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VIBRATION POWER FLOW MODELS OF ORTHOTROPIC PLATES

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ABSTRACT
In this paper, approximate power flow models for the vibration of orthotropic plates are derived to be used to predict the time- and space-averaged energy density distribution and the power transmission path of plates in the middle-high frequency ranges. The derived models are applied for the case in which single rectangular orthotropic plates are vibrating by a harmonic point load and their boundaries are simply supported. The results of the approximate models are compared with modal solutions obtained from the equation of motion of the plate. The models for the orthotropic plates are also compared with those for isotropic plates. The computational examples show that the derived power flow models are effective in predicting the energy and intensity distributions of the orthotropic plates.

1 - INTRODUCTION
Many dynamic complex structures, ships and airplanes in particular, contain plate elements made of orthotropic materials and the stiffened or reinforced plates. These naturally and structurally orthotropic plates have different bending stiffnesses in two perpendicular directions, on which the energy distribution and the power transmission path depend greatly. Since the developments of PFA models have been mainly focused on isotropic structural elements, it is necessary that more general power flow formulations should be investigated to apply the PFA method to properly predict the energy and intensity field in orthotropic structures.
The aim of this work is to develop approximate power flow models that predict the energy distribution and power transmission path in the time- and locally space-averaged sense for orthotropic plates vibrating in the middle and high frequency ranges. Computations are performed for simple cases in which finite rectangular orthotropic plates are simply supported along the edges and excited by a transverse harmonic point force located in the middle of the plates.

2 - APPROXIMATE POWER FLOW MODELS OF ORTHOTROPIC PLATES
The equation of motion of thin orthotropic plates excited by a harmonic point force located at (x_o, y_o) of which the amplitude and frequency are F and ω, respectively, can be written by

$$D_{xc}\frac{\partial^4 w}{\partial x^4}+2H_c\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{yc}\frac{\partial^4 w}{\partial y^4}+m\frac{\partial^2 w}{\partial t^2}=F\delta\left(x-x_o\right)\delta\left(y-y_o\right)\ \mathrm{e}^{\mathrm{j}\omega\mathrm{t}} \tag{1}$$

where w is the transverse displacement and m the mass per unit area of the plate. D_{xc} and D_{yc} are the complex bending stiffnesses in the x- and y-directions, respectively:

$$D_{xc}=D_x\left(1+\mathrm{j}\eta\right),\ D_{yc}=\ \mathrm{D_y}\left(1+\mathrm{j}\eta\right) \tag{2}$$

where η is the hysteretic damping loss factor. When the thickness of the plate is constant and its transverse displacement is relatively very small, the complex effective torsional stiffness, H_c, can be assumed as

$$H_c = \sqrt{D_{xc} D_{yc}} \tag{3}$$

The effective torsional stiffness expressed by equation (3) makes it easy to mathematically handle the equation of motion given by equation (1). Hence, the authors of this paper investigate approximate power flow models from equation (1) using equation (3).
The energy models for isotropic plates derived by Bouthier and Bernhard are obtained from the far-field solution of the equation of motion and appear to be a good representation of the approximate response of the plates. In a similar manner, only the far-field solution of equation (1) is utilized for this investigation, which can be expressed as

$$w_{ff}(x,y,t) = \left(A\mathrm{e}^{-\mathrm{j}(\mathrm{k_x x + k_y y})} + \mathrm{B}\mathrm{e}^{\mathrm{j}(\mathrm{k_x x - k_y y})} + \mathrm{C}\mathrm{e}^{-\mathrm{j}(\mathrm{k_x x - k_y y})} + \mathrm{D}\mathrm{e}^{\mathrm{j}(\mathrm{k_x x + k_y y})} \right) \mathrm{e}^{\mathrm{j}\omega t} \tag{4}$$

where k_x and k_y are the complex wave numbers in the x- and y-directions, respectively.
In general, the energy in plates is transmitted by shear forces (Q_{xz} and Q_{yz}), bending moments (M_x and M_y) and twisting moments (M_{xy} and M_{yx}). The time-averaged total energy density of the orthropic plate is

$$\begin{aligned} < e >= (1/4)\ \mathrm{Re}\{\mathrm{D_{xc}} \left(\partial^2 \mathrm{w}/\partial \mathrm{x}^2 \right) \left(\partial^2 \mathrm{w}/\partial \mathrm{x}^2 \right)^* + 2\sqrt{\nu_\mathrm{x}\nu_\mathrm{y}}\sqrt{\mathrm{D_{xc}D_{yc}}}\ \left(\partial^2 \mathrm{w}/\partial \mathrm{x}^2 \right) \left(\partial^2 \mathrm{w}/\partial \mathrm{y}^2 \right)^* \\ + D_{yc} \left(\partial^2 w/\partial y^2 \right) \left(\partial^2 w/\partial y^2 \right)^* + 2 \left(1 - \sqrt{\nu_\mathrm{x}\nu_\mathrm{y}} \right) \sqrt{\mathrm{D_{xc}\ D_{yc}}}\ \left(\partial^2 \mathrm{w}/\partial \mathrm{x}\partial \mathrm{y} \right) \left(\partial^2 \mathrm{w}/\partial \mathrm{x}\partial \mathrm{y} \right)^* \\ + m \left(\partial w/\partial t \right) \left(\partial w/\partial t \right)^* \} \end{aligned} \tag{5}$$

where Re ( ) represents the real part, the bracket < > the time average and the asterisk * the complex conjugate. ν_x and ν_y are the effective Poisson's ratios of the orthotropic plate. The x component of the time-averaged intensity is expressed by

$$< q_x >= (1/2)\,\mathrm{Re}\left\{ -Q_{xz} \left(\partial w/\partial t \right)^* + M_x \left(\partial^2 w/\partial x \partial t \right)^* + M_{xy} \left(\partial^2 w/\partial y \partial t \right)^* \right\} \tag{6}$$

