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SOUND DIFFRACTION BY A SLIT EXISTING BETWEEN TWO HALF INFINITE PLANES

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ABSTRACT
This paper deals with the problem of sound diffraction by the slit which exists between two half infinite planes. The planes are located between a point source and a receiver in the arrangement that the edges of the planes are in parallel holding certain distance. At first, a rather complicated formula to predict the amount of diffraction effect is derived based on Keller's geometrical theory of diffraction almost exactly. Secondly an approximate formula, simpler and more practical, is derived from the above formula. Calculated values from these two formulae are almost the same. The results of calculation using the formulae agreed well with the results of calculation using a boundary element method in two dimensions.

1 - INTRODUCTION

Calculation methods were developed for the effect of the sound diffraction by a slit that exists between the edges of two half-infinite planes. In the formulation, Keller's geometrical theory of diffraction [1] is applied to the configuration of two planes that are arranged towards arbitrary direction. The transmitted wave through the slit is expressed by the theory as the sum of the direct wave and waves with multiple diffraction between the slit edges.

First, a precise formula to express the wave with slit diffraction was established. Secondly, a practical formula was derived from the precise version by the expressions of single diffraction treatment [2]. To check the validity of these models, numerical simulation by two dimensional BEM (Boundary Element Method) was carried out and the comparison was made for the calculated values.

2 - WAVE WITH THE DIFFRACTION BY SLIT

The sound wave at a receiver P is expressed as the sum of direct wave and the diffracted waves. In Fig. 1, S is the sound source, P is the receiver, A and B are the edges of the planes forming the slit, and each θ with suffix is the angle clockwise from the side of the plane to the line linking the point and the edge of the plane.

In Keller's theory of diffraction, an incident wave that hits the edges of the slit gives rise to a cone of diffracted waves. Some of the waves diffracted from one edge hit the other edge, and give rise to another cone of diffracted waves again, so the incidence and the diffraction are repeated between the edges. For simplicity, we neglect the waves diffracted over twice and the effect of the multiple reflection between the surface of planes, then the transmitted wave through the slit ϕ is shown as below (See Fig. 1).

$$\begin{aligned}\phi = \; & \eta\left(\pi-\theta_{\mathrm{PA}}+\theta_{\mathrm{SA}}\right)\cdot\eta\left(\pi-\theta_{\mathrm{SB}}+\theta_{\mathrm{PB}}\right)\phi_{\mathrm{g}}+ \\ & \eta\left(\theta_{\mathrm{BA}}-\theta_{\mathrm{SA}}\right)\left\{\eta\left(\theta_{\mathrm{PA}}-\theta_{\mathrm{BA}}\right)\phi_{\mathrm{dA}}+\eta\left(\theta_{\mathrm{AB}}-\theta_{\mathrm{PB}}\right)\phi_{\mathrm{dAB}}\right\}+ \\ & \eta\left(\theta_{\mathrm{SB}}-\theta_{\mathrm{AB}}\right)\left\{\eta\left(\theta_{\mathrm{AB}}-\theta_{\mathrm{PB}}\right)\phi_{\mathrm{dB}}+\eta\left(\theta_{\mathrm{PA}}-\theta_{\mathrm{BA}}\right)\phi_{\mathrm{dBA}}\right\}\end{aligned} \tag{1}$$

Where $\eta(x)$ is a step function which gives 1 for $x \geq 0$, and gives 0 for $x < 0$, ϕ_{g} is the direct wave from the source S to the receiver P, ϕ_{dA} and ϕ_{dB} are the single-diffracted waves from the edge A and B, respectively and, ϕ_{dAB} and ϕ_{dBA} are the double-diffracted waves from the edge A-B and B-A, respectively.

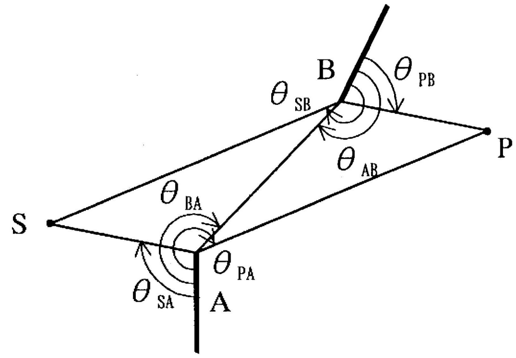

Figure 1: The arrangement of two half infinite planes.

