

**inter.noise 2000**

*The 29th International Congress and Exhibition on Noise Control Engineering  
27-30 August 2000, Nice, FRANCE*

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I-INCE Classification: 7.9

## DETERMINATION OF NEW EQUAL-LOUDNESS LEVEL CONTOURS BASED ON RECENT DATA WITH A LOUDNESS FUNCTION MODEL

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**Keywords:**

EQUAL-LOUDNESS LEVEL CONTOURS, LOUDNESS, LOUDNESS FUNCTION, ISO 226

**ABSTRACT**

A method of drawing equal-loudness level contours from discrete data points was developed based on a good knowledge of loudness perception. Four equations that express equal-loudness relation were deduced from four loudness functions, and an equation which best represents experimental results of equal-loudness relation was selected. Then, a new set of equal-loudness level contours was determined by fitting the equation to recent experimental data. The resultant equal-loudness level contours lie significantly above the contours of the international standard, ISO 226, in both low and high frequency regions.

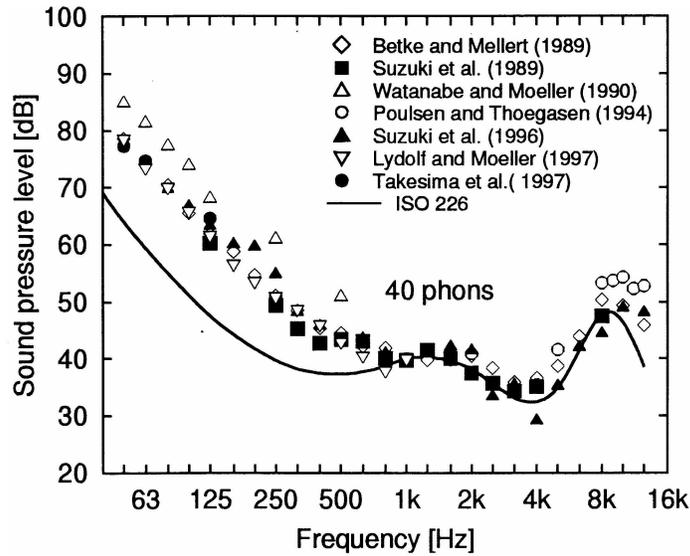
**1 - INTRODUCTION**

An equal-loudness level contour is the curve that ties up the sound pressure levels having equal loudness as a function of frequency. In other words, it is one of the expressions of frequency characteristic of loudness rating. Figure 1 shows the equal-loudness level contour at 40 phons specified in ISO 226, which is based on the proposal by Robinson and Dadson [1] published in 1956. In this figure, recent experimental data [2-9] for 40 phons obtained since 1989 are also plotted. Large discrepancies between the contour and the experimental data are clearly seen, and thus ISO 226 is now under full revision. The data obtained by the loudness-matching experiments are all discrete in phons and frequencies and include some errors. To draw equal-loudness level contours from such discrete data, therefore, interpolation and smoothing are inevitably required. In addition, to estimate reliable contours, it is important to apply good knowledge of loudness perception to the data processing. In this paper, a model for estimating equal-loudness level contours based on the knowledge of loudness functions is proposed and a set of new contours are presented.

**2 - MODEL FOR THE EQUAL-LOUDNESS RELATION****2.1 - Loudness function**

A loudness function represents the relation between sound intensity of a sound and its loudness. The most primitive loudness function for a pure tone was Stevens' power law [10]. However, since his loudness function cannot explain the loudness below 40 dB SPL, several modifications of that function have been proposed [11-14].

Atteneave [15] pointed out that there are two different processes in assessing loudness: one is a "loudness perception process" and the other is a "number assignment process." He also proposed a two-stage model in which the outputs of both processes are described by separate power transformations. Moreover, in



**Figure 1:** The equal-loudness level contour at 40 phons of ISO 226 (solid line) and recent experimental data.

an actual hearing system, the sound emitted from a sound source is transformed by a linear transfer function such as a head-related transfer function and transfer functions of the outer ear, the middle ear, and the anterior part of the inner ear. The linear transfer function describes a comprehensive transfer function between a sound source and the stage just before the loudness perception process.

