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USE OF RECIPROCITY PRINCIPLE IN A HYBRID MODELING TECHNIQUE (HMT) BASED ON INVERSE BOUNDARY ELEMENT METHOD (IBEM) FOR THE DETERMINATION OF THE OPTIMAL SPECTRAL CHARACTERISTICS OF A COMPLEX RADIATING NOISE SOURCE

M.A. Hamdi*, H. Defosse*, F. Damagnez*, T. Beauvilain*, P. Varet****

* STRACO, SA/Université de Technologie de Compiègne, 20 Rue du Fonds Pernants, Zac de Mercieres N°3, 60471, Compiègne, France

** Renault, IDVU, 42, route de Beynes, 78640, Villiers Saint Frederic, France

Tel.: 0033 (0)344304360 / Fax: 0033(0)344868777 / Email: hamdi@straco.fr

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ABSTRACT
A robust hybrid modeling technique (HMT) based on near field acoustic measurements combined with numerical results derived from an Inverse Boundary Element Method (IBEM) is presented. The developed hybrid technique takes advantage from reciprocity principle to efficiently determine the optimal spectral characteristics of very complex noise sources, typically encountered in transportation industries. The first section of the paper reminds the direct link between reciprocity principle and integral representations of the acoustic field radiated by a noise source at finite number of discrete points located outside a closed surface enveloping the source. The second section describes the parameters chosen to describe the radiating source and the numerical optimization procedure used to determine the optimal spectral characteristics of source parameters. The third and last section of the paper gives successful practical applications of the method in automobile industry to characterize the noise radiated by a car engine.

1 - INTEGRAL REPRESENTATION OF THE RADIATED ACOUSTIC PRESSURE

The acoustic pressure radiated by a complex acoustic source admits the following integral representation in the acoustic domain W_a occupying the exterior of a surface S,

$$p\left(x\right) = -\int_S \mu\left(y\right)\frac{\partial G\left(x,y\right)}{\partial n\left(y\right)}dS\left(y\right) \tag{1}$$

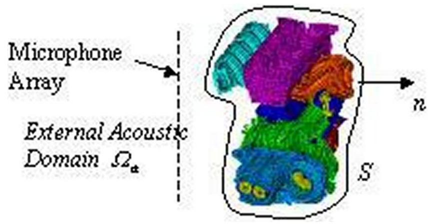
where, μ represents the density of a double layer potential distributed on the fictive surface S enveloping the complex acoustic source as shown in figure 1-a. The Green's function

$$G\left(x,y\right) = -\exp\left(ikR\left(x,y\right)\right)/\left(4\pi R\left(x,y\right)\right)$$

corresponds to the fundamental solution of the harmonic (time dependence $\exp\left(-i\omega t\right)$) acoustic wave equation in free field conditions, and where $R(x,y)$ is the distance separating two points x and y located in the external acoustic domain.
The density μ is related to the normal component γ of the acoustic acceleration of the surface S by the Fredholm integral equation of first kind,

$$\rho\gamma\left(x\right) = \int_S \mu\left(y\right)\frac{\partial^2 G\left(x,y\right)}{\partial n\left(x\right)\partial n\left(y\right)}dS\left(y\right) \tag{2}$$

where ρ is the mass density of the fluid, $k = \omega/c$ the acoustic wave number, ω the circular frequency and c is the sound velocity in the acoustic domain.



(a): Noise source enveloped by a fictive surface S.

(b): Experimental set up of a car engine.

Figure 1: IBEM model and experimental set up.

The integral equation (2) could be solved by a Boundary Element Method (BEM), which leads to the variational equation [1] satisfied by the density μ for any trial function μ' defined on S,

$$D\left(\mu',\mu\right) = C\left(\mu',\gamma\right) \tag{3}$$

where,

$$D\left(\mu',\mu\right) = \int_{S\times S} \mu'\left(x\right)\mu\left(y\right)\frac{1}{\rho}\frac{\partial^2 G\left(x,y\right)}{\partial n\left(x\right)\partial n\left(y\right)}dS\left(y\right)dS\left(x\right) \tag{4}$$

is the surface acoustic admittance operator and,

$$C\left(\mu',\gamma\right) = \int_{S} \mu'\left(x\right)\gamma\left(x\right)dS\left(x\right) \tag{5}$$

the coupling operator associated to the normal component γ of the surface acceleration.
The discrete algebraic system associated to the variational equation (3) is given by,