The y component of intensity is represented by Q_{yz}, M_y and M_{yx} as a similar form of equation (6). Although the displacement solution (equation (4)) is substituted into equation (5) and (6), no obvious relations between the energy density and intensity can be found at this stage. Thus, the time-averaged far-field energy density and intensity are spatially averaged over a half wavelength for small damping in the following manner:

$$< \tilde{e} >= \left(k_{xl} k_{yl} / \pi^2 \right) \int_0^{\pi/k_{yl}} \int_0^{\pi/k_{xl}} < e > \mathrm{dx}\ \ \mathrm{dy} \tag{7}$$

Neglecting all of the second and higher terms of the damping loss factor yields the simplified expression for the energy density and the x and y components of the intensity as follows:

$$< \tilde{e} >= (1/2)\ \omega^2 m \left(|A|^2 e^{--} + |B|^2 e^{+-} + |C|^2 e^{-+} + |D|^2 e^{++} \right) \tag{8}$$

$$< \tilde{q}_x >= \omega k_{xl} \sqrt{D_x} \sqrt{\omega^2 m} \left(|A|^2 e^{--} - |B|^2 e^{+-} + |C|^2 e^{-+} - |D|^2 e^{++} \right) \tag{9}$$

$$< \tilde{q}_y >= \omega k_{yl} \sqrt{D_y} \sqrt{\omega^2 m} \left(|A|^2 e^{--} + |B|^2 e^{+-} - |C|^2 e^{-+} + |D|^2 e^{++} \right) \tag{10}$$

where $e^{\pm\pm}$ is defined as $\exp\{\pm(\eta/2)\,k_{xl}x \pm (\eta/2)\,k_{yl}y\}$ for simplicity. The total energy density is represented as the sum of the energy densities of four plane wave components. The net intensity is represented as the subtraction of the intensities of the waves propagating in the negative direction from those of the waves propagating in the positive direction. From equations (8)-(10), it can be found that the x and y components of intensity are proportional to the first derivative of energy density with respect to x and y, respectively:

$$< \tilde{\mathbf{q}} >= -\left(\left(c_{gx}^2/\eta\omega \right) \left(\partial/\partial x \right)\ \mathbf{i} + \left(c_{gy}^2/\eta\omega \right) \left(\partial/\partial y \right)\ \mathbf{j} \right) < \tilde{e} > \tag{11}$$

where $c_{gx} = 2\left(\omega^2 D_x/m \right)^{1/4}$ and $c_{gy} = 2\left(\omega^2 D_y/m \right)^{1/4}$.

For the steady state elastic system, the power balance equation can be written as

$$\nabla \cdot \mathbf{q} + \pi_{diss} = \pi_{in} \tag{12}$$

where π_{diss} and π_{in} are the dissipated and input power, respectively. Cremer and Heckl showed that the time-averaged dissipated power in an elastic medium with small damping ($\eta \ll 1$) is proportional to the time-averaged total energy density in the form:

$$< \pi_{diss} >= \eta\omega < e > \tag{13}$$

Finally, the combination of equation (12) with equations (11) and (13) yields the second order partial differential equation that takes the total energy density as a primary variable:

$$-\left(\left(c_{gx}^2/\eta\omega\right)\left(\partial^2/\partial x^2\right) + \left(c_{gy}^2/\eta\omega\right)\left(\partial^2/\partial y^2\right)\right) < \tilde{e} > +\eta\omega < \tilde{e} >=< \tilde{\pi}_{in} > \tag{14}$$

If the bending stiffnesses D_x and D_y are equal to D, the above equation then becomes the energy governing equation for the isotropic plate derived by Bouthier and Bernhard as

$$-\left(c_g^2/\eta\omega\right)\left(\partial^2/\partial x^2 + \partial^2/\partial y^2\right) < \tilde{e} > +\eta\omega < \tilde{e} >=< \tilde{\pi}_{in} > \tag{16}$$

where c_g is the group velocity of the corresponding isotropic plate.

3 - COMPUTATIONAL EXAMPLES

The computations are performed for the finite rectangular orthotropic plate simply supported along its edges and excited by a transverse harmonic point force at the center of the plate. It is assumed that the plate is stiffened along the y-direction.

The exciting frequency and the damping of the plate are $f = 500$ Hz and $\eta = 0.02$. The energy density distributions obtained from the classical modal displacement solution and from the approximate power flow model are shown in Figs. 1 and 2, respectively. When the damping of the plate is increased to $\eta = 0.2$ with the same frequency, the classical modal solution and approximate solution of the energy density are shown in Figs. 3 and 4, respectively. If the frequency is $f = 5$ kHz and the damping loss factor is $\eta = 0.2$, the classical energy solution is more smoothed and becomes similar to the approximate energy solution, as shown in Figs. 5 and 6. and the corresponding intensity fields are illustrated in Figs. 7 and 8. Since the orthotropic plate has different bending stiffnesses in two perpendicular directions, its energy distribution differs from that of the isotropic plate, as shown in Figs. 9 and 10.

4 - CONCLUSIONS

The approximate power flow model for transversely vibrating orthotropic plates has been developed to apply the PFA method to the middle-high frequency vibration of the orthotropic plates. To derive the model, the far-field solutions were used and time- and space-averaging were performed. Computations were performed for the verification of this model. From the results, it is shown that the developed model well represents the global response of orthotropic plates vibrating at middle-high frequencies.

