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NOISE TRANSMISSION AND NONLINEAR VIBRATION OF CURVED PLATES

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ABSTRACT
A simple dynamic model of a general buckled plate using a single mode representation is used to present the typical trend of non-linear behavior. The highly nonlinear motion of snap-through and its effects on the overall dynamic response can be easily interpreted using the single-mode formula. The important non-linear dynamic characteristics of curved plates are found to be the transition from softening spring effects to hardening spring effects through the intermediate region of chaotic motion. The chaotic motion is a large amplitude dynamic snap-through motion between the two static buckled positions of the plate. The theoretical predications are in agreement with the experimental results. The experience from the studies can also be used to improve the testing and monitoring techniques for noise transmission and nonlinear vibration of curved plate under acoustic or air flow load.

1 - INTRODUCTION
A number of structural failures, vibration fatigue and noise transmission problems in building roof panels, aircraft fuselage, membrane cover of acoustic materials and ventilation ducts were found to be caused by the large amplitude movement of snap-through (which can changed the shape of the plate curvature from concave to convex and vice versa) under combined in-plane and transverse loading. The design of structures against snap-through are based on the static analysis and the use of approximate correction factor to account for dynamic effects. The study aims to develop experimental and theoretical techniques for non-linear dynamic test on flat and curved plates and to obtain the dynamic characteristics of the transverse motion under static in-plane loading.
Theoretical and experimental results of large-amplitude vibrations of post-buckled plates under sinusoidal excitation were obtained by Yamaki and Chiba [3]. The study did not include snap-through motion. The characteristics of snap-through motion in a bucked beam under sinusoidal excitation were studied by Tseng and Dugundji [2]. A theoretical study of the random response of an initially curved beam including snap-through motion was done by Seide [1]. The nonlinear acoustic response of thermally buckled plates was investigated by Ng [4]. A straightforward analysis of nonlinear snap-through behavior of post-buckled plates by simple analytical solution (using a single-mode analysis method) was done by Ng [4]. The paper showed the theoretical analysis method and experimental investigation with sinusoidal excitation forces on a buckled plate.

2 - THEORETICAL MODEL FOR DYNAMIC BEHAVIOR OF A BUCKLED PLATE
Using the von Karman equation for nonlinear analysis and the Galerkin's principle for the formulation, the following modal (non-dimenisonal) equations were obtained for a plate with modal displacement Q under modal force P (equation for equilibrium in the transverse direction).
For static equilibrium:

$$q^3 - Rq = p \tag{1}$$

For dynamic motion:

$$\frac{\ddot{q}}{\Omega^2} + \frac{2\varsigma\dot{q}}{\Omega} + \left(q^3 - Rq\right) = p \tag{2}$$

where

- q, nondimensional displacement parameter, = Q/Qp
- Q, modal displacement
- Qp, value of Q at R = 1
- $R = \lambda - 1$
- $\lambda = u/u_c$
- u = in - plane edge shortening displacement
- u_c = value of u at which buckling starts
- Ω = linear natural circular frequency of the flat configuration
- ς = modal damping coefficient
- p = nondimensional force parameter, $= \dfrac{P}{KQp}$
- P = externally applied modal force
- K = linear modal stiffness of the flat plate

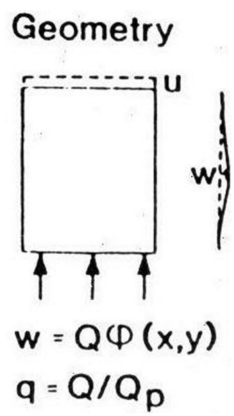

Figure 1.

Q_p, u_c, Ω, ς, K depend on the assumed shape function of the mode and other plate parameters. The nondimensional parameters, q, R, P, can be evaluated after Q_p, u_c, K are found by experiments or theories. The equation (2) involves only nondimensional parameters and is therefore independent of the plate parameters. Using equations (1)&(2), the nonlinear static and dynamic behaviors of a plate can be predicted and they are applicable to plates of any size, boundary conditions, materials properties.

