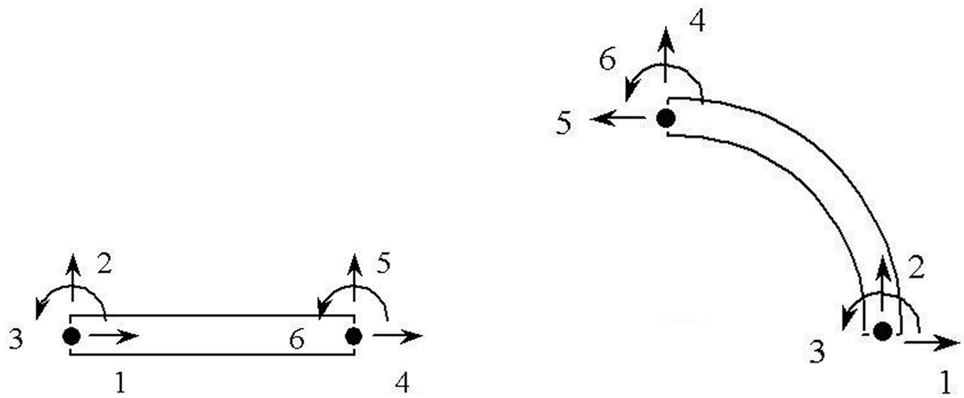
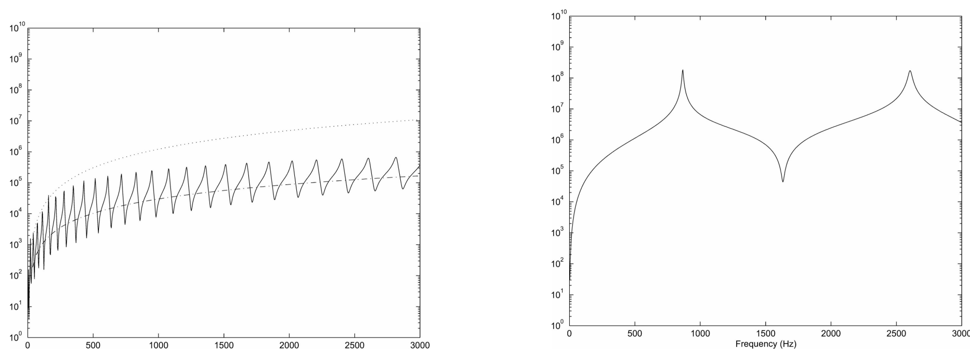
ABSTRACT
The response of a structural-acoustic system in the mid-frequency range typically consists of both long and short wavelength behavior. Modeling the short-wavelength behavior deterministically is usually computationally prohibitive and structural-acoustic techniques such as statistical energy analysis (SEA) are often adopted. However, SEA cannot adequately capture the long-wavelength global behavior of the system. Recent work aimed at addressing the mid-frequency problem has led to the development of a hybrid approach based on a wavenumber partitioning scheme [1]. One of the requirements of the approach is the automatic generation of a set of long-wavelength global basis functions from a finite element (FE) model. These basis functions are used to represent the global dynamic behavior of a structure when the local short-wavelength behavior is suppressed. This paper describes how the basis functions may be obtained using a wavefield suppression approach based on a modified form of Guyan reduction. The approach is described and a number of numerical examples are presented.

1 - INTRODUCTION
Consider the straight beam illustrated in Figure 1a. The dynamic behavior of the beam is fully described by the dynamic stiffness matrix associated with the physical degrees of freedom (dofs) at the ends of the beam. In many instances the physical and material properties of the beam are such that the transverse motion of the beam has a much shorter wavelength than the axial motion of the beam. A numerical evaluation of the dynamic stiffness matrix using a FE model therefore requires a large number of elements in order to fully capture the short wavelength behavior. A significant reduction in computational expense can be achieved by adopting a hybrid analysis which employs an approximate statistical description of the short wavelength behavior and a deterministic description of the long wavelength behavior [1]. The dynamic stiffness of the beam is then obtained by (a) generating a coarse FE mesh of the beam, (b) designating the transverse displacements and rotations associated with the interior nodes of the beam to be Guyan slave degrees of freedom [2], while retaining the in-plane dofs as Guyan masters (c) evaluating the dynamic stiffness of the beam using this reduced model and (d) applying a fuzzy correction to the dynamic stiffness which accounts for the bending wavefield. In effect the approach adopts a form of fuzzy component mode synthesis for the transverse response of the beam in which the constraint modes are obtained deterministically using Guyan reduction, while the component modes are described statistically. Figure 2a compares the transverse dynamic stiffness of an example beam obtained using a detailed FE model, a Guyan reduced FE model and a Guyan reduced model with an added fuzzy correction. The properties of the beam are listed in Table 2. The Guyan reduced model is representative of a coarse FE model in which the number of elements is insufficient to resolve the response. In this example the fuzzy correction was obtained by assuming an asymptotic form for the spatial correlation field within the beam and performing a double line integral over the domain of the beam as described in [1]. The fuzzy



(a) **(b)**

Figure 1: Degree of freedom numbering for straight and curved beams.