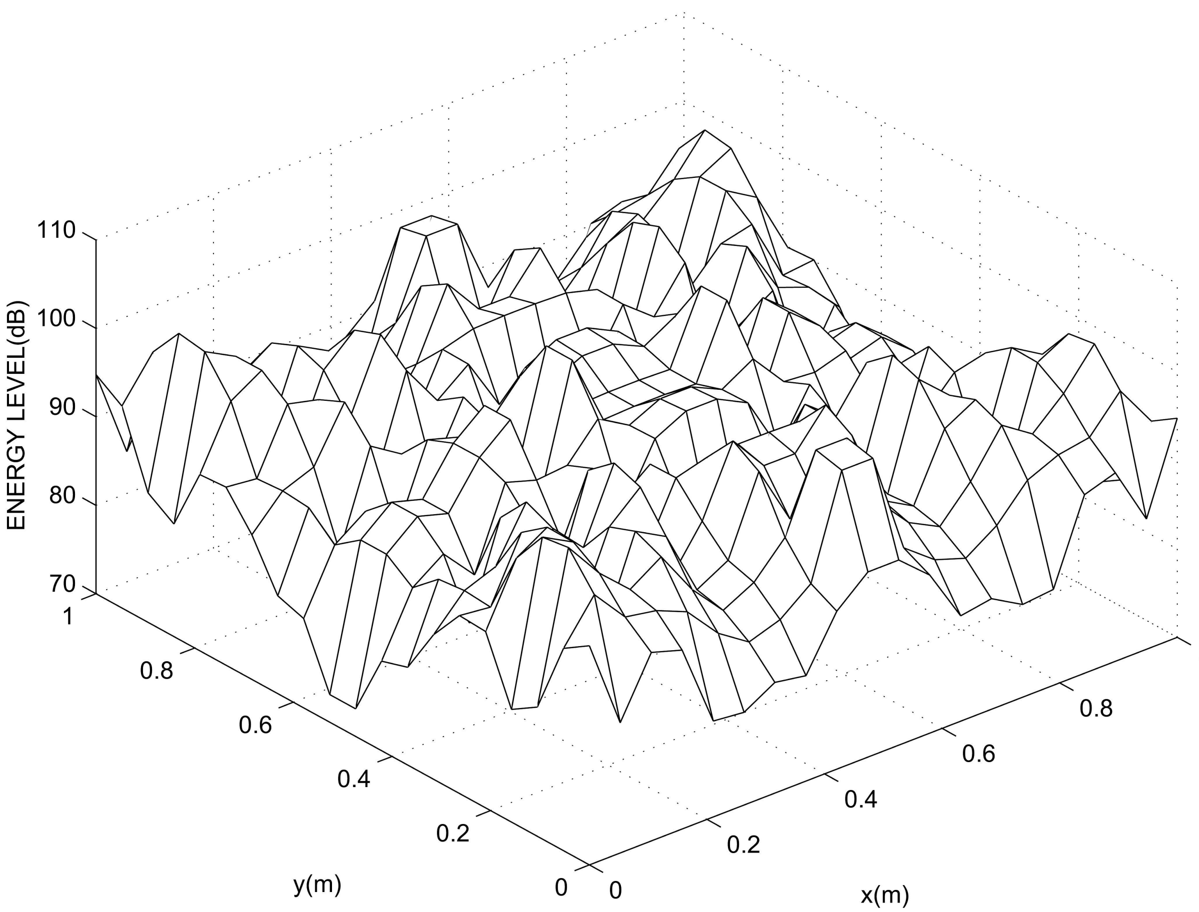

Figure 1: Classical modal energy density (f = 500 Hz, η = 0.02).

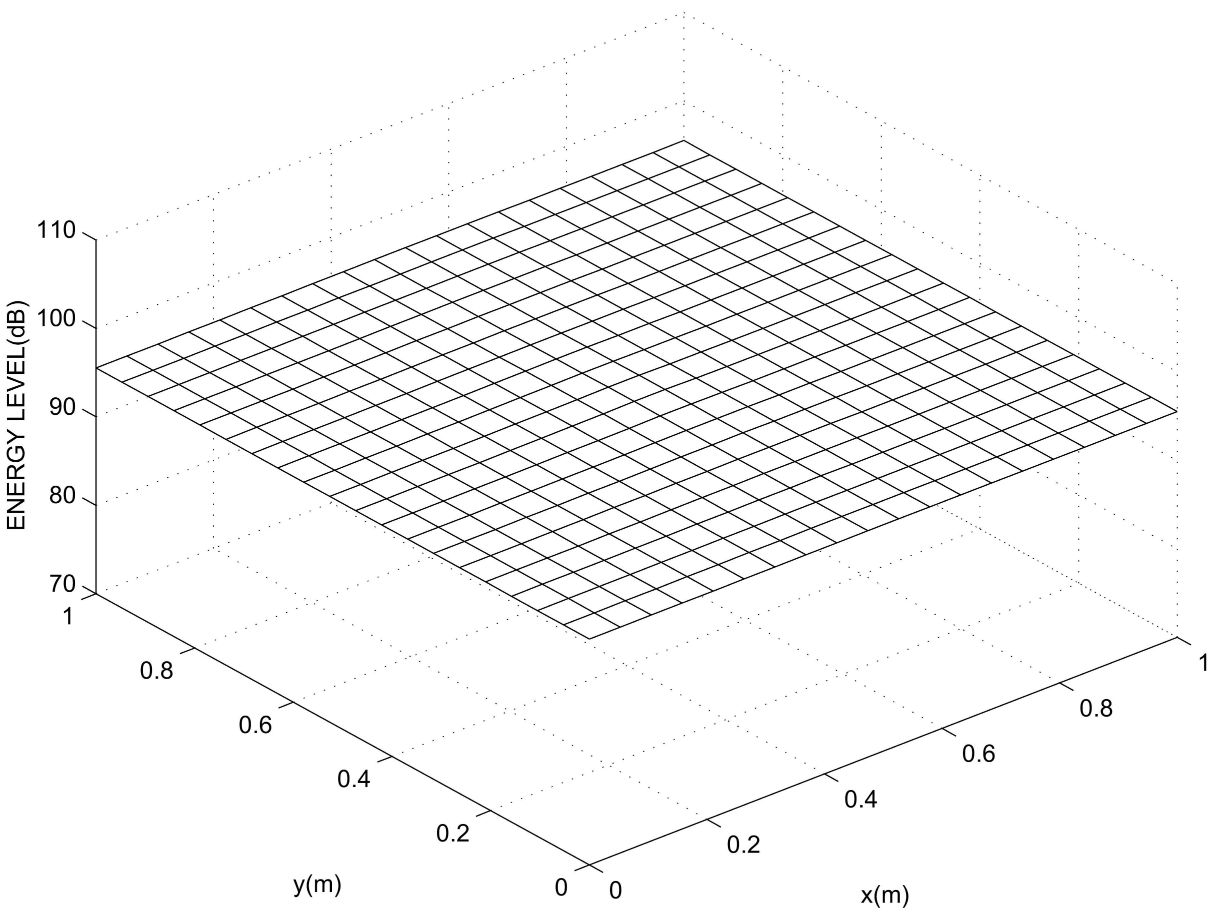

Figure 2: Approximate energy density (f = 500 Hz, $\eta = 0.02$).