Based on these ideas, the process of loudness rating consists of three parts: a linear transfer function part, a loudness perception part, and a number assignment part. Figure 2 shows a block diagram describing this model. Four loudness functions expressing those proposed by Stevens [10], Sharf and Stevens [11], Zwislöcki and Hellman [12] and Lochner and Burger [13], and Zwislöcki [14] were examined. They are expressed on the basis of the present ideas as follows:

Stevens:

$$n = b \left\{ c (UP)^{2\alpha} \right\}^{\beta} \quad (1)$$

Sharf:

$$n = b \left[ c \left\{ (UP)^2 - (UP_0)^2 \right\}^{\alpha} \right]^{\beta} \quad (2)$$

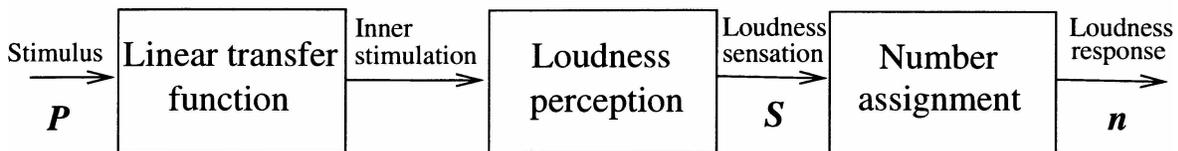
Zwislöcki, Lochner:

$$n = b \left[ c \left\{ (UP)^{2\alpha} - (UP_0)^{2\alpha} \right\} \right]^{\beta} \quad (3)$$

Zwislöcki:

$$n = b \left( c \left[ \left\{ (UP)^2 + C(UP_0)^2 \right\}^{\alpha} - \left\{ C(UP_0)^2 \right\}^{\alpha} \right] \right)^{\beta} \quad (4)$$

where  $n$  is the loudness for a pure tone,  $P$  is its sound pressure,  $P_0$  is the threshold of hearing in terms of sound pressure,  $U$  is an extended linear transfer function,  $c$  and  $\alpha$  are an extended dimensional constant and an exponent for the "loudness perception process," respectively, and  $b$  and  $\beta$  are those for the "number assignment process," respectively.  $C$  in Eq. (4) is the noise-to-tone energy ratio required for a just detectable tone embedded in intrinsic or masking noise.



**Figure 2:** A block diagram of a loudness-rating-process model.

## 2.2 - Equations describing the equal-loudness relation

When the loudness of a 1-kHz pure tone is equal to the loudness of an  $f$ -Hz pure tone, the following equations can be derived from the loudness functions shown above, respectively.

$$P_f^2 = \frac{1}{U_f^2} P_r^{2(\alpha_r/\alpha_f)} \quad (5)$$

$$P_f^2 = \frac{1}{U_f^2} (P_r^2 - P_{r0}^2)^{\alpha_r/\alpha_f} + P_{f0}^2 \quad (6)$$

$$P_f^2 = \frac{1}{U_f^2} \left\{ (P_r^{2\alpha_r} - P_{r0}^{2\alpha_r}) + (U_f P_{f0})^{2\alpha_f} \right\}^{1/\alpha_f} \quad (7)$$

$$P_f^2 = \frac{1}{U_f^2} \left[ (P_r^2 + C_r P_{r0}^2)^{\alpha_r} - (C_r P_{r0}^2)^{\alpha_r} + \left\{ C_f (U_f P_{f0})^2 \right\}^{\alpha_f} \right]^{1/\alpha_f} - C_f P_{f0}^2 \quad (8)$$

where  $P_f$  is the sound pressure of an  $f$ -Hz pure tone when its loudness is equal to that of a 1-kHz pure tone with sound pressure  $P_r$ ,  $P_{f0}$  and  $P_{r0}$  are the thresholds of hearing at the frequency of  $f$  Hz and 1 kHz, respectively.  $\alpha_f$  and  $\alpha_r$  are exponents for  $f$ -Hz and 1-kHz pure tones, respectively.  $U_f$  is a coefficient of the linear transfer function normalized at 1 kHz, that is,  $U$  at 1 kHz is set at 1. In those derivations it is assumed that the variables for the "number assignment process,"  $b$  and  $\beta$ , do not depend on the frequency. With these equations, the sound pressure level of an  $f$ -Hz pure tone whose loudness is equal to that of a 1-kHz pure tone can be calculated. That is, applying these equations enable us to the equal-loudness level contours.

## 3 - SELECTION OF THE MODEL EQUATIONS

The equation that best represents the equal-loudness relation should be selected from Eqs. (5)-(8). The differences between the four equations lie only below the loudness level of 40 phons. Therefore, a detailed experiment to measure equal-loudness levels was carried out for pure tones, especially at low loudness levels.

**Experiment:** Equal-loudness levels for a 125-Hz pure tone at loudness levels of 5, 10, 15, 20, 25, 30, 40, 50, and 70 phons were determined, as well as the thresholds of hearing. The randomized maximum likelihood sequential procedure [7] was used for the measurement of equal-loudness levels, and the bracketing method was used for the measurement of thresholds of hearing. This experiment was conducted in an anechoic room, and the stimulus sounds were given to a subject through a loudspeaker. The distance between the loudspeaker and the subject was 2 m. The number of subjects was eleven, and all of them had normal hearing.

