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THE CHARACTERIZATION OF VIBRATION SOURCES AND MEASUREMENT OF FORCES USING MULTIPLE OPERATING CONDITIONS AND MATRIX DECOMPOSITION METHODS

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ABSTRACT
The problem of understanding the sources in a machine is fundamental to building a model of the system including models of the forces that excite the system and models of the transfer paths between these forces and the responses of interest. Once the sources are understood, proper measurements can be made to experimentally determine the forces and the associated models can be developed to predict the response of the system to such forces. In this paper, strategies are described to formally characterize the sources to determine how many independent sources occur within the system. Additional strategies are developed to measure these forces indirectly using external measurements and to build a multiple-input, multiple-output model of the system. Both sets of strategies depend heavily on linear algebra to extract the independence of data gathered on the system. These data are interpreted to better understand the system and specify models.

1 - INTRODUCTION
In many machinery applications, it is difficult to locate transducers at the source of vibration and to measure a noise-free source signal that is characteristic of the system and independent of other mechanical sources in the system. In addition, the various sources exciting a system are often synchronized to certain events within the machine and are statistically coherent, and thus indistinguishable using simple spectral information. This makes it difficult to use direct implementation of multiple-input, multiple-output signal processing techniques on machinery [1]. With the availability of large-channel count data acquisition systems it is reasonable to do an initial large-scale investigative measurement of a machine under various operating conditions to characterize the machine. Linear algebra techniques are useful for this purpose since it is possible to evaluate a large amount of data simultaneously and sort these data according to their fundamental characteristics. This characterization process is useful both for the establishment of a strategy for indirect determination of forces and to identify a proper input-output model of the machine. Each phase of the process and the utility of linear algebra for each phase will be discussed in this paper.

2 - SOURCE CHARACTERIZATION
In many machines the sources of excitation are buried within the machine in an environment which is unfriendly to transducers and measurement systems. Thus, it is difficult to locate a transducer to directly measure a signal characteristic of a single source of vibration and noise. Surrounding structures, including those very close to the source, often respond to all sources in the system. Thus, using a motion response to characterize the source is difficult. The development of input-output models of a machine is made difficult by this unavailability of independent and dedicated transducers that directly measure each source in isolation.

The sources in most machines also tend to be synchronous. Thus, response transducers mounted on the structure of the machine will not only sense the response of the system to multiple sources but the user of these data will be unable to distinguish these sources based on spectral characteristics. However, different sources tend to be distinctive under different operating conditions. So while sources in a synchronously operating machine tend to be coherent, they will be distinguishable under different operating conditions. This type of behavior suggests that a strategy be developed whereby a system is observed in an extensive manner under different operating conditions and that a data reduction tool be used to resolve how many independent mechanisms are observable in this data set.

Kompella suggested that this characterization task could be done in the frequency domain using an extensive data set gathered on the exterior of the machine under different operating conditions [2,3]. Using modern data acquisition systems, an "overmeasured" system is recommended since redundant information is identifiable and is not harmful to the characterization process. The data would be structured in such a way that auto-spectra and/or cross-spectra can be evaluated for independence. The characterization process would be done using rank determination methods from linear algebra. Kompella suggests three analyses to help characterize the behavior of the system, NIS (number of incoherent sources) analysis [4], NIOC (number of independent operating conditions) analysis, and NIIT (number of independent input transducer) methods. The NIIT analysis is the most general and will be discussed here.

For NIIT analysis a large rectangular matrix is prepared from the measured data using the cross spectrum matrix at each operating condition for one spectral line (in either the frequency domain or the order domain, which ever is appropriate). This matrix has the form

$$[S_{xx}]_{NIIT} = \begin{bmatrix} [S_{xx}]_1 \\ [S_{xx}]_2 \\ [S_{xx}]_3 \end{bmatrix}$$

where the subscripts refer to the operating condition number. The rank of this matrix is the number of independent sources. The method is capable of identifying both coherent and incoherent sources as independent sources. The subset of transducers with proper rank and the lowest condition number is the set of transducers that most clearly observes the independent sources.

Using this scheme an estimate of the number of independent sources is made at each frequency. These results must be interpreted by the user. Generally, the interpretation is only done at spectral lines associated with synchronous events in the machine where the signal levels are significant. While there may be some inconsistency between the interpretation of data at different frequencies this was not found to be a significant difficulty for the applications studied.

The noise in these measurements is significantly higher than generally occurs in traditional rank determination processes. For classical rank analysis, the noise in the matrix is due to the numerical precision of the computer. Rank determination is done a threshold of the matrix singular values or eigenvalues based on this noise level. For vibration and noise applications the precision of the data is determined by the measurement chain. The range of singular values will be relatively narrow. A threshold must be chosen based on the precision of the measurements. For well-controlled applications, the range of singular values was generally only five orders of magnitude. The rank determination threshold must be set based on this type of precision.