With disregard the time factor $e^{-i\omega t}$, one obtains the direct wave:

$$\phi_{\rm g} = {\rm C} \cdot e^{ikR_{\rm SP}}/R_{\rm SP} \tag{2}$$

where C is the constant concerning with sound amplitude, $R_{\rm SP}$ is the distance from S to P, and k is the wave number. The single-diffracted wave $\phi_{\rm dA}$ is given as below:

$$\phi_{\rm dA} = {\rm C} \cdot D_{\rm S\ AP} \cdot e^{ikR_{\rm dSAP}}/R_{\rm dSAP} \tag{3}$$

where $R_{\rm dSAP}$ is the length of the diffracted path S-A-P, and $D_{\rm SAP}$ is the diffraction coefficient, which is derived from the Kouyoumjian's approximation [3] for the diffraction effect of wedge in case where exterior angle is 2π.
In Keller's theory of diffraction, the double-diffracted wave $\phi_{\rm dAB}$ is expressed as the product of diffraction coefficients, so it becomes:

$$\phi_{\rm dAB} = {\rm C} \cdot D_{\rm\ SAP'} \cdot D_{\rm S'BP} \cdot e^{ikR_{\rm dSABP}}/R_{\rm dSABP} \tag{4}$$

where $R_{\rm dAB}$ is the length of the double-diffracted path S-A-B-P, and $D_{\rm SAP'}$ and $D_{\rm S'BP}$ are the diffraction coefficients of the diffracted path S-A-P ' and S'-B-P, respectively (See Fig. 2).

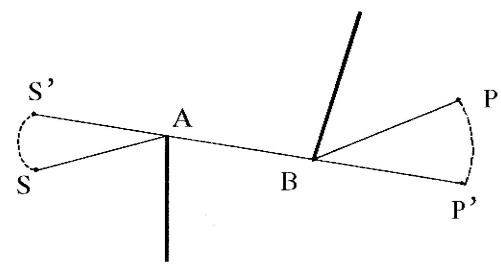

Figure 2: Replacing S and P by S' and P'.

3 - PRACTICAL CALCULATION METHOD

One can approximates the formula Eq. 1 and proposes more practical formula to predict the effect of sound diffraction.
The sound attenuation by single diffraction $\Delta L_{\rm dSAP}$ is derived from Eqs. (2) and (3):

$$\Delta L_{\rm dSAP} = -10\log_{10}\left|\frac{\phi_{\rm dA}}{\phi_{\rm g}}\right|^2 = -10\log_{10}\left(|D_{\rm SAP}| \cdot \frac{R_{\rm SP}}{R_{\rm dSAP}}\right)^2 \tag{5}$$

The sound attenuation by double diffraction $\Delta L_{\rm dSABP}$ is derived from Eqs. (2), (4) and (5):

$$\begin{aligned}\Delta L_{\rm dSABP} &= -10\log_{10}|\phi_{\rm dAB}/\phi_{\rm g}|^2 = -10\log_{10}\left(|D_{\rm SAP'} \cdot D_{\rm S'BP}| \cdot R_{\rm SP}/R_{\rm dSABP}\right)^2 \\ &= \Delta L_{\rm dSAP'} + \Delta L_{\rm dS'\ BP} - 10\log_{10}\left\{R_{\rm SP} \cdot R_{\rm dSA\ BP}/\left(R_{\rm SP'} \cdot R_{\rm S'P}\right)\right\}^2\end{aligned} \tag{6}$$

where $R_{\rm dSABP} = R_{\rm dSAP'} = R_{\rm dS'BP}$ (See Fig. 2).