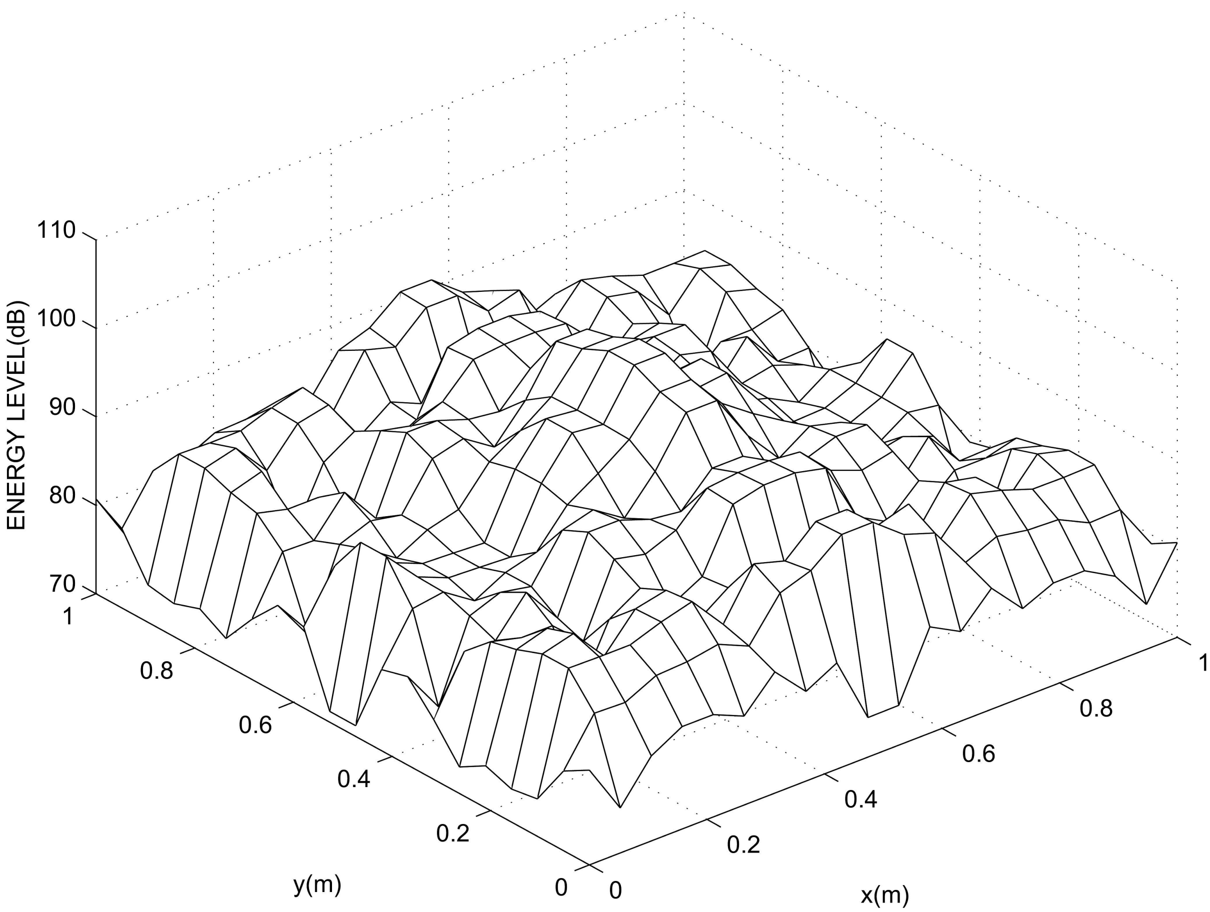

Figure 3: Classical modal energy density (f = 500 Hz, η = 0.2).