3 - FREQUENCY RESPONSE OF A BUCKLED PLATE

For the case Ω=63, ξ=0.01, R=1 q(t=0)=1, and sinusoidal force, the nonlinear response of q is obtained by time integration procedures. Three regions of different nonlinear dynamic behavior:
1. **Softening spring** – the resonance frequency decrease with amplitude and the jump behavior is to the left-hand side of the resonance frequency. Dynamic displacement $\Delta q < 0.3$, mean displacement q = ~1.
2. **Chaotic** – the response is non-periodic, there are intermittent dynamic motion which overshoots the other static position q = -1. This is called snap-through motion between the static equilibrium positions. The snap through is of very low frequency and large amplitude compared with the oscillation around the static equilibrium position. Dynamic displacement $0.3 < \Delta q < 1$, mean displacement $0 < q < 1$. The chaotic region serves as the transition between two paths.

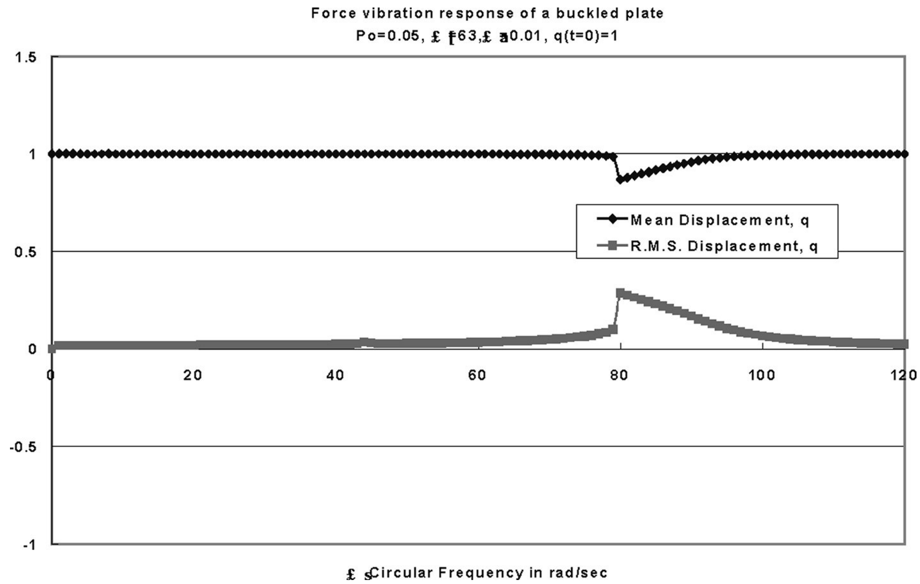

Figure 2: Soft spring behavior at 80 rad/s with Po = 0.05.

3. **Hardening spring** – when the snap-through motion gets more frequency and of larger amplitude, it becomes periodic again. The snap-through motion occurs in every cyclic of vibration. The frequency is initially low and increase rapidly with amplitude. The jump behavior is to the right hand side of the resonance frequency. Dynamic displacement $\Delta q > 1$, mean displacement q(mean) = 0.

A 25.4×254 ×0.813 mm aluminum alloy beam was tested with base excitation using 45kg shaker attached to the supporting frame. Super-harmonic response, chaotic motion, and softening and hardening spring effects are found in the frequency responses (Fig. 7) which is in agreement with the theoretical results.

4 - CONCLUSIONS

The important result of large amplitude vibrations of curved plates is the snap-through motion due to instability. The intermittent snap-through serves as a transitional region from the softening spring behavior to the hardening spring behavior. The presence of snap-through motion requires special care in measuring the mean values and low frequency of the dynamic displacement responses. Also it will give rise to problems in vibration and noise emission at low frequency.

