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THEORETICAL COMPARISON OF ACTIVE NOISE CONTROL USING POTENTIAL ENERGY ERROR SENSING AND INTENSITY ERROR SENSING

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ABSTRACT
This paper presents a theoretical comparison between pressure and intensity-based control strategies for active noise control in ducts. The control methods used for each strategy are the Frequency-Domain, Filtered-X LMS, which uses pressure as the cost function, and a method named ASIC (Active Sound Intensity Control), which uses mean active sound intensity as the cost function. A duct with two types of termination − open-ended, and rigidly terminated − was modeled using the Spectral Element Method. These models allowed the evaluation of the performance of the controllers in environments with propagation and reverberation. The criteria used to analyze the results were the potential energy density, the mean active intensity, and the total acoustic power radiated by the sources working together, compared with the primary source alone.

1 - INTRODUCTION
Lately, there have been several studies about methods that use intensity as the cost function. Sommerfeldt and Nashif [1] used a method based on the Filtered-X LMS algorithm, where the adaptive updating of the filter weights is done in the time domain, thus minimizing both the active and reactive power. Kang and Kim [2] also use a method based on the Filtered-X LMS algorithm, where the gradient of the product of the instantaneous pressure and velocity is computed, again in the time domain. Qiu and Hansen [3] use as cost function the intensity in the frequency domain, but there is no reference signal in their method. Because of this, the controller will try to cancel all the sound intensities in the environment, not only the intensity that is coherent with the reference signal. Swanson at al. [4] use the filtered-X LMS as an intensity control method by replacing pressure for intensity as the error signal and using the transfer function between the reference signal and the measured intensity as the filtered path. The proposed method originally proposed by Arruda et al. [5] uses the mean active intensity as the cost function. Furthermore, the method was implemented in the frequency domain, which has some advantages over the methods in the time domain, namely the possibility of controlling each frequency line independently. This method has been compared with the frequency domain filtered-X LMS [6,7] by Pereira et al. [8].
In this paper, an ideal duct with two types of termination − open-ended and rigidly terminated − is investigated. The criteria used to analyze the results were the potential energy density, the mean active intensity, and the total acoustic power radiated by the sources working together.

2 - THE SPECTRAL ELEMENT METHOD
The Spectral Element Method (SEM) [9] has the advantage that it is an exact solution. Therefore, only one element is necessary in each span between two impedance discontinuities. Furthermore, localized and lumped impedances can be easily added. It is much easier to use SEM to model acoustic ducts than to use other analytical models, as those presented in acoustics textbooks [10]. The one-dimensional duct element matrix of this method is given by:

$$\left\{ \begin{array}{c} \mathrm{U_1} \\ \mathrm{U_2} \end{array} \right\} = -\frac{\mathrm{S}}{\mathrm{c}\rho}\frac{1}{1-\mathrm{e}^{\mathrm{i2KL}}} \left[\begin{array}{cc} 1+\mathrm{e}^{\mathrm{i2KL}} & -2\mathrm{e}^{\mathrm{iKL}} \\ -2\mathrm{e}^{\mathrm{iKL}} & 1+\mathrm{e}^{\mathrm{i2KL}} \end{array} \right] \left\{ \begin{array}{c} \mathrm{P_1} \\ \mathrm{P_2} \end{array} \right\} \quad (1)$$

In the case of a rigid termination boundary condition, the element matrix in Eq. (1) applies directly. For open-end termination, the impedance at the end of the duct can be approximated by the radiation impedance, which must be added to the corresponding diagonal term of the global duct SEM matrix. This can be done using the following relationships [10]

$$\mathrm{U_1} = \frac{\mathrm{S}^2}{\mathrm{Z_m}}\mathrm{P_1} \qquad \text{where} \qquad \frac{\mathrm{Z_m}}{\rho \mathrm{cS}} = \frac{1}{4}\left(\mathrm{Ka}\right)^2 + \mathrm{i}0.6\mathrm{Ka} \tag{2}$$

3 - FREQUENCY-DOMAIN FILTERED-X LMS

The frequency domain filtered-X LMS is an adaptive control method that minimizes the square of the error function. Its equation in normalized form is given by [6,7]:

$$\mathrm{W}(\mathrm{n}+1) = \mathrm{W}(\mathrm{n}) - \frac{\mu}{\alpha + |\mathrm{X}(\mathrm{n})|^2}\left(\mathrm{X}'(\mathrm{n})\right)^* \mathrm{E}(\mathrm{n}) \tag{3}$$

where X'(n) is the filtered reference, E(n) is the error function (in this case, sound pressure), m is the step size (between 0 and 1), and α is a small constant used to avoid singularity (e.g., 1×10^{-15}). When applied to active noise control, the error function is pressure; so it controls the square of the pressure or, in other words, it controls the *potential energy density* at the point where the error signal is measured.

4 - ACTIVE NOISE INTENSITY CONTROL

In the Active Noise Intensity Control (ASIC) [5], the cost function is the square of the mean active intensity. In other words, it controls the power that is flowing at the error sensor location. Here, the intensity approximates by finite differences is used:

$$\mathrm{I} = -\frac{1}{2\omega\rho\Delta \mathrm{x}}\Im\left\{\mathrm{E}_1^*\mathrm{E}_2\right\} \tag{4}$$