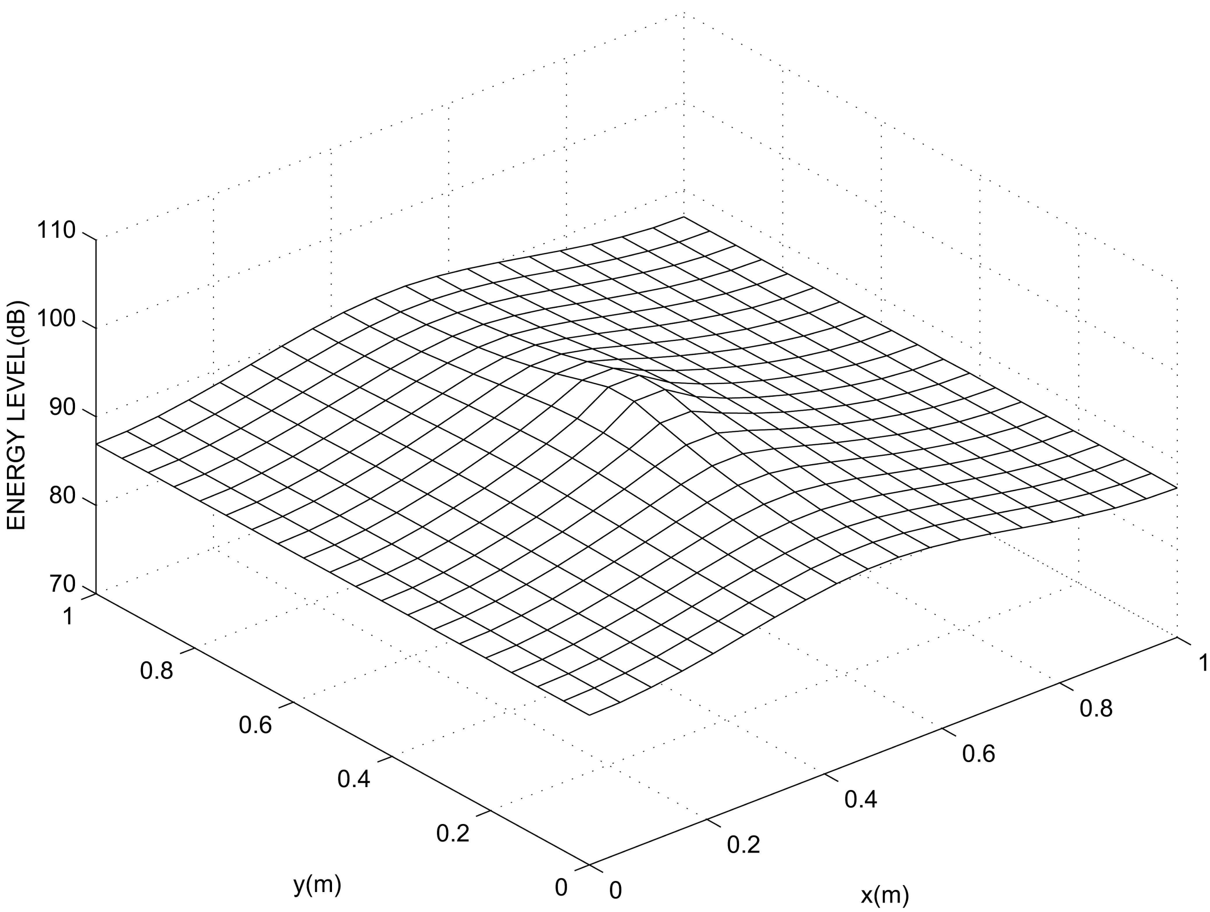

Figure 4: Approximate energy density ($f = 500$ Hz, $\eta = 0.2$).

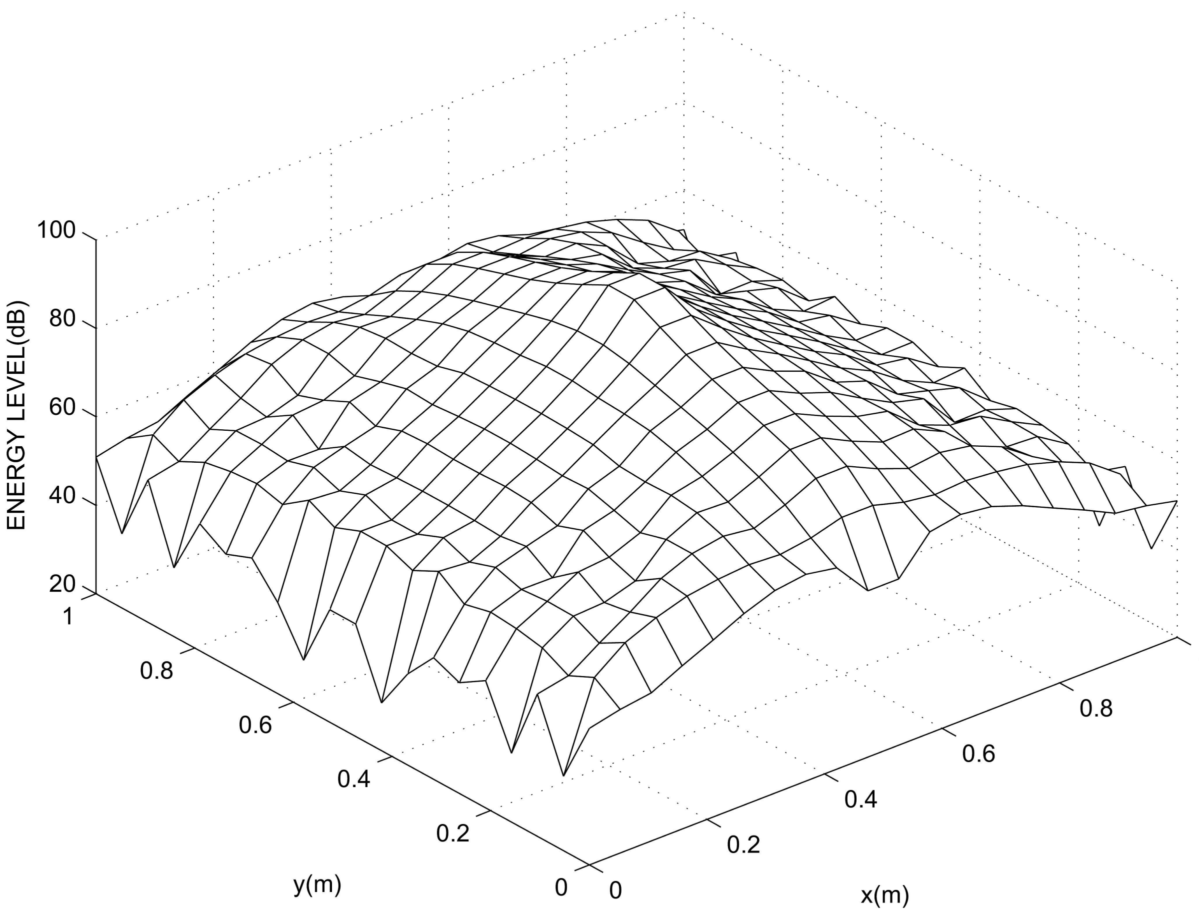

Figure 5: Classical modal energy density (f = 5 kHz, η = 0.2).