The approach suggested by Kompella, and similar approaches, are important tools and should be used as a first step towards understanding a machine or building a model of the machine. This gives us a tool to understand the sources even under conditions where the excitation sources might be coherent. Once the machine is better understood, it is relatively straightforward to build models.

3 - INDIRECT FORCE MEASUREMENT

For some situations it is desirable to be able to estimate excitation forces within a machine where it is impossible to measure such forces directly. One approach for such applications is to use measured responses and known frequency response function characteristics between the forces and the response locations as an inverse approach to estimate these forces. For the indirect force measurement problem, the frequency response functions of the system between the excitation forces and the response locations must be measured under conditions of a "free body". The frequency response functions must be an accurate representation of the system under operating conditions.

Inverse methods are prone to large errors if the problem is not well conditioned. For the inverse problem, a machine characterization exercise is important not only to determine whether sources are independent, but how the inverse problem can be constructed such that it is robust to noise and small variations in the system due to operating conditions. Roggenkamp proposed that the characterization could be done using an "overmeasured" set of frequency response functions [5]. The sub-matrix of the frequency response function matrix with the lowest condition number across the frequency band of interest is the best set to use for the inverse force estimation process. Roggenkamp's approach included estimation of all of the cross spectral terms of the force vector using the equation

$$[S_{ff}] = [H_{fa}]^{H+} [S_{aa}] [H_{fa}]^{+}$$

where S_{ff} is the matrix of force auto- and cross-spectra, S_{aa} is the matrix of response (typically acceleration) auto- and cross-spectra, H_{fa} is the matrix of frequency response functions between the forces and responses, and the superscript H designates the matrix Hermetian (conjugate transpose). In general, more response measurements than forces can be used. Thus, the inverse problem is solved in a least squares sense. The superscript + indicates the pseudo-inverse matrix operation for least squares solutions.

Roggenkamp also developed a useful indicator of potential problems in the inverse force determination process. The relative error in the force matrix, dS_{ff} can be bounded based on the relative errors in the response matrix, dS_{aa}, the errors in the frequency response function matrix, dH_{fa}, and the condition number of the frequency response function matrix, $\kappa(H_{fa})$,

$$\frac{\|dS_{ff}\|_F}{\|S_{ff}\|_F} \leq \frac{\|dH_{fa}\| \left\|H_{fa^+}\right\|}{1 - \|dH_{fa}\| \left\|H_{fa^+}\right\|} + \frac{\kappa(H_{fa}) \|dH_{fa}\| \left\|H_{fa^+}\right\|}{\left(1 - \|dH_{fa}\| \left\|H_{fa^+}\right\|\right)^2} + \frac{\kappa(H_{fa})^2}{\left(1 - \|dH_{fa}\| \left\|H_{fa^+}\right\|\right)^2} \frac{\|dS_{aa}\|_F}{\|S_{aa}\|_F}$$

where $\|\|_F$ is the Frobenius norm of a matrix and $\|\|$ is the 2-norm. While this equation is written in terms of norms of the various matrices, the relationships generally hold reasonably true for the individual terms of the matrices as well, particularly if terms of the matrix are scaled to be the same order of magnitude. For the relatively simple case where the errors in the frequency response function matrix are negligible, the last term in the equation is the only term remaining. The errors in the inverse estimation of force auto- and cross-spectra are proportional to the square of the condition number of the frequency response function matrix and the errors in the response auto- and cross-spectra. Thus, if the condition number is appreciable, any errors in the response spectra will be amplified significantly by the inverse process. In both analytical and experimental testing Roggenkamp found that it was difficult to make good estimates of forces by the inverse technique when the condition number of the frequency response function matrix was greater than 10 due to the errors generally inherent in the spectral estimates of the response variables. A condition number of 10 is difficult to achieve in many practical applications. This type of problem is believed to dominate many applications of force estimation using inverse methods.

The techniques developed by Roggenkamp were utilized in modified form by Abram [6,7], and Gilmer [8] on engines. The applications here were more straightforward than the general case considered by Roggenkamp. In addition, time gating and filtering were used to condition the signals to reject response data known to be due to other forces. The input transducers were placed close to the source and relatively few transducers were used.