$$\mathbf{D}\vec{\mu} = \mathbf{C}\vec{\gamma} \tag{6}$$

where, $\vec{\mu}$ is the density vector defined on S and $\vec{\gamma}$ is the vector of the normal accelerations defined at the center of gravity of the boundary elements.
The integral representation (1) leads to the following algebraic equation,

$$P_k = -\mathbf{U}^t\mathbf{C}\vec{\mu} \tag{7}$$

where P_k is the acoustic pressure measured by the microphone (k) located at x_k, and where each component U_j of the vector $\mathbf{U}$ appearing in equation (7) is given by,

$$U_j = \frac{\partial G\left(x_k,y_j\right)}{\partial n_{y_j}} \tag{8}$$

The component U_j represents the normal derivative at the center of gravity y_j of the boundary element e_j, of the acoustic incident field induced by an unitary point source placed at the microphone location x_k. Eliminating the vector $\vec{\mu}$ from (6) and substituting in to (7) gives,

$$P_k = -\mathbf{b}_k^t\mathbf{q} \tag{9}$$

where,

$$\mathbf{b}_k = \mathbf{D}^{-1}\mathbf{C}^t\mathbf{U} \tag{10}$$

corresponds to the 'blocked' acoustic pressure vector induced by an unitary point source placed at x_k. The surface S is considered as rigid. The vector,

$$\mathbf{q} = \mathbf{C}^t\vec{\gamma} \tag{11}$$

represents the acceleration flux at nodes of the surface S.
Equation (9) shows that the acoustic pressure at the microphone x_k is given by the inner scalar product between the blocked pressure vector induced by an unitary point source placed

at the microphone location x_k and the acoustic acceleration flux vector q of the surface S. So equation (9) simply translates the reciprocity principle for acoustics.
Finally the acceleration flux vector q could be chosen as the state vector characterizing the complex acoustic source occupying the interior of the envelope surface S.
For an array of M microphones, the vector P of acoustic pressures is given by,

$$\mathbf{P} = -\mathbf{B}^t \mathbf{q} \tag{12}$$

where each column k of the reciprocal transfer matrix B corresponds to the blocked pressure induced on the surface S by an unitary point source placed at the microphone location x_k.

2 - INVERSE TECHNIQUE FOR THE COMPUTATION OF THE ACCELERATION FLUX VECTOR

The spectral characteristics of the state flux vector $\mathbf{q}$ should be deduced from the spectral characteristics of the acoustic pressure vector $\mathbf{P}^{\text{measure}}$ of dimension M measured by the microphone array (M microphones). The developed method consists of representing the state flux vector $\mathbf{q}$ on an ad hoc matrix basis $\mathbf{Q}$ composed by m wave-envelope vectors corresponding to the eigen-vectors of a Surface-Laplace operator defined on S. The dimension m should remain small compared to the number M of microphones ($m<<M$), such that:

$$\mathbf{q} = \mathbf{Q}\vec{\alpha} \tag{13}$$

where the vector $\vec{\alpha}$ of dimension m represents the generalized components of the state flux vector $\mathbf{q}$ on the basis $\mathbf{Q}$. Substituting (13) into equation (12) gives,

$$\mathbf{P} = -\mathbf{H}\vec{\alpha} \tag{14}$$

where the rectangular matrix,

$$\mathbf{H} = -\mathbf{B}^t \mathbf{Q} \tag{15}$$

of dimension ($M \times m$), corresponds to the generalized ***Reciprocal Transfer Matrix*** (RTM) relating the pressure vector $\mathbf{P}$ at the microphone array to the generalized components (vector $\vec{\alpha}$ of the state vector $\mathbf{q}$). The inverse acoustic problem [2] consists of searching the components of the vector $\vec{\alpha}$ which minimize the quadratic cost function,

$$\mathbf{J}(\omega) = \|\mathbf{H}\vec{\alpha} - \mathbf{P}^{\text{me asure}}\| \tag{16}$$

The minimization of the cost function $\mathbf{J}(\omega)$ with respect to $\vec{\alpha}$ (in the Least Mean Square sense) leads to,

$$\vec{\alpha} = \mathbf{H}^+ \mathbf{P}^{\text{measure}} \tag{17}$$

where the rectangular ($m \times M$) matrix,

$$\mathbf{H}^+ = \left[\mathbf{H}^h \mathbf{H}\right]^{-1} \mathbf{H}^h \tag{18}$$