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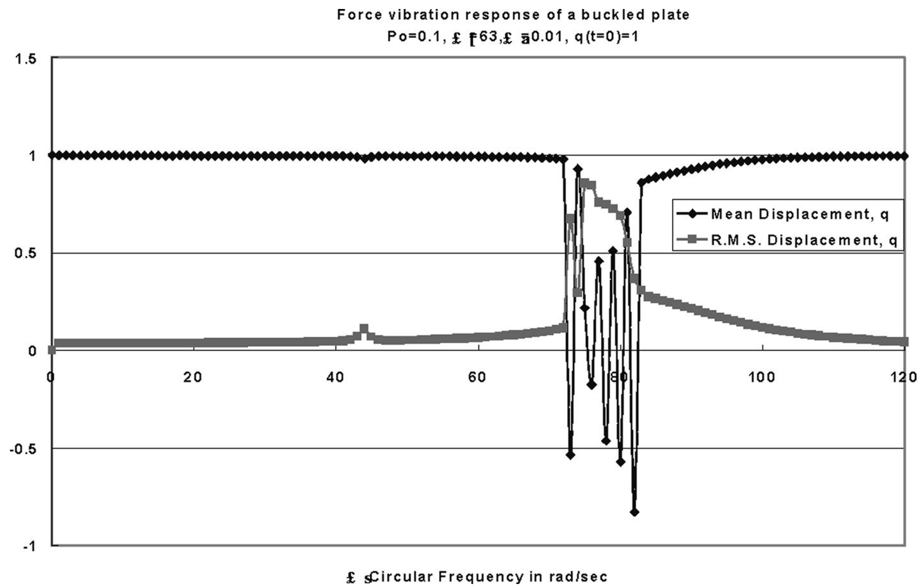

Figure 3: Chaotic behavior at 72 to 82 rad/s with Po = 0.1.

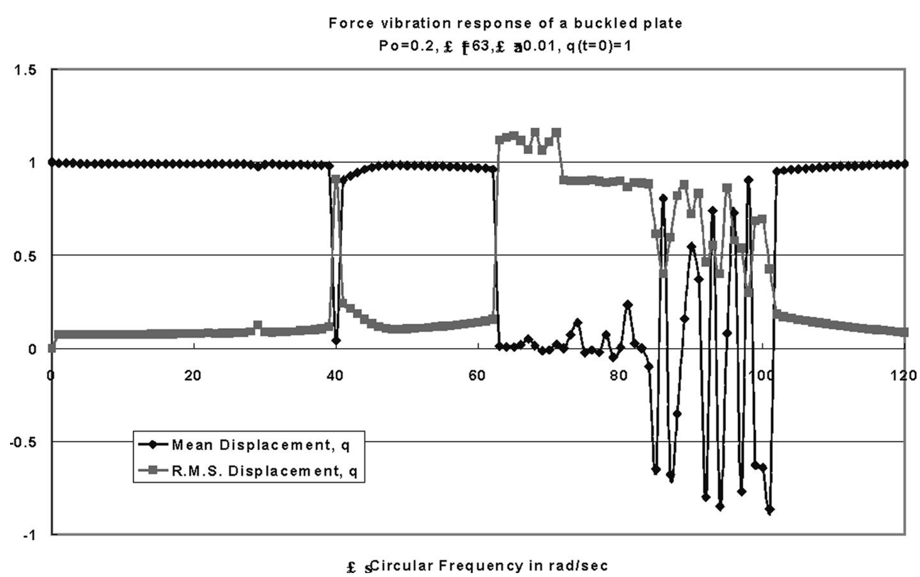

Figure 4: Chaotic behavior wit Po=0.2.

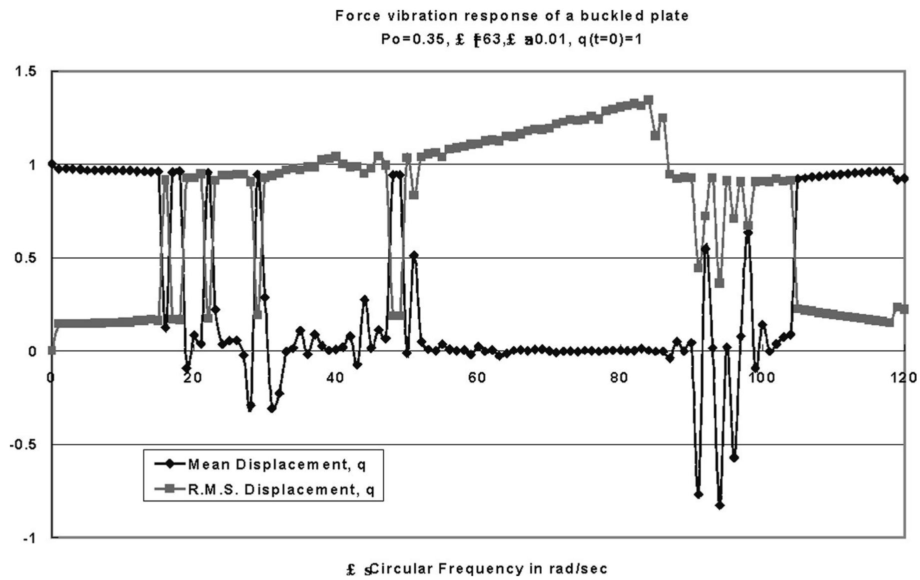

Figure 5: Chaotic behavior with Po = 0.35.

