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NOVEL MIXED FINITE ELEMENT FORMULATION FOR THE ANALYSIS OF SOUND ABSORPTION BY POROUS MEDIA

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ABSTRACT
This paper presents a novel mixed finite element formulation very well adapted for the analysis of the absorption of elastic and acoustic waves propagating in porous or fibrous media. The proposed formulation is based on modified Biot's equations [1] written in terms of the skeleton displacement *(U)* and the acoustic pressure p in the pores of an inhomogeneous media. It generalizes the previous formulation proposed by Professor Atalla & Co. [2], and has the great advantage of automatically satisfying all boundary conditions without having to compute surface coupling integrals at porous sub-domain interfaces. When elastic forces in the skeleton are neglected, the formulation automatically degenerates to an equivalent fluid model taking into account inertial coupling with the skeleton. In addition the present formulation has the advantage over existing formulations [3] of reducing the number of degrees of freedom per node (dof/node) to four (4) and only to one (1) when elastic forces are negligible in the skeleton. The implementation of this new generalized formulation and associated equivalent fluid finite element model in RAYON-PEM Solver developed and integrated by STRACO in the graphical environment of I-DEAS Vibro-Acoustics, solves several practical problems in transportation industry. It is shown in this paper that the results obtained with the new formulation agrees very well with the experimental results [4] using impedance tube and vibro-acoustic measurements on treated multi-layered structures.

1 - MODIFIED BIOT'S EQUATIONS

Propagation of elastic and acoustic waves in porous media are governed by Biot's equations [1], which could be written in the following form for time harmonic waves (time dependence $\exp(-i\omega t)$):

$$\frac{\partial\left(\sigma_{kl}^{s}\right)}{\partial x_l} = -\omega^2\left(\tilde{\rho}_{11}U_k^s + \tilde{\rho}_{12}U_k^f\right) \tag{1.1}$$

$$\frac{\partial\left(-\phi p \delta_{kl}\right)}{\partial x_l} = -\omega^2\left(\tilde{\rho}_{12}U_k^s + \tilde{\rho}_{22}U_k^f\right) \tag{1.2}$$

where, ω is the circular frequency, U_k^s and U_k^f represent respectively the skeleton and fluid displacement components. The mass density coefficients appearing in equations (1.1) and (2.2) are given by the following formulas:

$$\tilde{\rho}_{11} = (1-\phi)\,\rho_s - (1-\tau)\,\phi\rho_f - \phi b/j\omega \tag{2.1}$$

$$\tilde{\rho}_{22} = \tau\phi\rho_f - \phi b/j\omega \tag{2.2}$$

$$\tilde{\rho}_{12} = (1-\tau)\,\phi\rho_f + \phi b/j\omega \tag{2.3}$$

where ϕ is the porosity, τ the turtuosity, ρ_s and ρ_f are the mass density of the skeleton material and of the interstitial fluid. The coefficient b represents the viscous coupling between solid and fluid phases given by [1],

$$b = \phi\sigma\sqrt{1 - 4j\omega\eta\rho_f\left(\frac{\tau}{\Lambda\phi\sigma}\right)} \tag{3}$$

where σ is the flow resistivity, η is the viscosity of the interstitial fluid and Λ is the viscous characteristic length of the media. σ^s_{kl} and $\sigma^f_{kl} = -\phi p\delta_{kl}$ represent the components of the stress tensors respectively in the skeleton and in the interstitial fluid where p is the pressure. The stress tensor components are related to the strain tensor components by the constitutive laws of the porous media,

$$\sigma^s_{kl} = \left(Adiv\left(U^s\right) + Qdiv\left(U^f\right)\right)\delta_{kl} + G\left(\frac{\partial U^s_k}{\partial x_l} + \frac{\partial U^s_l}{\partial x_k}\right) \tag{4.1}$$

$$-\phi p = Qdiv\left(U^s\right) + Rdiv\left(U^f\right) \tag{4.2}$$

where, G is the skeleton shear modulus, Q is the coupling modulus, A and R are the first Lamé coefficient and the bulk modulus of the porous media. Coefficient A, Q and R are related to the bulk modulus K_s of the skeleton material, the bulk modulus K_b of the skeleton in vacuum and to the bulk modulus K_f of the interstitial fluid by the formulas given in reference [1].
The relative displacement vector $(U^f - U^s)$ is given from equation (1.2) by,

$$U^f - U^s = \frac{1}{\omega^2\tilde{\rho}_f}grad\left(\phi p\right) - \tilde{\beta}U^s \tag{5}$$

where $\tilde{\beta} = (1 + \tilde{\rho}_{12}/\tilde{\rho}_{22})$ is the inertial coupling factor. Modified Biot's equations are derived by substituting constitutive laws (4.1) and (4.2) into equation (1.1) and into the divergence of equation (5):

$$\tilde{\rho}_s\omega^2U^s + div\left(\tilde{\sigma}^s_{kl}\left(U^s\right)\right) - \tilde{\alpha}\phi p\delta_{kl} \tag{6.1}$$