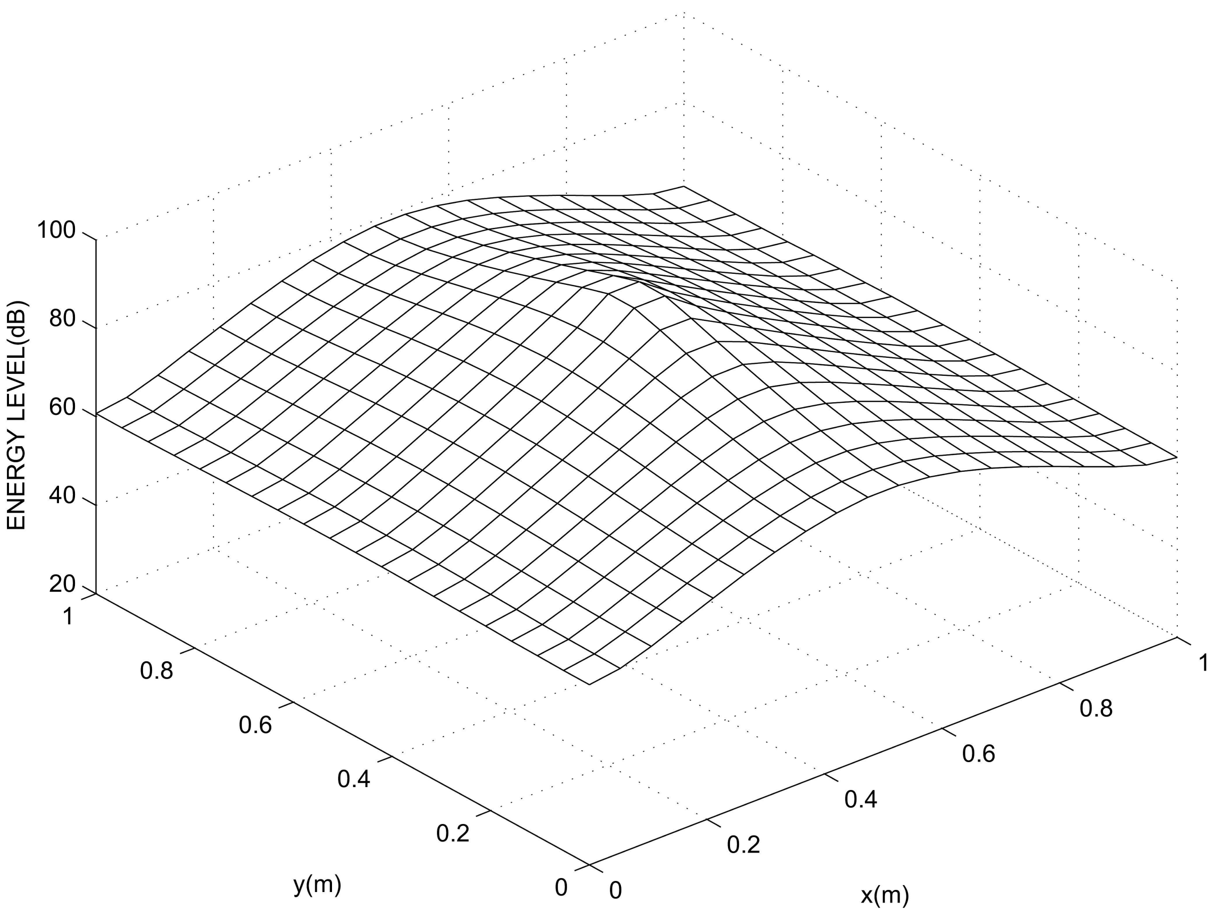

Figure 6: Approximate energy density ($f = 5$ kHz, $\eta = 0.2$).

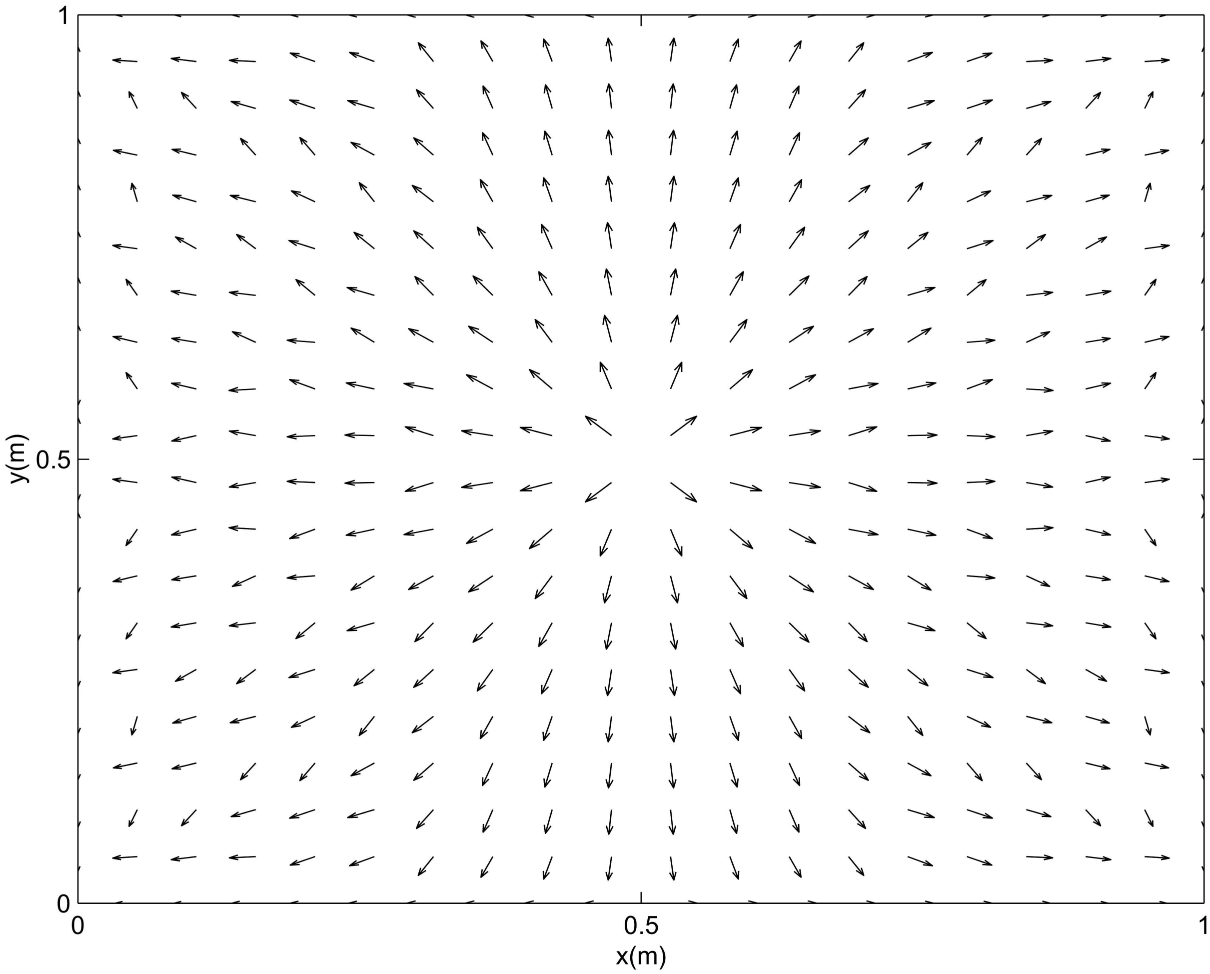

Figure 7: Classical modal intensity field ($f = 5$ kHz and $\eta = 0.2$).

