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ON THE USE OF PROCESSING VIBRATING VELOCITY IN WAVENUMBER DOMAIN FOR COMPUTATION OF SOUND RADIATION OF A PLANAR PLATE

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ABSTRACT

Sound radiation from planar radiators can be computed by processing the vibrating velocity in wavenumber domain. As Fourier transform is applied to a finite aperture, wavenumber processing is often accompanied with distortions, resulting errors in calculations. The objective of this paper is to show some techniques to solve the problems encountered in wavenumber processing. Two modified Green function in k -space are given and compared. Sound radiation fields parallel and perpendicular to the plate plane can respectively be obtained from normal velocity measured with a scanning laser vibrometer by k -space signal processing and forward sound field propagation. Investigations show that wavenumber-processing technique is of efficiency in the calculation of sound radiation from vibrating velocity.

1 - INTRODUCTION

It has been shown that structural intensity and its divergence used for an analysis of energy flow can be computed from the distribution of a vibrating complex velocity measured by the holographic interferometry or scanning laser vibrometry techniques using wave number processing [1]. The present work was motivated by a need for a software *VIPART* [2] to evaluate a sound radiation field of a planar source by processing experimental vibrating velocity in wavenumber domain. Helmholtz integral that is the solution of the wave equation in a sound field by an arbitrary source is a fundamental formula for computing the sound radiation from vibrating sources. This method, however, is time consuming. The Fast Fourier transform was used for evaluation of the radiation field near a thin plate source [3]. It is shown that this method increases the speed of computations with respect to the calculation of the integral. However, the discrete Fourier transform can results bias (aliasing) errors in results of sound fields. The significant bias errors are due to a singularity containing in the k -space Green function. To remedy the bias errors, two modified Green functions were used, the averaged Green function [3] and the Green function with the complex acoustic wave number [4]. This work shows the application of the technique of wavenumber processing in the computation of sound radiation fields from normal vibrating velocity. An equivalent condition for the two types of the modified Green function is derived. From the normal velocity measured by scanning vibrometry technique, radiation fields such as the sound pressure, the intensity, are calculated using wavenumber processing together with NAH. Examples of results are given.

2 - TECHNIQUES OF WAVENUMBER PROCESSING

2.1 - Basic formulations

Calculation of sound radiation from the distribution of the normal velocity on an infinite plane using wavenumber processing is based on the angular spectrum form of the radiation integral involving the Green function that satisfies the Neumann boundary conditions. It is written as [3]

$$P(k_x, k_y, d) = jk\rho_0cV_n(k_x, k_y)G_v(k_x, k_y, d), \quad p(x, y, d) = F^{-1}[P(k_x, k_y, d)] \quad (1)$$

where $P(k_x, k_y, z)$ and $V_n(k_x, k_y)$ are respectively the Spatial Fourier Transform (SFT) of the sound pressure and the normal vibration velocity in the plane. F^{-1} is the inverse Fourier transform. $p(x, y, d)$ is the acoustic pressure in the plane $z = d$. $G_v(k_x, k_y, d)$ is the SFT of the Green function and is given by

$$G_v(k_x, k_y, d) = -j\exp(-jk_z d)/k_z, \quad k_z = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2} & k^2 \geq k_x^2 + k_y^2 \\ -j\sqrt{k_x^2 + k_y^2 - k^2} & k^2 \leq k_x^2 + k_y^2 \end{cases} \quad (2)$$

The recent techniques of the holographic interferometry or scanning laser vibrometry allow the normal velocity in the plane to be obtained experimentally [1,2]. The pressure field in any plane in front of the source plane can be found from the knowledge of the normal velocity in the plane $z = 0$ using Eq. (1). The sound pressure and the vector of the particle velocity $\mathbf{v}(x, y, z)$ in any plane can be obtained using forward propagation in the wavenumber domain, which is then used to compute the energetic quantities such as the active sound intensity $\mathbf{I} = \text{Re}p\mathbf{v}/2$.