$$div\left(\frac{1}{\omega^2\tilde{\rho}_f}grad\left(\phi p\right) - \tilde{\beta}U^s\right) + \frac{\phi p}{R} + \tilde{\alpha}div\left(U^s\right) = 0 \tag{6.2}$$

where $\tilde{\alpha} = (1 + Q/R)$ is the stiffness coupling factor between the skeleton and the fluid.
The equivalent masses $\tilde{\rho}_f$ and $\tilde{\rho}_s$ appearing in modified Biot's equations (6.1) and (6.2) are given by:

$$\tilde{\rho}_f = \phi\rho_e \text{ and } \tilde{\rho}_s = (1-\phi)\,\rho_s + \phi\rho_f\left(1 - \frac{\rho_f}{\rho_e}\right) \tag{7}$$

where $\rho_e = \rho_f\tau - b/j\omega$ is the effective mass of the interstitial fluid which includes the viscous coupling factor b with the skeleton. Modified Biot's equations (6.1) and (6.2) have the great advantage of involving explicitly the total stress tensor, $\sigma^{tot}_{kl} = \tilde{\sigma}^s_{kl} - \tilde{\alpha}\phi p\delta_{kl}$ where $\tilde{\sigma}^s_{kl}$ is the stress tensor of the skeleton with vacuum inside.

2 - MIXED VARIATIONAL FORMULATION OF MODIFIED BIOT'S EQUATIONS

The mixed variational formulation associated to modified Biot's equations could be simply derived by multiplying equation (6.1) by a virtual displacement vector V^s and equation (6.2) by a virtual pressure ϕq, and by integrating over the domain Ω occupied by the porous media,

$$Z\left(U^s, V^s\right) + A\left(p, q\right) - \hat{C}\left(p, V^s\right) - \hat{C}\left(q, U^s\right) = \tilde{C}_S\left(T, V^s\right) + C_S\left(q, W_n - U^s_n\right) \tag{8}$$

for admissible $V^s(\Omega)$ and $q(\Omega)$ satisfying prescribed boundary conditions.
$Z\left(U^s, V^s\right) = K\left(U^s, V^s\right) - \omega^2M\left(U^s, V^s\right)$ is the mechanical impedance operator of the skeleton where K and M are the stiffness and mass operators given by,

$$K\left(U^s, V^s\right) = \int_\Omega \left(\tilde{\sigma}^s_{kl}\left(U^s\right)\tilde{\varepsilon}^s_{kl}\left(V^s d\,\Omega\right. \;;\; M\left(U^s, V^s\right) = \int_\Omega \tilde{\rho}_s\left(U^s, V^S\right)d\Omega \tag{9}$$

$A\left(p, q\right) = H\left(p, q\right) - Q\left(p, q\right)/\omega^2$ is the acoustic admittance operator of the interstitial fluid where H and Q are the stiffness and inertial operators given by,

$$H\left(p, q\right) = \int_\Omega \frac{1}{\tilde{\rho}_f}\left(grad\left(\phi p\right), grad\left(\phi q\right)\right)d\Omega \;;\; Q\left(p, q\right) = \int_\Omega \frac{\phi^2 pq}{R}d\Omega \tag{10}$$

$\hat{C}(p,V^s)$ is the volume coupling operator given by,

$$\hat{C}(p,V^s) = \int_\Omega \left\{ \tilde{\alpha}\phi p\, div\,(V^s) + \left(\tilde{\beta}V^s, grad\,(\phi p)\right) \right\} d\Omega \tag{11}$$

The volume coupling operator is composed by the sum of the volume stiffness coupling proportional to $\tilde{\alpha}$ and of the volume inertial coupling operator proportional to $\tilde{\beta}$.
$\tilde{C}_S(T,V^s) = \int_S (T,V^s)\,dS$ is the surface loading operator, where $T_k = \sigma_{kl}^{tot} n_l$ represent the component on the coordinate axis x_k of the total surface stress vector T.
Finally, $C_S(q, W_n - U_n^s) = \int_S q\,(W_n - U_n^s)\,dS$ is the surface cinematic coupling operator, where n is the outgoing unitary vector normal to the boundary S of the porous domain Ω. As shown in figure 1-a, for practical applications the porous media is attached on a part S_1 of its boundary to an impervious master structure and is coupled on another part S_0 to an acoustic cavity. The master structure communicates to the porous media a total displacement vector $W{=}U^s$. Reciprocally the porous component applies to the master structure a surface loading T. At the interface S_0 with the acoustic cavity the normal component W_n of the total acoustic displacement and the normal components U_n^s and U_n^f of the skeleton and of the fluid are related by the equation, $W_n - U_n^s = \phi\left(U_n^f - U_n^s\right)$. For impervious surfaces the relative displacement is null, and for perforated surfaces there exist a relative displacement between the skeleton and the fluid. So the acoustic cavity applies to the porous component a surface pressure loading $T_k = -pn_k$, and reciprocally the porous media communicates to the cavity a relative displacement $W_n - U_n^s$. *In summary the master structure communicates to the porous component a cinematic displacement and the acoustic cavity apply on it an acoustic surface loading. The porous component reacts by applying a surface loading on the master structure, and by communicating a displacement to the cavity.*
In the configuration of figure 1, equation (9) could be rewritten in the following form,

