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A NUMERICAL ANALYSIS ON VISCO-THERMAL DISSIPATION OF RESONATOR NECKS

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ABSTRACT
To investigate extrapolation of the Ingard's explicit representation [1] for acoustic dissipation around a circular aperture to a non-circular one, rigorous experiments were carried out. Consequently the dissipation given by the explicit representation became less than that of experiment as the aperture shape became apart from circular. To have more reliable estimation of the dissipations of non-circular apertures, a boundary element numerical approach for acoustic mode field along with the admittance representation of the visco-thermal boundary layer was introduced and their effectiveness was confirmed.

1 - INTRODUCTION
On acoustic energy dissipation of resonators of Helmholtz type, the viscosity and thermal conductance near the solid surface of the neck plays the leading role. Explicit expressions have been given by many researchers for the acoustic resistance of the neck of simple shape such as a long circular tube. However for a neck of non-circular and finite length, we have to solve implicit relationships between acoustic mode wave and the visco-thermal boundary layer. We employed a boundary element numerical approach for the acoustic mode fields and introduced the effective acoustic mode admittance [2] for the visco-thermal boundary layer to couple the acoustic mode and the visco-thermal mode.

2 - EXPLICIT ACOUSTIC REPRESENTATIONS OF APERTURES
We consider a Helmholtz resonator at the end of a long straight duct (of cross-sectional area S_D) as shown in Fig. 1. This consists of a rigid-walled cavity (of volume V_{cav}) and a neck of area S_N and length l_N. The acoustic impedance of the resonator is represented as

$$Z_{HR} = p_F/U_F = R_{HR} + jX_{HR} \tag{1}$$

where p_F denotes the complex sound pressure amplitude just outside of the resonator aperture. U_F is the volume velocity flowing into the resonator. $X_{HR} = \omega M_{HR} - 1/\omega C_{HR}$. $C_{HR} = V_{cav}/\rho c^2$ is the acoustic compliance associated with the cavity volume. $M_{HR} = \rho\left(l_N + l_M\right)/S_N$ is the acoustic inertance associated with the aperture. l_M is the mass correction length. In the estimation of l_M, we employed the expressions by S.N. Rschevkin for circular apertures [3], and by C.S. Kosten and I.M. Smits for slits [3]. The mass correction length l_M for non-circular apertures, the Rschevkin's representation also applied by employing the equivalent radius $\sqrt{S_N/\pi}$ which gives the same sectional aria as the aperture.
The acoustic resistance is represented as

$$R_a = 2R_v\left(l_N + l_R\right)/S_N a \tag{2}$$

where R_v is a surface resistance and will be defined in the succeeding section 4. l_R denotes the resistance end correction and was given empirically by U. Ingard [1] as $l_R \simeq 2a$ for circular apertures. We apply this also to non-circular aperture (of perimeter length L_P) by introducing the equivalent hydraulic radius $2S_N/L_P$, and we have an expression as

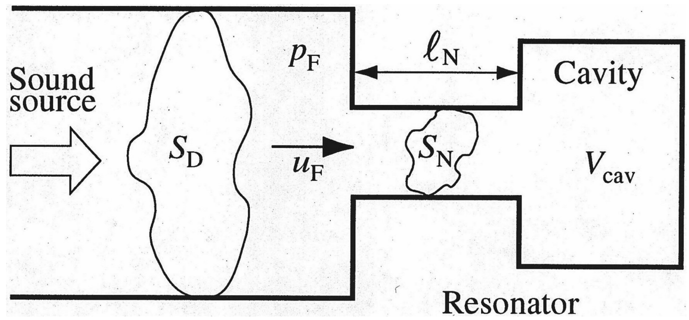

Figure 1: A Helmholtz resonator at an end of a straight duct.

$$R_a = R_v \left(l_N L_P / S_N + 4 \right) / S_N \tag{3}$$

Introducing a normalized resistance $R_a^* = R_a / (\rho c / S_D)$, the power absorption factor α_F (which coincides with the acoustic dissipation factor δ_{HR} in this case), i.e. the fraction of incident power absorbed by the resonator, is represented as

$$\alpha_F = \delta_{HR} = 4R_a^* / \left\{ (1 + R_a^*)^2 + X_{HR}^2 \right\} \tag{4}$$

3 - EFFECTIVENESS OF THE EXPLICIT REPRESENTATION ON NON-CIRCULAR APERTURES

In the experiments, the cavities and the straight ducts are made of 10mm thick steel and 30mm thick PVC plates. Several thick steel, brass and PVC plates with test apertures of several shapes were inserted between them by bolting or welding. The sectional center of each aperture was fitted to those of the duct and the cavity. The absorption factors α_F were determined by using the transfer function (two microphone) method. Fig. 2 shows the influence of materials (and test setups) on the absorption factor. The deviations caused from the materials are small.