**Results:** The results are shown in Fig. 3. The open circles show the average of equal-loudness levels, the filled circle shows the threshold of hearing, and the error bars show the 95% confidence intervals.

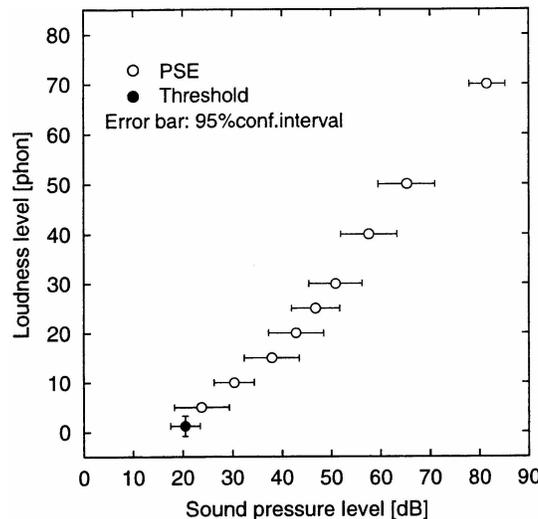
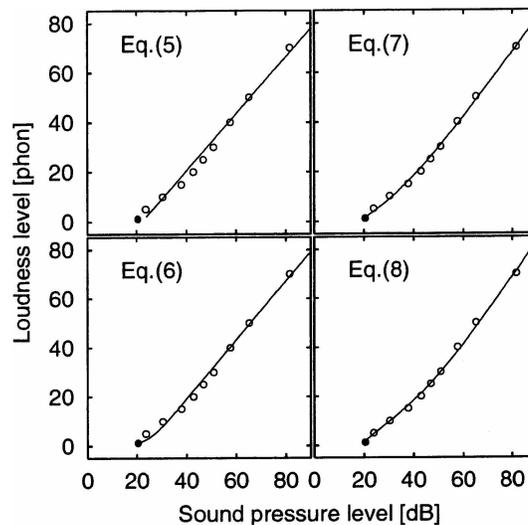


Figure 3: Equal-loudness levels of 125-Hz pure tone.

**Discussion:** To apply the equations, the exponent at 1 kHz should be determined. The typical value obtained by means of the AME method was 0.27 [13]. Loudness obtained by an AME experiment seems to be suitable for the output of the two-stage model. Thus, the exponent of 0.27 is adopted in this paper as the value, which corresponds to  $\alpha_r\beta$  in the equations. As for  $\beta$ , Zwislocki [16] found that  $\beta = 1.08$  by experiments. Therefore, the exponent at 1 kHz,  $\alpha_r$ , is assumed to be 0.25 ( $=0.27/1.08$ ) in this paper. The curves yielded by application of the four equations to the experimental data with the non-linear least squares method are drawn in Fig. 4. Differences can be seen among the curves below 40 phons. The residual sums of squares of the curves are 1.63, 1.13, 0.36 and 0.31 for Eqs. (5) to (8), respectively. The goodness of fit for Eqs. (7) and (8) is far better than that for Eqs. (5) and (6). The authors selected Eq. (7) for further consideration in this paper for the following reasons: First, Eq. (8) contains one more parameter than Eq. (7) and the estimation of  $C_f$  by use of recent experimental data was quite unstable in a preliminary calculation. Second, data of the threshold of hearing, which are stable, cannot be used for Eq. (8).



**Figure 4:** Comparison between the model equations fitted to the experimental data shown in Fig. 3.

#### 4 - ESTIMATION OF EQUAL-LOUDNESS LEVEL CONTOURS

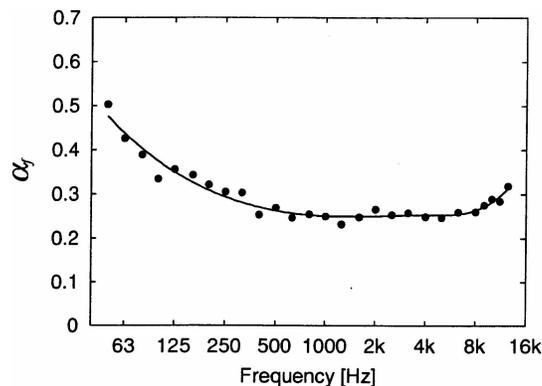
Equal-loudness level contours are estimated here by applying Eq. (7) to the recent data [2-9] between 50 Hz and 12.5 kHz. The procedure for estimating the equal-loudness level contours was as follows:

1. Thresholds of hearing measured by the recent studies were averaged and then smoothed by a cubic B-spline function. The result was used as  $P_{f0}$ .
2. Equation (7) was fitted to the experimental data at each frequency by the nonlinear least squares method to estimate  $\alpha_f$  and  $U_f$ . Reciprocal numbers of the standard errors were used as the weighting in the nonlinear least squares method. The obtained  $\alpha_f$ 's were smoothed by the cubic B-spline function, assuming that  $\alpha_f$  would not change abruptly as a function of frequency. The result is indicated as a solid line in Fig. 5.
3.  $U_f$ 's were then re-estimated by Eq. (7) using the smoothed  $\alpha_f$  for each frequency. The re-estimated  $U_f$ 's were smoothed by the cubic B-spline function in log-log coordinates as shown by the solid line in Fig. 6.
4. A set of equal-loudness level contours was drawn by using Eq. (7) for the whole frequency range with the smoothed and interpolated  $P_{f0}$ ,  $\alpha_f$  and  $U_f$ .

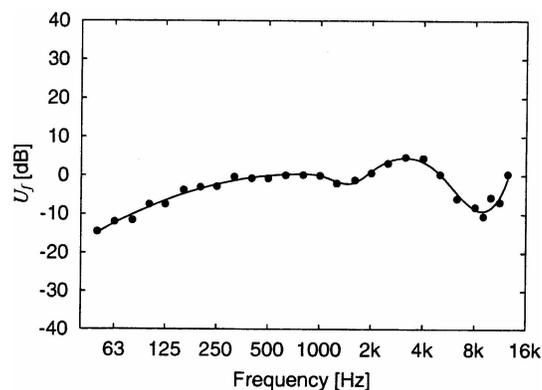
The equal-loudness level contours obtained here are illustrated in Fig. 7. Figure 8 shows a comparison between the contours and those given in the present ISO 226. Estimated contours lie significantly above the contours of ISO 226 in both low and high frequency regions.

#### 5 - CONCLUSION

A method for drawing equal-loudness level contours from discrete data points based on the loudness function was proposed, and equal-loudness level contours were actually estimated by applying the method



**Figure 5:** Estimated  $\alpha_f$  by the method of non-linear least squares; solid line shows a smoothed line by a cubic B-spline function.



**Figure 6:**  $U_f$ 's translated into dB re-estimated by using the interpolated  $\alpha_f$ , i.e., the solid line in Fig. 5; solid line shows a smoothed line by a cubic B-spline function.

to recently obtained experimental data. This method based on application of a good knowledge on loudness perception provides a reasonable way to estimate equal-loudness level contours from limited experimental data.

#### ACKNOWLEDGEMENTS

This study was partly supported by a Grant-in-Aid from the Ministry of Education, Science, Culture and Sports of Japan, No. 08650073. A part of this study was supported by the NEDO International Joint Research Grant Program.

#### REFERENCES

1. **D. W. Robinson and R. S. Dadson**, A re-determination of the equal-loudness relations for pure tones, *Br. J. Appl. Phys.*, Vol. 7, pp. 166-181, 1956
2. **K. Betke and V. Mellert**, New measurements of equal-loudness level contours, In *Inter-noise 89*, pp. 793-796, 1989
3. **S. Suzuki, Y. Suzuki, S. Kono, T. Sone, M. Kumagai, H. Miura and H. Kado**, Equal-loudness level contours for pure tone under free field listening conditions (I) - Some data and considerations on experimental conditions -, *J. Acoust. Soc. Jpn. (E)*, Vol. 10, pp. 329-338, 1989
4. **H. Fastl, A. Jaroszewski and E. Zwicker**, Equal loudness contours between 100 and 1000 Hz for 30, 50, and 70 phon, *Acustica*, Vol. 70, pp. 197-201, 1990
5. **T. Watanabe, and H. Møller**, Hearing threshold and equal loudness contours in free field at frequencies below 1 kHz, *J. Low Frequency Noise and Vibration*, Vol. 9, pp. 135-148, 1990