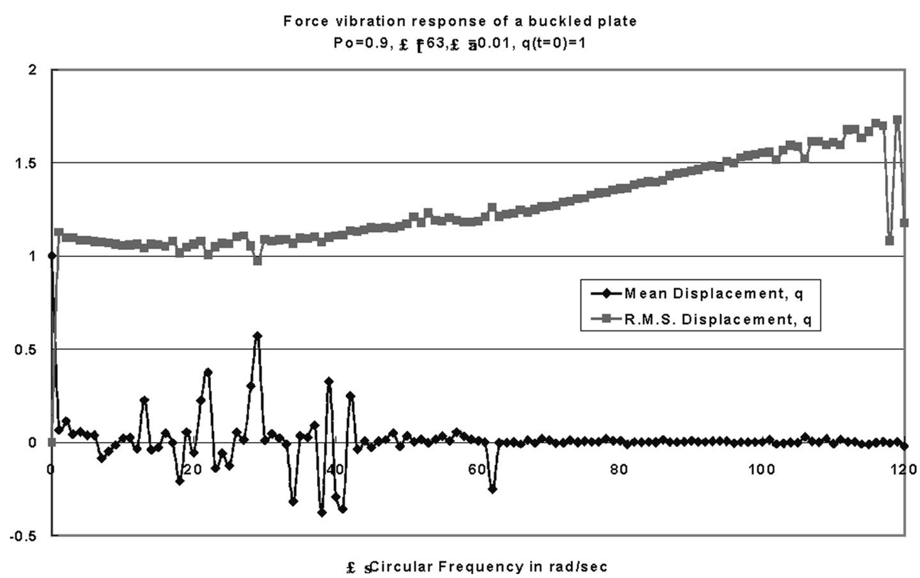

Figure 6: Chaotic behavior with Po=0.35.

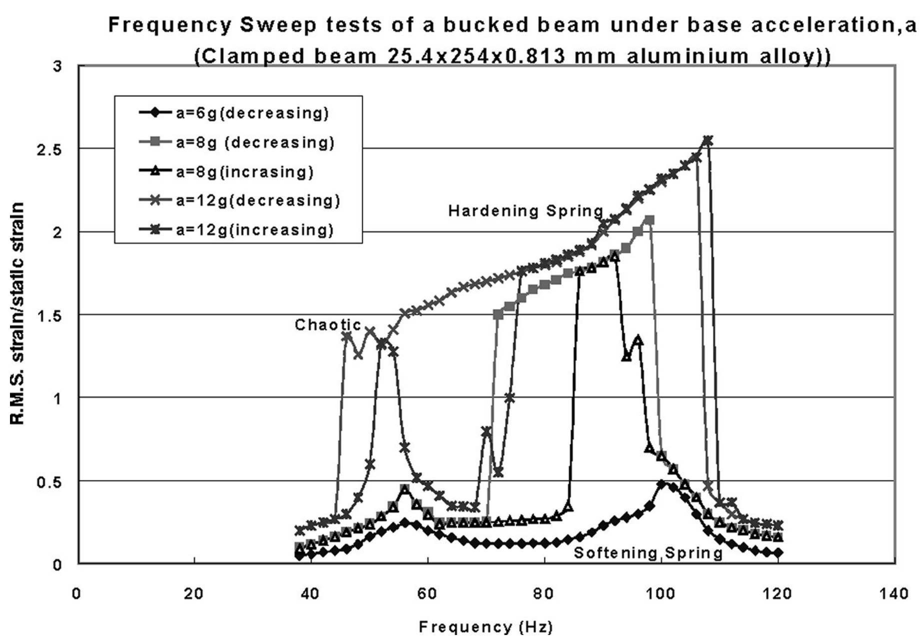

Figure 7: Frequency sweep tests of a buckled beam.