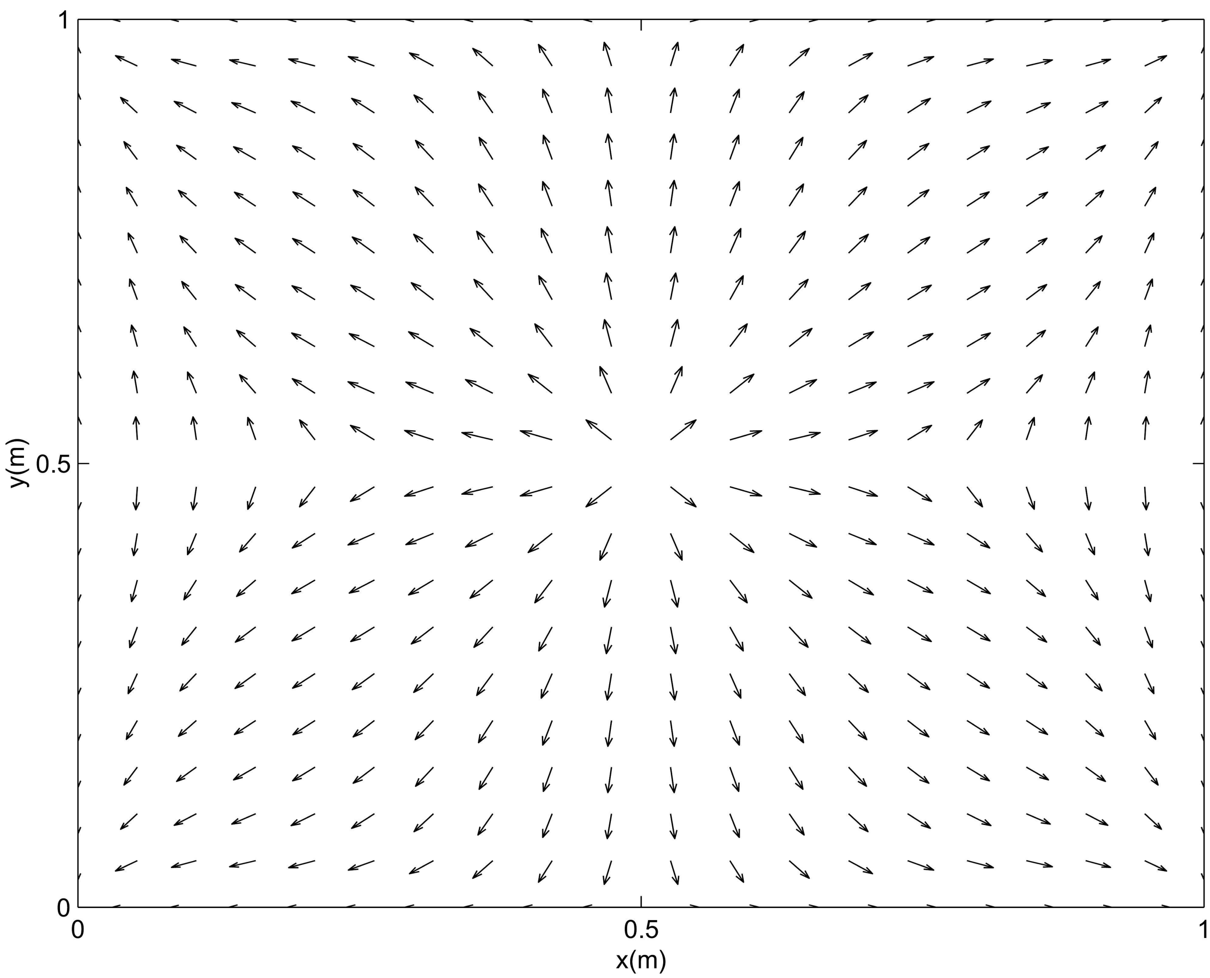

Figure 8: Approximate intensity field ($f = 5$ kHz and $\eta = 0.2$).