4 - INPUT/OUTPUT MODEL

To be robust to changes in operating condition, a multiple-input, multiple-output model must use the independent sources as inputs, not simply the incoherent sources. The schematic shown in Figure 1 is an illustration from Kompella [2,3] of the relationship between the real system and the model to be developed. For this modeling process it is assumed that the input transducers, X, are not able to measure the excitation sources, U, directly and are not isolated from the influence of the remainder of the physical sources or the independent mechanisms internal to the physical sources. However, some subset of these measurements must be used as the input to the model. There is a matrix of undetermined frequency response functions, G, between the sources, U, and the measured "inputs", X. The estimation of the frequency response function matrix, H, is critical to the development of a robust model. The composite frequency response function GH must match the frequency response function T as closely as possible. If too few X-inputs are used, the model will not be robust to changes in the operating condition. If too many transducers are used, the frequency response function estimation process will be rank deficient and not solvable.

In addition, when some of the independent mechanisms are coherent, traditional frequency response function estimation techniques are not possible. If only partially incoherent X-inputs can be used, too few H-frequency response functions will be developed to fully model the machine.

Kompella proposed that once the characterization process is complete and the number and location of independent input, the frequency response functions should be estimated using a $[S_{xx}]_{NIIT}$ submatrix with full rank and lowest possible condition number for the equation

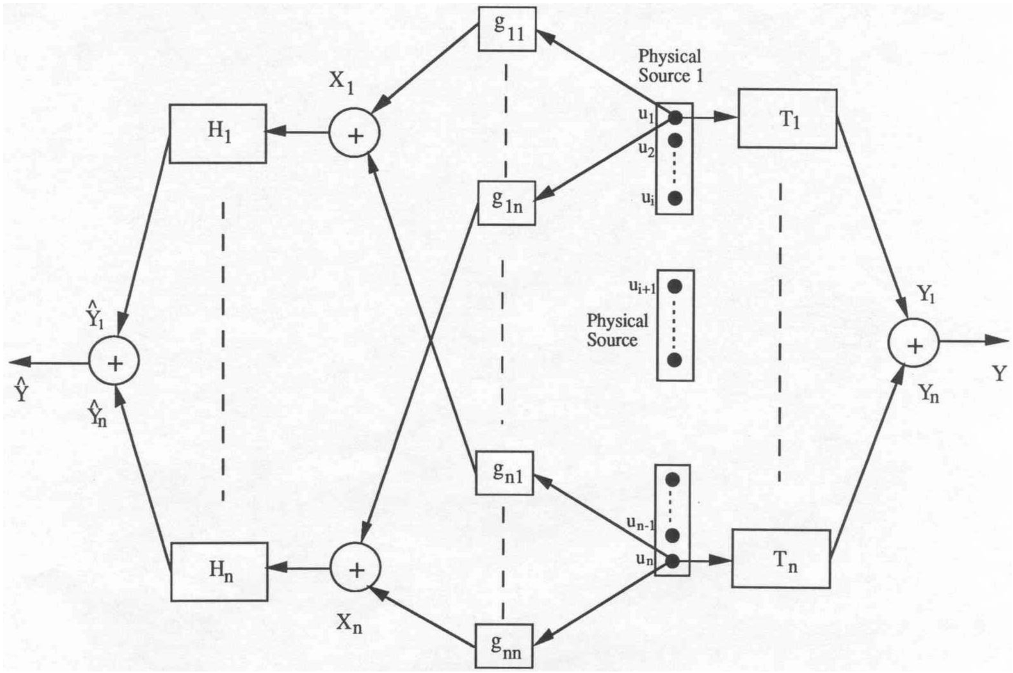

Figure 1: Schematic representation of the real system (T) and the model of the system (H) for machinery with imbedded physical sources and multiple independent mechanisms (from Kompella [2,3]).

$$\{H\} = [S_{xx}]^{+}_{NIIT}\,\{S_{xy}\}_{NIIT}$$

This frequency response function estimation process uses all of the operating condition information. With this approach it is possible to estimate frequency response functions when sources are coherent but independent. Several case studies were done with partially coherent sources under a range of operating conditions [2,3], [9]. The models were found to be straightforward to estimate, physically sensible, and robust to operating condition changes.

5 - CONCLUSIONS

The model development and frequency response function estimation process is significantly improved and more robust when the machine is well understood. A pre-model evaluation of the machine using a source characterization process with different operating conditions is very useful for understanding the behavior of the machine. With current hardware, overmeasurement of the machine for purposes of characterization is feasible. In these situations, the large data set can be evaluated in a relatively straightforward manner using linear algebra. Thus all of the tools are in place to revolutionize our process of understanding a machine and building a proper multiple-input, multiple output model.

When force estimation is desired, a pre-test and evaluation of the condition number of the frequency response function matrix can be used to improve the robustness of the inverse force process. Thus the indirect force measurement techniques can be better understood and improved using linear algebra.

In this paper, the methods for source characterization, indirect force estimation, and model development have been presented in an idealized and relatively formal form. Simpler and less formal variations of these approaches are possible, and recommended in many cases. However, the general approach and strategies are useful as a guideline and represent a good beginning point for design of an experiment approach for model development.

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