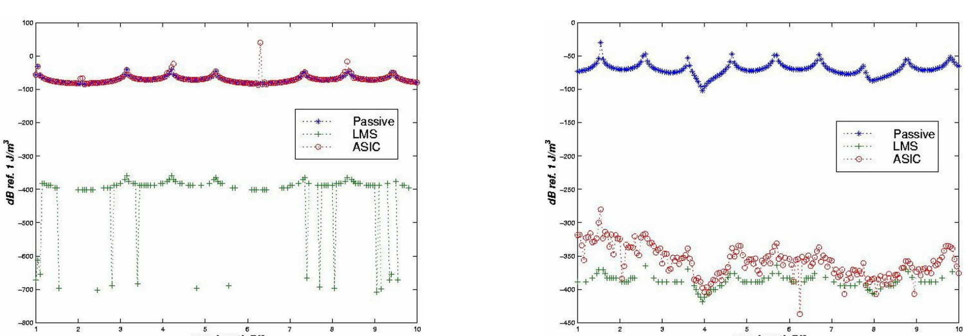
So, the equation of ASIC in its normalized form is given, for each frequency line, by [5], [8]:

$$\mathrm{W}(\mathrm{n}+1) = \mathrm{W}(\mathrm{n}) + \mu\frac{\mathrm{X}^*(\mathrm{n})}{|\mathrm{X}(\mathrm{n})|^2}\mathrm{i}\frac{\left[\mathrm{E_1}(\mathrm{n})\mathrm{S}_2^*(\mathrm{n}) - \mathrm{E_2}(\mathrm{n})\mathrm{S}_1^*(\mathrm{n})\right]}{\Im\left\{\mathrm{S_1}(\mathrm{n})\mathrm{S}_2^*(\mathrm{n})\right\}} \tag{5}$$

where X(n) is the reference, $\mathrm{E_1}$(n) and $\mathrm{E_2}$(n) are the measured sound pressures, $\mathrm{S_1}$(n) and $\mathrm{S_2}$(n) are the control path frequency response functions, m is the step-size (between 0 and 1), i is imaginary number. All variables are in the frequency domain.

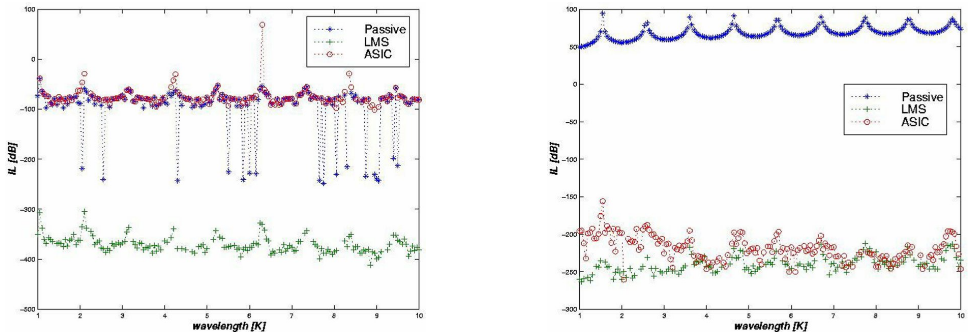
5 - SIMULATIONS

The simulations were performed in a theoretical duct with length L = 3 meters, radius a = 3 inches. The primary source was at position $\mathrm{x_p} = 0$ and the volume velocity was simulated in the frequency range 50-550 Hz with 2 Hz resolution, below the cut-off frequency of the duct, which was approximately 1.3 kHz. The secondary source was located at position $\mathrm{x_s} = \mathrm{L}/2$, the error sensor $\mathrm{E_1}$ was at position 3L/4, and the error sensor $\mathrm{E_2}$ was placed at 5 centimeters from the first error sensor, $\mathrm{E_1}$. A step-size of μ = 0.5 and 100 iteration steps were used. First, the duct was simulated with the rigid termination, closed in x = L, and, then, the case of the duct with an open end at x = L was considered.



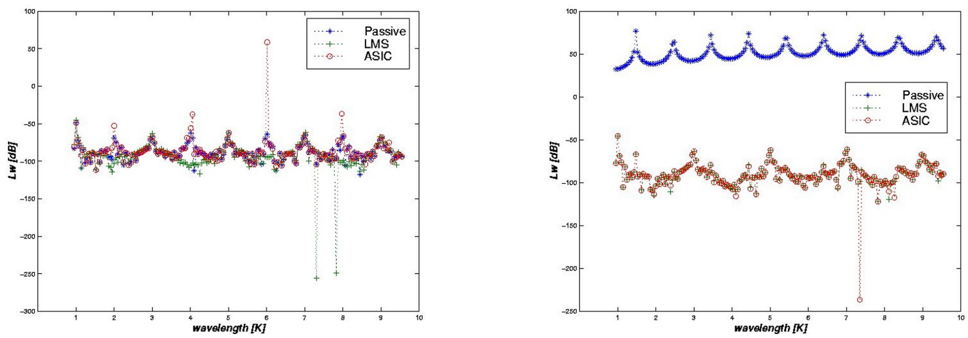
(a): Rigidly terminated. **(b):** Open-ended.

Figure 1: Potential energy density.



(a): Rigidly terminated. **(b):** Open-ended.

Figure 2: Mean active intensity.



(a): Rigidly terminated. **(b):** Open-ended.

Figure 3: Sound power radiated by the sources.

6 - CONCLUSIONS

Exact simulations of a duct system were implemented using the Spectral Element Method. This method is a very useful simulation tool for investigating active noise control strategies in duct systems. The simulations showed that when the control methods cancel the noise downstream the sources. In a not purely reverberant duct (open-ended case), the performances of both methods were the same. Both lead to the optimal solution, either using potential energy density or mean active intensity as the cost function. In short, there is no difference between them in this situation. However, as expected, when the methods are applied to a purely reverberant duct (rigidly terminated), only the LMS was effective, as there is no mean (active) intensity to be controlled in the duct. Another interesting remark is that neither the LMS nor the ASIC controllers could control in wavelengths $K = (2n - 1)\,\pi/L$, because at these wavelengths there are pressure nodes at the secondary source location.

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