2.2 - Modified Green functions

Processing in k -space is the use of a finite discrete Fourier transform. Errors associated with the discrete Fourier transform such as the bias caused by the replicated sources and by the singularity in transform space distort results [3]. Eq. (2) shows that $G_v(k_x, k_y, d)$ has very large values at $k_z=0$ whenever the radiation circle $k_x^2 + k_y^2 = k^2$ coincides with a mesh point. These errors can be reduced by modification of Green function in wave number space $G_v(k_x, k_y, z)$ [3], [7]. An averaged Green function proposed by Williams [3] is obtained by the calculation of the integration over a square area of length Δk centered on the mesh point. By assuming that the velocity spectral $V(k_x, k_y)$ is slowly varying, the averaged Green function over a square area of length Δk centered on the mesh point is expressed as ($d = 0$).

$$\bar{G}_v(k_x, k_y, 0) = \begin{cases} -j2 \left(\sqrt{k^2 - k_1^2} - \sqrt{k^2 - k_2^2} \right) / (k_2^2 - k_1^2), & k_2 \leq k_1 \\ 2 \left(-j\sqrt{k^2 - k_1^2} + \sqrt{k_2^2 - k^2} \right) / (k_2^2 - k_1^2), & k_1 \leq k \leq k_2 \\ 2 \left(\sqrt{k_1^2 - k^2} - \sqrt{k_2^2 - k^2} \right) / (k_2^2 - k_1^2), & k \leq k_1 \end{cases} \quad (3)$$

where $k_1 = k_r - \Delta k/2$, $k_2 = k_r + \Delta k/2$, $k_r^2 = (m^2 + n^2)\Delta k^2$, ($m, n = -N+1, -N, \dots, 0, 1, 2, \dots, N-1, N$). $\Delta k = 2\pi/L$, L is the aperture window. At the point ($k_x=0, k_y=0$), $\bar{G}_v(0, 0, 0) = -j/k$. The second modified Green function is to add damping in the acoustic wave number, $k_\alpha = k(1 - j\alpha)$, with $0 < \alpha < 1$, where α is the attenuation coefficient. Substituting k_α in Eq. (2) yields:

$$G_v(k_x, k_y, d) = -j\exp\left(-jd\sqrt{k_\alpha^2 - k_x^2 - k_y^2}\right) / \sqrt{k_\alpha^2 - k_x^2 - k_y^2} \quad (4)$$

The effect of adding damping factor α in the acoustic wave number is to smooth the Green function in k -space as well as to avoid the singularity in Eq. (2), resulting in a point source losses its energy as its sound field propagated out. The averaged Green function can be made equivalent to the Green function of Eq. (4). The equivalent condition is obtained by considering the following expression,

$$\frac{2}{(k_2^2 - k_1^2)} \left[-j\sqrt{k^2 - k_1^2} + \sqrt{k_2^2 - k^2} \right] = \frac{-j}{\sqrt{k_\alpha^2 - k_r^2}} \quad (5)$$

substituting $k_r=k$ and neglecting terms $(\Delta k/k)^2$ in Eq. (5) yields the equivalent condition $\alpha = \Delta k/4k$. It is shown that α depends on the sampling number. Figure 1 shows the nonaveraged Green function, the averaged Green function and the Green function with the complex acoustic wave number where $k = 25 \approx k_r(20, 20)$ rad/m ($k_r = \sqrt{k_x^2 + k_y^2}$). It is shown that both the averaged Green function and that with the complex wavenumber have smoothing effects on the radiation circle so that they avoid from the singularity occurring in the nonaveraged Green function.

3 - EXAMPLES OF SOUND RADIATIONS FROM A PLANAR SOURCE

For the acquisitions, a scanning vibrometer was used, together with LMS software. It uses a OFV 300 optical head from Polytec, which comprises a single point interferometer with a motor-driven control focusing device. Two galvo-driven mirrors direct the laser beam horizontally and vertically. It allows one

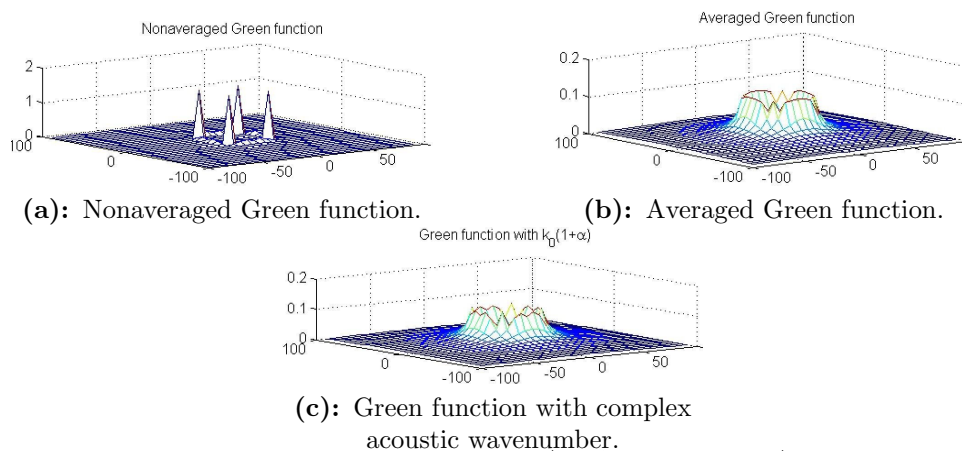


Figure 1: $\alpha=0.055$, $k=25$ rad/m, $\Delta k=6.0868$ rad/m.