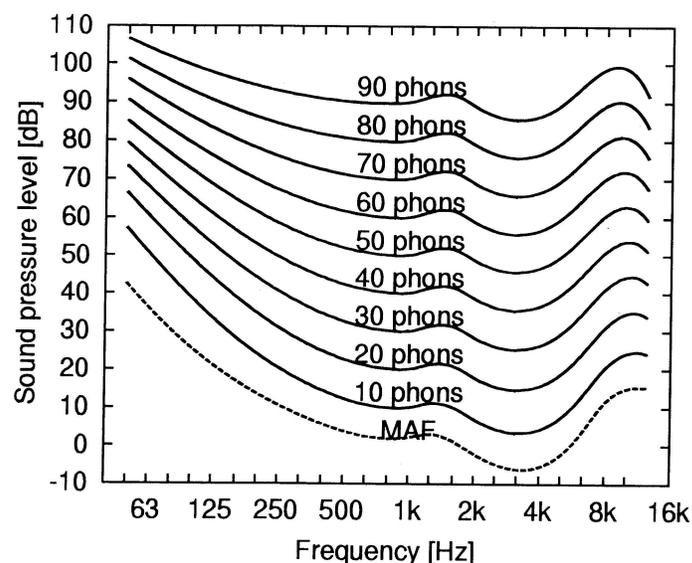
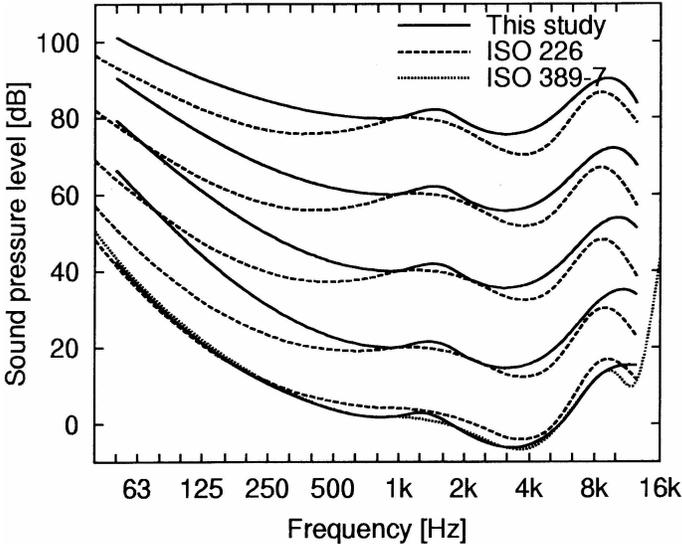


Figure 7: Estimated equal-loudness level contours.

6. **T. Poulsen, and L. Thøgersen**, Hearing threshold and equal loudness level contours in a free sound field for pure tones from 1 kHz to 16 kHz, In *Nordic Acoust. Meeting*, pp. 195-198, 1994
7. **Y. Suzuki, H. Takeshima, H. Fujii, M. Kumagai and T. Sone**, *Equal-loudness level measurement by the randomized maximum likelihood sequential procedure*, ISO/TC 43/WG 1 N246, pp. 1-9, 1996
8. **M. Lydolf, H. Møller**, New measurements of the threshold of hearing and equal-loudness contours at low frequencies, In *The 8th International meeting on Low Frequency Noise and Vibration*, pp. 76-84, 1997
9. **H. Takeshima, Y. Suzuki, M. Kumagai, T. Sone, T. Fujimori and H. Miura**, Equal-loudness level measured with the method of constant stimuli -Equal-loudness level contours for pure tone under free-field listening conditions (II)-, *J. Acoust. Soc. Jpn. (E)*, Vol. 18, pp. 337-340, 1997
10. **S. S. Stevens**, On the psychophysical law, *Psychological Review*, Vol. 64, pp. 153-181, 1957
11. **B. Sharf and J. C. Stevens**, The form of the loudness function near threshold, In *Int. Congr. Acoust., 3rd, Stuttgart, 1959, L. Cremer Ed. (Elsevier Publishing Co., Inc., Amsterdam)*, pp. 80-82, 1961
12. **J. J. Zwislocki and R. P. Hellman**, On the psychophysical law, *J. Acoust. Soc. Am.*, Vol. 32, pp. 924, 1960
13. **J. P. A. Lochner and J. F. Burger**, Form of the loudness function in the presence of masking noise, *J. Acoust. Soc. Am.*, Vol. 33, pp. 1705-1707, 1961
14. **J. J. Zwislocki**, *Analysis of some auditory characteristics*, Handbook of mathematical psychology, R. D. Luce, R. R. Bush, and E. Galanter, Wiley, New York, pp. 1-97, 1965
15. **F. Atteneave**, *Perception and related areas*, A study of science, 4, S. Koch Ed., McGraw Hill, New York, 1962
16. **J. J. Zwislocki**, Group and individual relations between sensation magnitudes and their numerical estimates, *Perception & Psychophysics*, Vol. 33, pp. 460-468, 1983



**Figure 8:** Comparison between equal-loudness level contours specified in ISO 226 (broken lines) and those estimated in this study (solid lines).