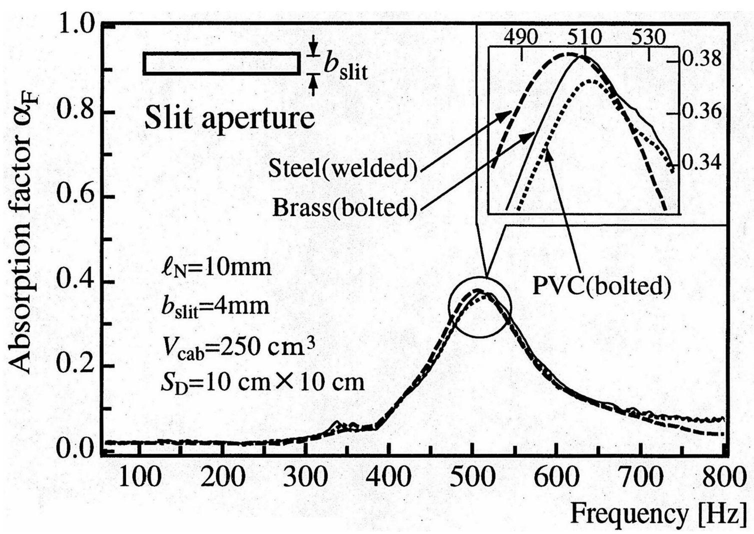

Figure 2: Dependence of the absorption factor on materials.

Fig. 3 gives the typical ones of the experimental results for non-circular apertures. We can see a tendency that the dissipation factor given by the explicit representation (3) becomes smaller than that by the experiment as the aperture shape becomes apart from circular.

4 - NUMERICAL ANALYSIS OF APERTURES WITH DISSIPATION

Subdividing the acoustic field into sub-regions, a boundary element formulation was employed for the acoustic wave mode in each sub-region and matching of the field variables between their interfaces was

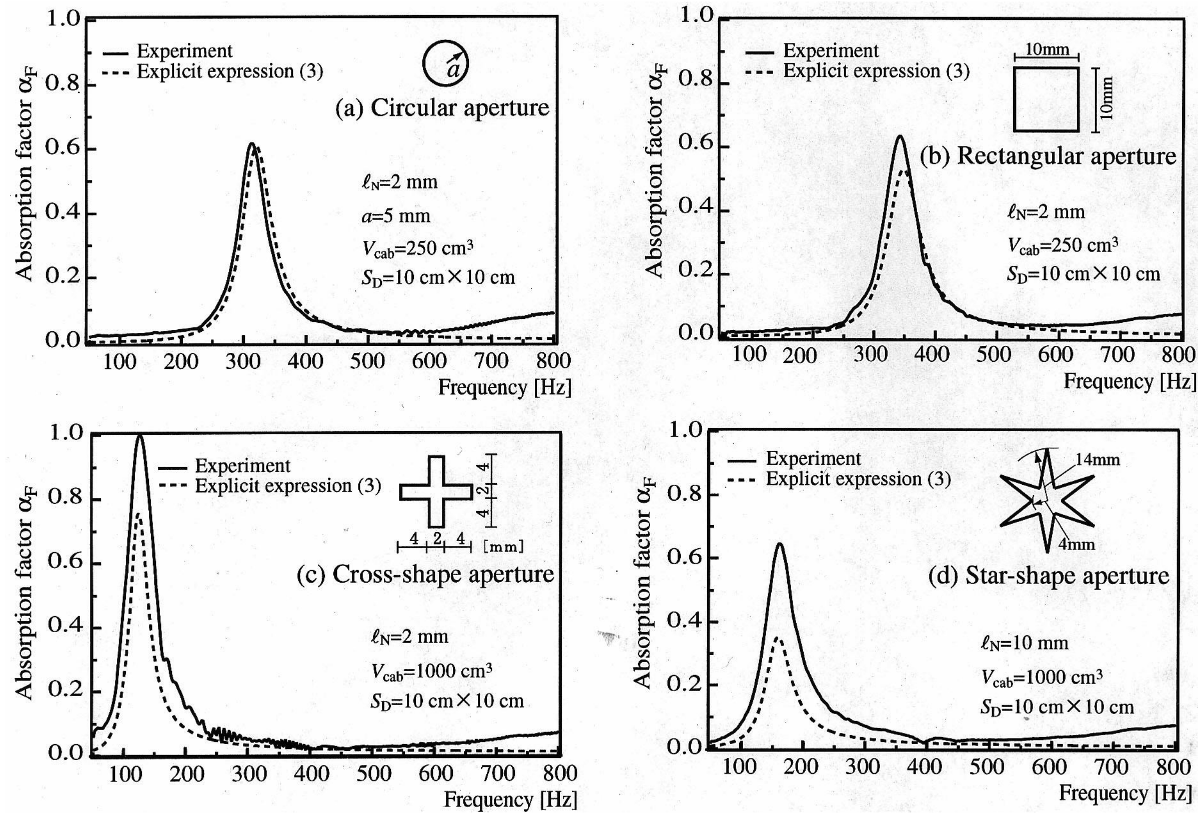

Figure 3: Effectiveness of the explicit representation (3).