The sound attenuation by slit diffraction $\Delta L_{\mathrm{d,slit}}$ is expressed using the ratio of $|\phi|$ to $|\phi_{\mathrm{g}}|$. Now we hypothesize that we disregard the phases of diffracted waves in Eq. 1, then $|\phi|$ is considered as the sum of the absolute values of each wave in Eq. (1), i.e, $|\phi_{\mathrm{g}}|$, $|\phi_{\mathrm{dA}}|$, $|\phi_{\mathrm{dAB}}|$, etc. But the formula obtained is complicated and should be approximated and simplified to make it suitable for practical use.
First, the ΔL_{dSAP} in Eq. (5) was related to the calculation of Maekawa's chart [4] according to the geometrical arrangements of S, P and the plane:

$$\Delta L_{\mathrm{d}}\left(N_{\mathrm{SAP}}\right)=\begin{cases}\Delta L_{\mathrm{dSAP}} & \text{for } N_{\mathrm{SAP}}\geq 0\\ -10\log_{10}\left(1-10^{-\Delta L_{\mathrm{dS\ AP}}/10}\right) & \text{for } N_{\mathrm{SAP}}<0\end{cases}\tag{7}$$

where $\Delta L_{\mathrm{d}}(N)$ is the sound attenuation by single barrier according to the Fresnel Number N.
Secondly, S and P was categorized as belonging to zone I or zone II. In zone I, the slit AB is visible and in zone II, invisible then one can classify the combinations of the arrangement of the points into four cases (See Fig. 3). And $\Delta L_{\mathrm{d,slit}}$ is derived from Eqs. (5), (6), and (7):
Case 1: both S and P are in the zone I

$$\begin{aligned}\Delta L_{\mathrm{d,slit}} &= -10\log_{10}\left(1-10^{-\Delta L_{\mathrm{dS\ AP}}/10}-10^{-\Delta L_{\mathrm{dSBP}}/10}+10^{-\Delta L_{\mathrm{dSABP}}/10}+10^{-\Delta L_{\mathrm{dSBAP}}/10}\right)\\ &\approx -10\log_{10}\left\{\left(1-10^{-\Delta L_{\mathrm{dSAP}}/10}\right)\cdot\left(1-10^{-\Delta L_{\mathrm{dSBP}}/10}\right)\right\}\\ &= \Delta L_{\mathrm{d}}\left(N_{\mathrm{SAP}}\right)+\Delta L_{\mathrm{d}}\left(N_{\mathrm{SBP}}\right)\end{aligned}\tag{8a}$$

Case 2: S is in the zone I; on the other hand, P is in the zone II

$$\begin{aligned}\Delta L_{\mathrm{d,slit}} &= -10\log_{10}\left(10^{-\Delta L_{\mathrm{dSBP}}/10}-10^{-\Delta L_{\mathrm{dSABP}}/10}\right)\\ &\approx -10\log_{10}\left(10^{-\Delta L_{\mathrm{dSBP}}/10}-10^{-(\Delta L_{\mathrm{dSBP}}+\Delta L_{\mathrm{dSAP'}})/10}\right)\\ &= \Delta L_{\mathrm{d}}\left(N_{\mathrm{SAP'}}\right)+\Delta L_{\mathrm{d}}\left(N_{\mathrm{SBP}}\right)\end{aligned}\tag{8b}$$

Case 3: S is in the zone II; on the other hand P is in the zone I

$$\begin{aligned}\Delta L_{\mathrm{d,slit}} &= -10\log_{10}\left(10^{-\Delta L_{\mathrm{dSAP}}/10}-10^{-\Delta L_{\mathrm{dSABP}}/10}\right)\\ &\approx -10\log_{10}\left(10^{-\Delta L_{\mathrm{dSAP}}/10}-10^{-(\Delta L_{\mathrm{dSAP}}+\Delta L_{\mathrm{dS'BP}})/10}\right)\\ &= \Delta L_{\mathrm{d}}\left(N_{\mathrm{SAP}}\right)+\Delta L_{\mathrm{d}}\left(N_{\mathrm{S'BP}}\right)\end{aligned}\tag{8c}$$

Case 4: both S and P are in the zone II

$$\Delta L_{\mathrm{d,slit}}=\Delta L_{\mathrm{dSABP}}=\Delta L_{\mathrm{d}}\left(N_{\mathrm{SAP'}}\right)+\Delta L_{\mathrm{d}}\left(N_{\mathrm{S'BP}}\right)-10\log_{10}\left(\frac{R_{\mathrm{SP}}\cdot R_{\mathrm{dSABP}}}{R_{\mathrm{SP'}}\cdot R_{\mathrm{S'P}}}\right)^{2}\tag{8d}$$