(a): D_{22} obtained using: – detailed FE model, ... coarse FE model -.- hybrid model.

(b): D_{11} obtained using coarse FE model; (see Fig. 1 for definition of dof).

Figure 2: Dynamic stiffness of straight beam.

correction clearly improves the estimate of the transverse dynamic stiffness. The axial dynamic stiffness is plotted in Figure 2b and is accurately estimated using a coarse FE mesh.

2 - CURVATURE

Consider now the curved beam structure illustrated in Figure 1b. The curvature introduces coupling between the tangential and radial degrees of freedom which becomes significant below the ring frequency of the beam. The waves that propagate in the beam are typically dominated by either tangential or radial motion. In order to apply the hybrid approach to this structure it is necessary to suppress the radial wavefield and replace it with a fuzzy description, while maintaining a deterministic description of the tangential wavefield using a coarse FE model.

At first sight it would appear that the approach described in Section 1 could be used to suppress the radial motion of the beam. In this approach the radial displacements and rotations are designated as Guyan slave degrees of freedom, while the tangential degrees of freedom are designated as Guyan masters. However, when implemented it is found that the Guyan reduction fails to suppress the radial wavefield of the beam. In fact, performing a modal analysis reveals that the Guyan reduced model still attempts to resolve the short wavelength radial modes of the beam. This can be seen in Table 1 which contrasts the first six non-zero natural frequencies of the beam obtained with and without Guyan reduction. The 10^{th} mode shape of the Guyan reduced beam is shown in Figure 3a and is clearly dominated by radial motion.

3 - A MASS BASED REDUCTION SCHEME

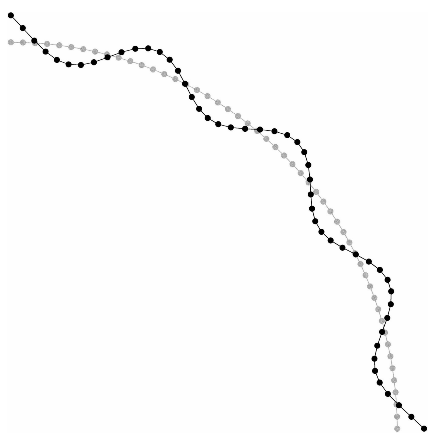
The radial wavefield typically has a much higher wavenumber than the tangential wavefield. If the beam is driven at a wavenumber associated with tangential motion then the radial wavefield will be mass controlled. Similarly, if the beam is driven at a wavenumber associated with radial motion then the tangential wavefield will be stiffness controlled. Applying standard Guyan reduction to the beam enforces a stiffness relationship between the radial and tangential dofs. This promotes motion in which the radial wavefield is resonant while the tangential response is stiffness controlled. This is the opposite of what is required in the hybrid approach.

In order to suppress the radial wavefield and obtain motion dominated by the tangential wavefield it is necessary to enforce an inertial relationship between the tangential and radial degrees of freedom. It can be shown that, to first order, the required relationship is given by

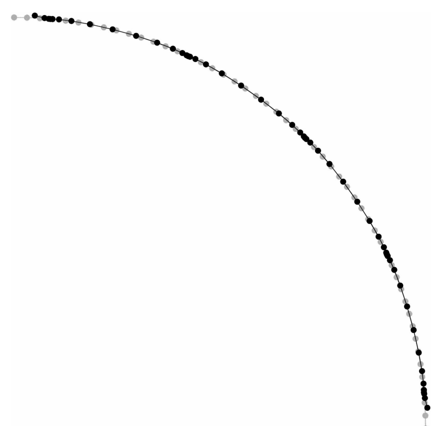
$$\mathbf{q}_r = -\mathbf{M}_{rr}^{-1}\mathbf{M}_{rt}\mathbf{q}_t \tag{1}$$

where $\mathbf{M}$ is the mass matrix associated with degrees of freedom $\mathbf{q}$ and the subscripts r and t represent radial and tangential dofs respectively (the rotational dofs are assumed to be associated with the radial wavefield). Equation (1) represents a modified form of Guyan reduction in which the radial slave dofs are related to the tangential master dofs by an inertial interpolation (rather than the stiffness interpolation $\mathbf{q}_r = -\mathbf{K}_{rr}^{-1}\mathbf{K}_{rt}\mathbf{q}_t$ employed in standard Guyan reduction). This modified reduction scheme is here termed 'mass reduction'. It is important to note that the mass reduction approach requires a consistent mass formulation in order to evaluate the inverse of the radial mass matrix partition.