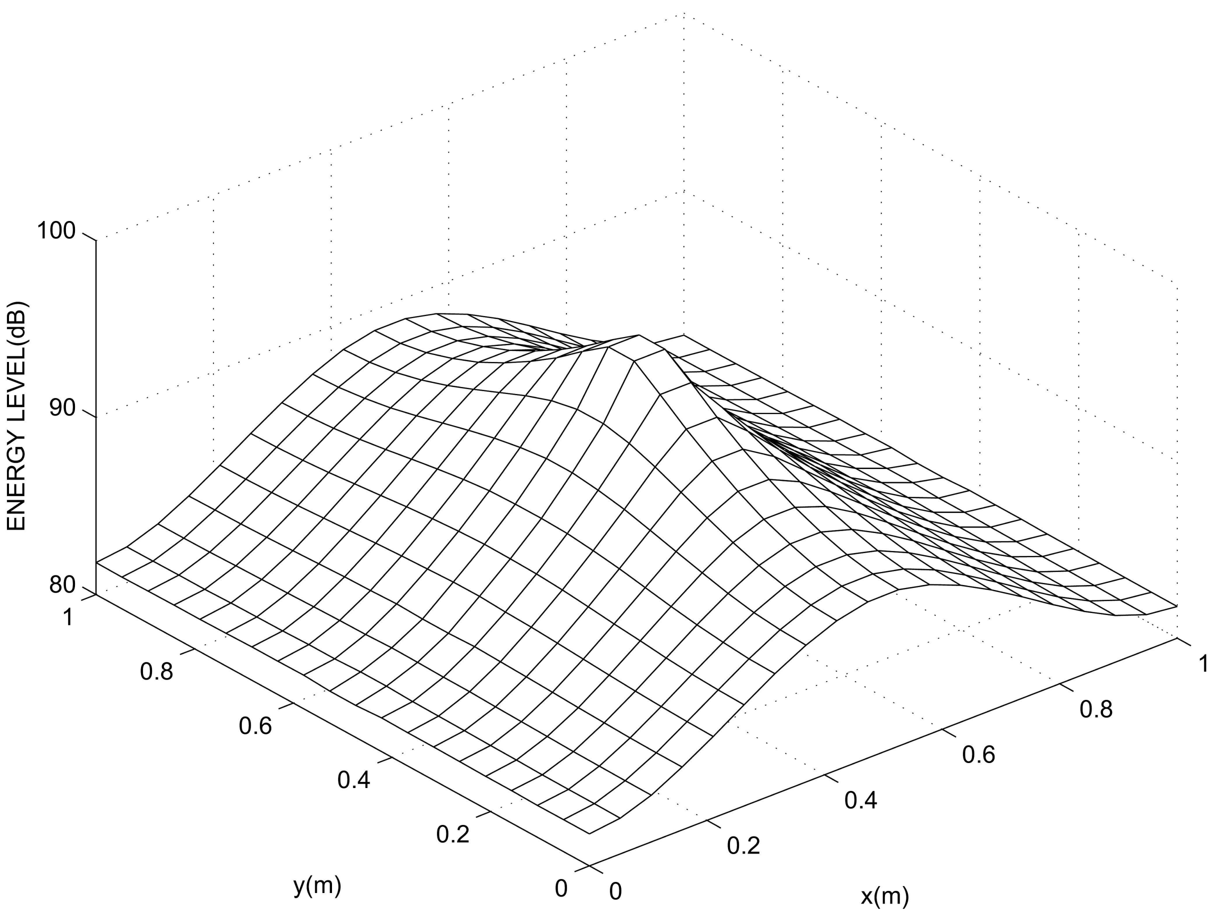

Figure 9: Approximate energy density of the orthotropic plate.

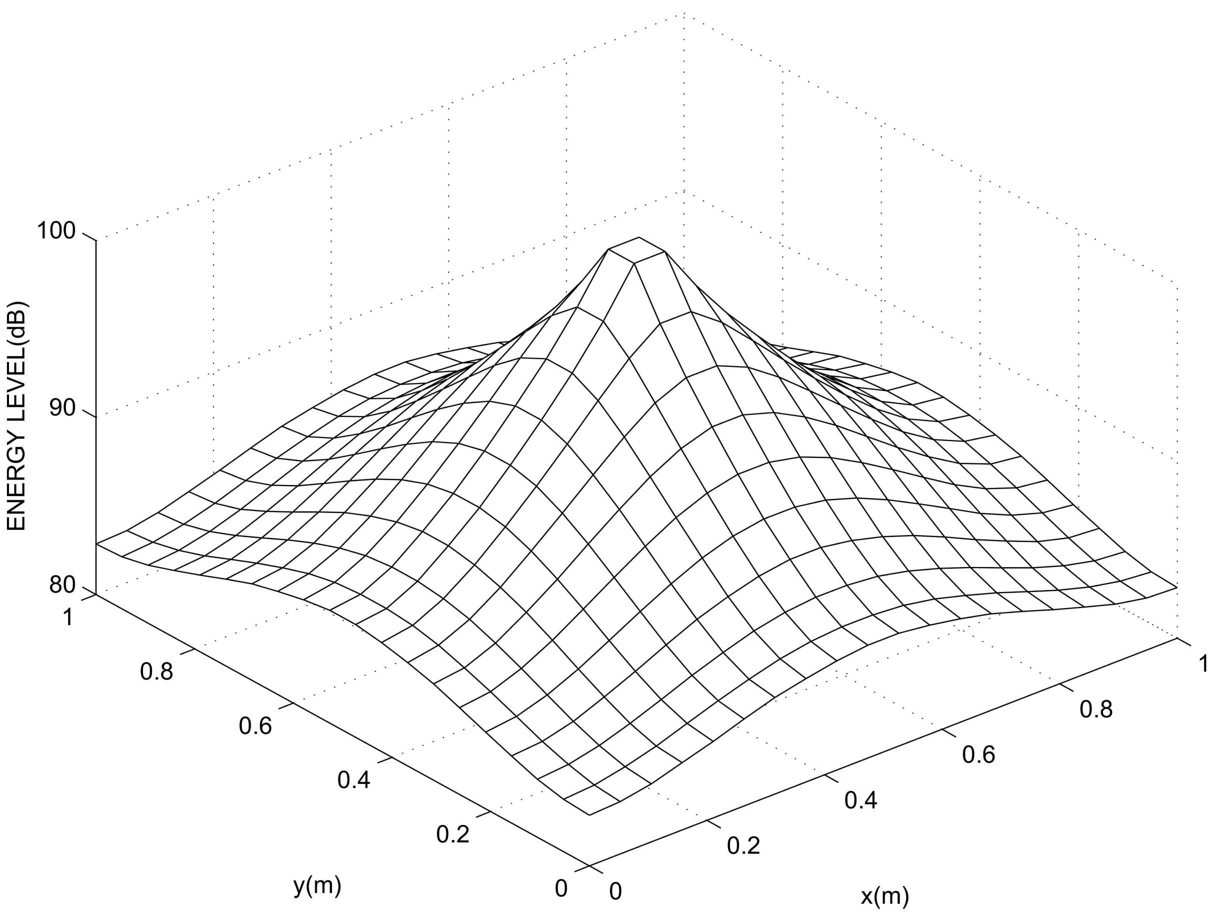

Figure 10: Approximate energy density of the isotropic plate.