to obtain easily the 1024 (32×32) measurement points with a high resolution on a large frequency range (see Figure 2c). The plate dimension is 350 mm \times 200 mm and its thickness is 3 mm. A pressure field was calculated from the vibrating velocity using Eq. (1). The averaged Green function was used in the evaluation of the pressure spectrum to solve the problem of singularity in Eq. (2). The acoustic radiation field in the plane perpendicular to the vibrating plate is obtained by successively forward propagating sound pressure of the plate to one's prescribed planes. This work was done using a user-friendly acoustic holography software [5] which provides one with the choices of k-space windows and the methods of wavenumber processing such as zero padding and mirror techniques [2] to reduce bias errors caused by direct discrete FFT. The square of amplitude of sound pressure and active intensity on a plane (x, z) at $y_p=0.14$ cm perpendicular to the vibrating plane are respectively shown in Figure 2a and Figure 2b. The energy flow can easily be analysed by these representations.

4 - CONCLUSION

From the knowledge of the normal velocity which can be obtained using the holographic interferometry and scanning laser vibrometry, the techniques of processing in wavenumber domain is a very efficient method, allowing one to compute structural intensity and divergence, the radiation sound fields such as sound pressure and velocity, sound intensity and the far-field directivity. To reduce the errors due to discrete Fourier transform in the calculation of sound radiation fields, two modified Green functions were given. Their equivalent conditions are evaluated. This allows us to use one of them in the calculation of sound radiation field. Examples show that 3-dimensional sound radiation fields can be obtained from normal vibrating velocity using wavenumber processing together with the technique of forward sound field propagation.

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REFERENCES

1. **J.-C. Pascal, J.-F. Li, X. Carniel**, Wavenumber processing techniques to determine structural intensity and its divergence from optical measurements without leakage effects, In *4th Int. Conf. on Vibration Measurements by Laser Techniques, (Proc. SPIE), Ancona (Italy), 2000*
2. **X. Carniel, J.-C. Pascal, J.-F. Li**, Structural Intensity and Acoustic Radiation from Optical Measurements: the VIPART Software, In *4th Int. Conf. on Vibration Measurements by Laser Techniques, (Proc. SPIE), Ancona (Italy), 2000*
3. **E. G. Williams, J.D. Maynard**, Numerical evaluation of the Rayleigh integral for planar radiators using the FFT, *Journal of the Acoustical Society of America*, Vol. 72 (6), pp. 2020-2030, 1982
4. **H. Fleisher, V. Axelrad**, Fourier acoustics: An approach to acoustic-field analysis, *Acustica*, Vol. 57 (2), pp. 51-61, 1985
5. **J.C. Pascal, J.-F. Li**, *Acoustic Holography Toolbox on Matlab*, 1999

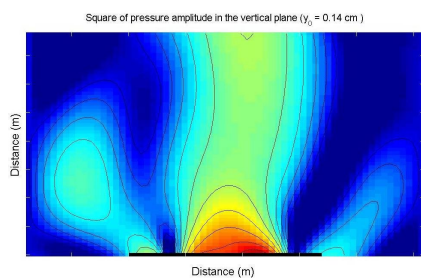


Figure 2(a): Square of amplitude of sound pressure.

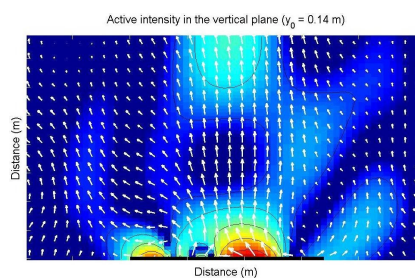


Figure 2(b): Active intensity in the plane vertical to the vibrating plane reconstructed using forward sound field propagation from the normal velocity at $f = 1160$ Hz.



Figure 2(c): Experimental set-up for technique of scanning laser vibrometry.