The first six natural frequencies of the curved beam obtained using the mass reduction scheme are listed in Table 1. For comparison a detailed FE analysis was performed and the natural frequencies associated with modes dominated by tangential motion are also tabulated. The mass reduction approach clearly suppresses the radial modes of the beam. This can also be seen in Figure 3b where the 10^{th} mode shape obtained using the mass reduced model has been plotted. The mass reduction approach however also suppresses the rigid body motion of the beam and introduces an additional low frequency tangential mode. A modified reduction approach which reintroduces the rigid body motion using the constraint modes of the beam is currently under investigation. The tangential dynamic stiffness of the beam obtained using the mass reduced model is compared with that obtained using the detailed FE model in Figure 4. There are small discrepancies in the location of the first anti-resonance (due primarily to the additional tangential mode), however the reduced model provides a good estimate of the tangential dynamics of the beam at higher frequencies.



(a): Standard Guyan reduction.



(b): Mass reduction.

Figure 3: Comparison of mode shapes

Full FE model (all modes)	8.56	24.41	48.77	81.35	122.11	171.06
Guyan reduction	8.56	24.42	48.86	81.75	123.51	174.92
Mass reduction	732	1828	3368	4968	6597	8221
Full FE (tangential modes)	-	1825	3366	4968	6589	8220

Table 1: Natural frequencies of curved beam (in Hz) obtained using various reduction schemes.

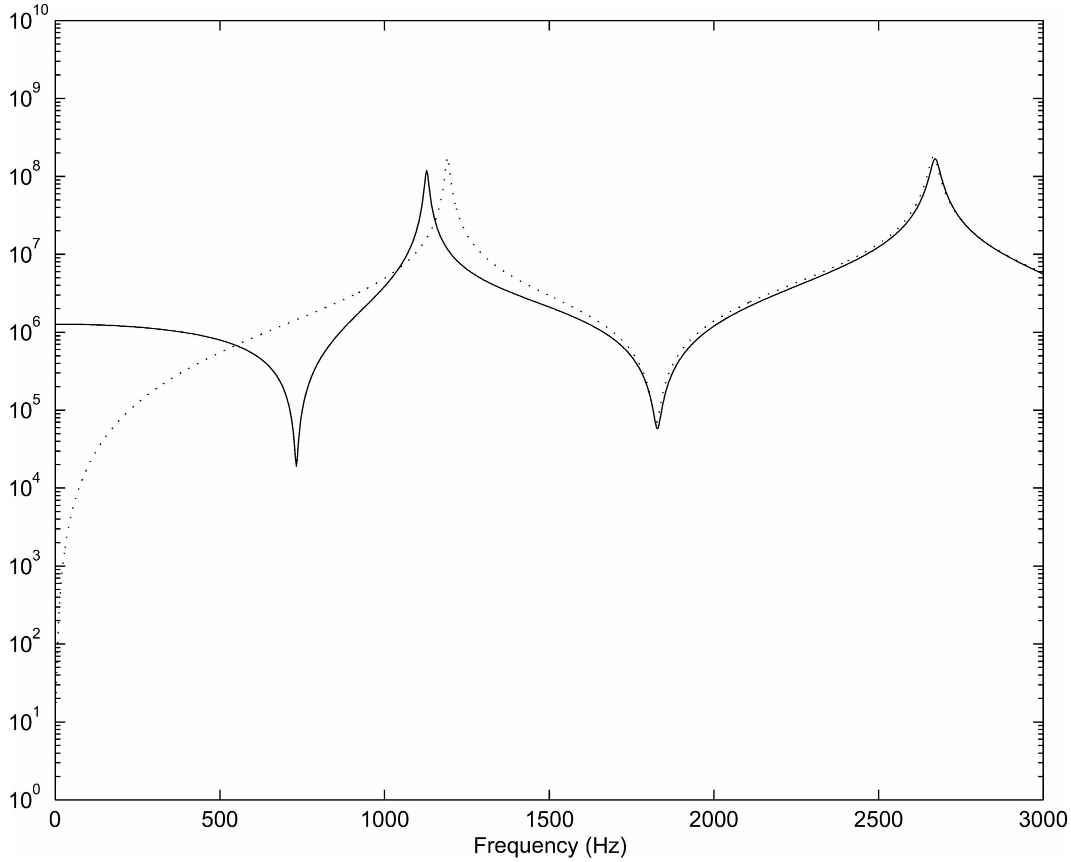


Figure 4: Comparison of D_{22} for curved beam; . . . detailed FE model; − Mass reduced FE model.

Density	2700 kg/m^3
Young's Modulus	71e9 Pa
Length	$\pi/2$ m
Cross section diameter	5e-3 m
Radius of curved beam	1 m
Loss factor	0.01

Table 2: Beam physical properties.

4 - CONCLUSIONS

This paper has described a mass based reduction scheme which can be used to suppress short wavelength motion within a finite element model. The examples presented in this paper were concerned with planar beam structures, however the technique is expected to be equally applicable to more general shell structures.

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