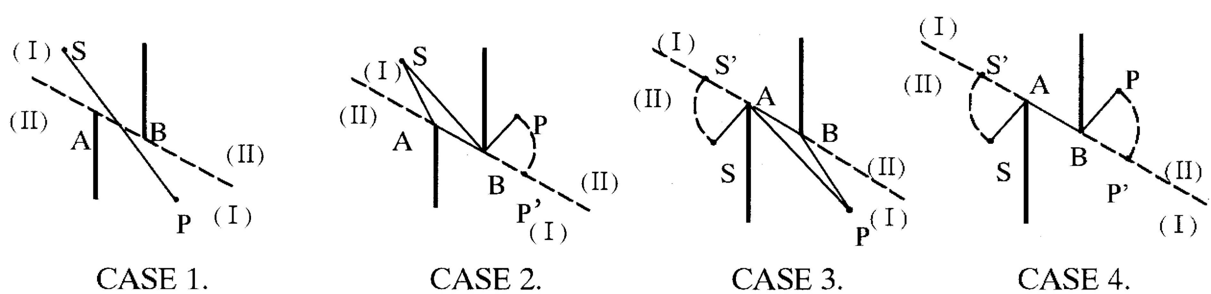

Figure 3: The modeling of the classification of the source and the receiver.

In Keller's geometrical theory of diffraction, Eq. (8) is also applicable to the calculation of the effect of diffraction by double barriers without any modification.

4 - COMPARISON OF CALCULATED RESULTS

Results calculated by Eq. (1) and Eq. (8) are compared with those by two-dimensional boundary element method (BEM). In BEM calculation, the size of elements was 2.5 cm, the both sides of the planes were perfectly reflective, and sound frequency was 500 Hz. The arrangement is set as shown in the Fig. 4. The width of the slit AB and the length from the plane B to the row of receivers is fixed to 5 m, and the angle θ is rotated clockwise from 0° to 120°. The results of the comparison in the cases of $\theta = 0°$, 60° are shown in Fig. 5. The calculated values of Eq. (1) and Eq. (8) agree well with the values of BEM when $\theta < 90°$.

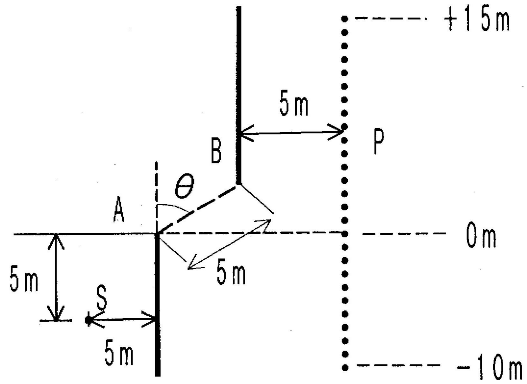

Figure 4: The arrangement of the planes used to comparison.

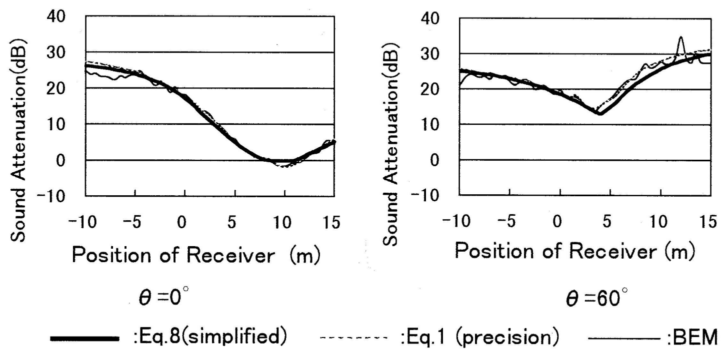

Figure 5: The comparison of the results calculated by Eq. 1, Eq. 8 and BEM.

5 - CONCLUSION

The calculation methods for the effect of diffraction by the slit between two half infinite planes have been developed. The formulae proposed in the present paper agree well with the calculated values by BEM. It is shown that the difference between the results obtained from the simplified formula Eq. (8) and the complicated formula Eq. (1) is within 2dB.

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