$$Z(U,V) + A(p,q) - \bar{C}(p,V) - \bar{C}(q,U) = \tilde{C}_1(T,V) + C_0(q,W_n) \tag{12}$$

where $\bar{C}(p,V) = C_0(p,V) - \hat{C}(p,V)$. In order to avoid the assemblage between the cavity and the porous component which allows the use of incompatible FEM meshes, the continuity of the pressure at the interface S_0 could be relaxed and imposed in a weak form by a Lagrange multiplier X_n:

$$C_0(X_n,(p-p^c)) = \int_{S_0} (p-p^c)\,X_n dS_0 = 0 \tag{13}$$

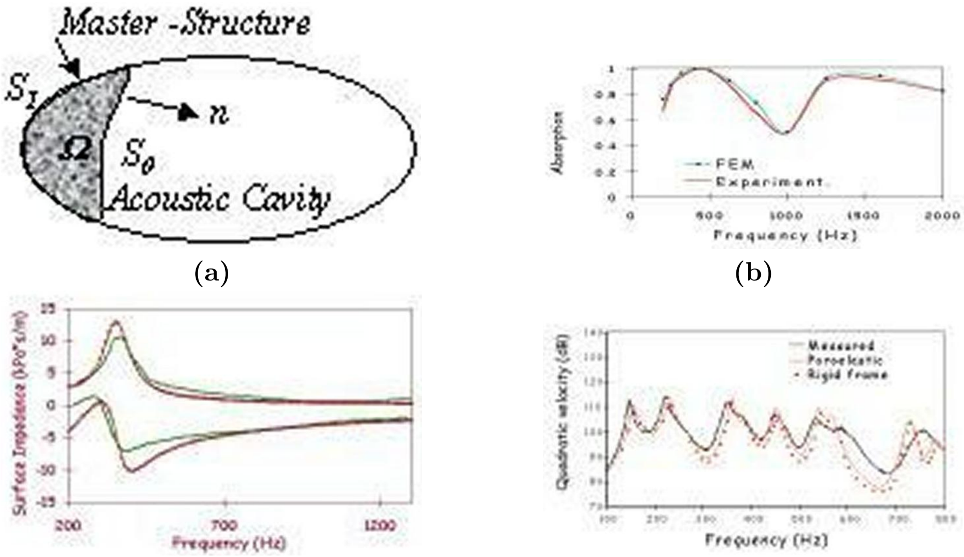
where p^c is the pressure applied by the cavity. Addition of equations (12) and (13) gives,

$$Z(U,V) + A(p,q) - \bar{C}(p,V) - \bar{C}(q,U) - C_0(q,W_n) - C_0(p,X_n) = \tilde{C}_1(T,V) - C_0(p^c,X_n) \tag{14}$$

The right hand side of equation (14) corresponds exactly to the total energy exchanged between the porous media and its surrounding environment: the mechanical energy absorbed by the porous component from the Master-Structure corresponds to the first term and the acoustic energy absorbed from the acoustic cavity corresponds to the second term.

3 - FINITE ELEMENT RESULTS

The FEM discretization of equation (14) allows the computation of the mixed impedance matrix of the porous component Ω by eliminating all internal degrees of freedom (dof) of the porous component except those dof's attached to the master structure and to the acoustic cavity. *This has the great advantage of not increasing the size of the global vehicle model and of allowing suppliers to compute separately and to deliver the impedance matrices of their components to vehicle manufacturers.* FEM results predicted by RAYON-PEM Solver developed by STRACO (France) in cooperation with the University of Sherbrooke (Canada) are presented. Figure 1.b shows an excellent agreement between simulation and measurement for an homogeneous limp wool sample, protected by a thin perforated screen and placed in impedance tube. Figure 1.c compares with very good agreement the real and imaginary parts of the surface impedance predicted by the proposed model and impedance tube measurements for a multi-layers simple including a septum [4]. Finally Figure 1-d shows the results for a system made up from two clamped plates separated by an unbounded foam. One plate is excited by a shaker and the normal quadratic velocity is measured on the facing plate. Good agreement is again achieved using both elastic and equivalent fluid models.



(c) (d)

Figure 1: FEM model and results.

4 - CONCLUSION AND PERSPECTIVES

The proposed new FEM formulation has the advantage of automatically managing all interior and exterior boundary conditions. In addition it reduces at minimum the modeling effort by allowing the direct computation of the impedance matrices added by porous components to the matrix impedance of the vehicle body. The resulting RAYON-PEM Solver developed by STRACO and commercially available in the environment of I-DEAS Vibro-acoustics software package constitutes a very powerful analysis tool, which could be advantageously used to optimize the acoustic treatment of vehicles.

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