taken. For the acoustic mode field, the boundary conditions were given by introducing the admittance representation with visco-thermal boundary layer [2] as

$$\beta = \frac{(p - p_{driv})}{u_n} = \frac{1}{Z_w} + (1+j)\left\{\frac{R_h}{\rho^2 c^2} - \frac{R_v \nabla^2_{tan}}{\rho^2 \omega^2}\right\} \tag{5}$$

where p and u_n denote the sound pressure and outward normal velocity of the acoustic mode. p_{driv} and Z_w are the driving pressure and specific impedance of the wall. $R_h = (\gamma - 1)\sqrt{\rho\omega\kappa/2c_p}$, $R_v = \sqrt{\rho\omega\mu/2}$. κ and μ denote the thermal conductivity and viscosity of the air. c_p and γ are the specific-heat coefficient at constant pressure and the specific-heat ratio of the air. The tangential Laplacian ∇^2_{tan} stands for the sum of the second derivatives with respect to the two coordinates tangential to the boundary surface, and $\nabla^2_{tan}p$ of each surface element was approximated in a finite difference expression by using the pressures of the neighboring elements.

To investigate the effectiveness of the numerical simulation, we conducted a series of numerical simulations and experiments for the dissipation factors of slit resonators, and compared the results.

From the numerical simulations changing the admittance of each of the surfaces to be with and without visco-thermal dissipation, it was confirmed that the dissipation factors of all the surfaces except the neck in the resonator side of the microphone was so small (say, about 2% at the resonant frequency in terms of the absorption factor) that we may regard the resultant absorption factor to be almost the neck dissipation factor unless the resultant absorption factor at the resonant frequency is very small.

The boundary element size governs the accuracy of the simulation. Coarse elements gave a negative large absorption when the loss-free conditions were imposed to all the surfaces. Fine elements of 0.1mm were employed here, but the negative absorption factor of a few percent compared to the resultant absorption factor was still remained. For this numerical error in each case, we compensated the resultant absorption, through subtracting from it the corresponding negative absorption appeared at loss-free conditions. Fig. 4 shows a typical. The agreement between the absorption factor by the numerical simulation and that by the experiment was excellent.

5 - CONCLUSION

The explicit representation based on U. Ingard [1] for acoustic dissipation around a circular aperture gives

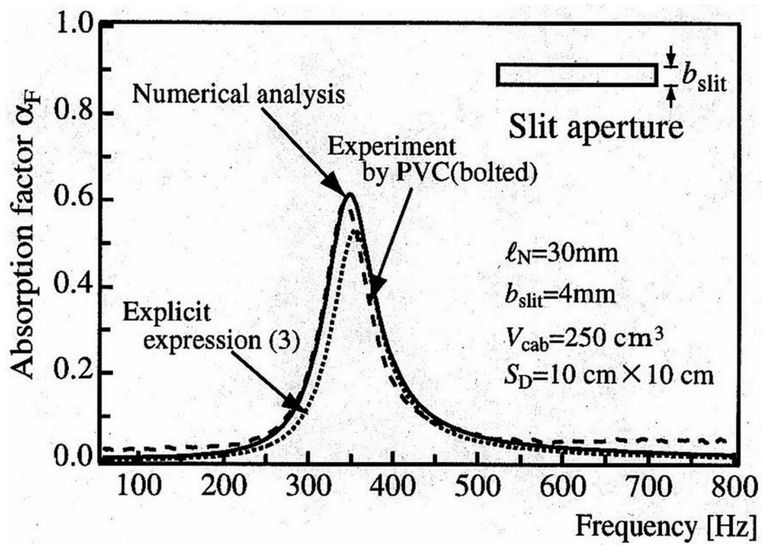

Figure 4: Effectiveness of the numerical simulation.

a significant under-estimation of the dissipation when it applied to a non-circular aperture. A boundary element approach along with the admittance for the visco-thermal boundary layer was implemented, and the agreement between the dissipation factor by this numerical simulation and that by the experiment was excellent as far as the boundary element dimension was 0.1 mm or finer.

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