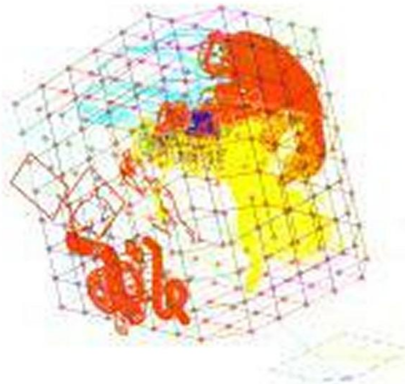
is the pseudo-inverse of the reciprocal transfer matrix $\mathbf{H}$, and where the symbol h corresponds to the complex conjugate of a transposed matrix.
From (17) the spectral matrix $\mathbf{S}_{\alpha\alpha}$ relative to the vector $\vec{\alpha}$ is related to the spectral matrix $\mathbf{S}_{\text{pp}}$ of the measured pressure vector $\mathbf{P}^{\text{measure}}$ by the matrix equation,

$$\mathbf{S}_{\alpha\alpha} = \mathbf{H}^+ \mathbf{S}_{pp} \mathbf{H}^{+h} \tag{19}$$

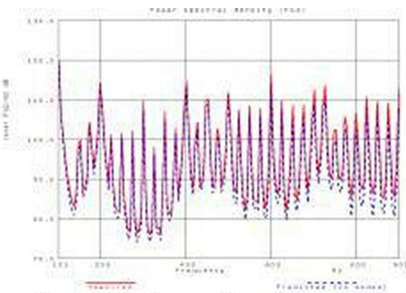
Knowing $\mathbf{S}_{\alpha\alpha}$, it is very easy from equation (13) to compute the spectral matrix $\mathbf{S}_{\text{qq}}$ of the acceleration flux vector $\mathbf{q}$, then the predicted spectral matrix ($\mathbf{S}_{\text{pp}}$) estimated by the model using the matrix relation (14).

3 - RESULTS

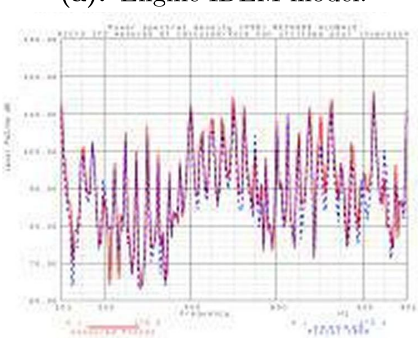
The proposed Inverse Boundary Element Method (IBEM) has been applied to characterize a car engine in the low frequency band $f < 1000$Hz. Figure 1-b shows the experimental set up where a microphone array (60 microphones) is placed on the top of a running car engine at 3000 rpm. The microphone array has been placed on right, left, top and front sides of the engine. Figure 2-a shows the IBEM model corresponding to the fictive surface enveloping the car engine which is nearly a closed box meshed with approximately 250 quadrangular boundary elements. Figure 2-b shows the comparison between measured and predicted averaged sound pressure levels (240 microphones), where 20 eigen-functions (wave-envelopes) have been used to represent the acceleration flux through the enveloping surface S. Figure 2-c shows the comparison between measured and predicted sound pressure levels at the position of a microphone not included in the global cost function. The error between measured and predicted results remains less than 2 dB. This confirms the robustness of the proposed inverse method to predict and extrapolate the acoustic pressure radiated by the engine. Finally Figure 2-d shows the measured and the predicted acoustic pressures radiated by the engine at 600 Hz.



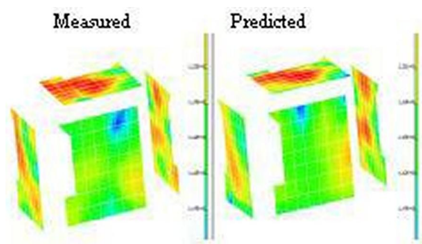
(a): Engine IDEM model.



(b): Average sound pressure level.



(c): Local sound pressure level.



(d): Radiated acoustic pressure at 600 Hz.

Figure 2: Results.

4 - CONCLUSION

A hybrid inverse boundary element method based on the exploitation of the reciprocity principle in acoustics combined with a wave envelope technique has been developed and implemented by STRACO in RAYON^{R}-IBEM solver of I-DEAS-Vibro-Acoustics^{TM}. This method has been successfully applied to characterize a real car engine as a distributed noise source which could be used to excite the car body and predict either interior and exterior noise (pass by noise). This method which has a big potential of applications, could be advantageously used to characterize and identify noise sources in transportation industries [3].

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