EXPERIMENTAL VALIDATION OF 3-D POROELASTIC FINITE ELEMENTS BASED ON BIOT THEORY

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ABSTRACT
The paper deals with experimental validation of 3-D poroelastic finite elements based on Biot displacement theory in the context of vibroacoustic applications. The validation experiment involves the impedance measurement of a resonant porous sample in a duct with lateral air gap, allowing a real 3-D motion of the material with skeleton motion and acoustic coupling. Emphasis is made on the mechanical properties of the skeleton: isotropic and axisymmetrical elasticity laws are tested. Good agreement is found in terms of resonance location and absorption efficiency, validating the numerical approach. Isotropic law using adequate characteristics gives better results, although the skeleton is not isotropic. This shows that the determination of the viscoelastic properties of the skeleton is of importance in the modeling of poroelastic materials.

1 - INTRODUCTION
Porous materials like glass wool and plastic foam are widely used for noise control in different areas such as building construction, aeronautics and automotive industries. In order to investigate the effect of such materials in finite size structures, finite element codes including 3-D poroelastic elements based on Biot displacement theory [1] have been recently developed [2-5]. However validation of these formulations is incomplete for two reasons. The first is that comparison is made with lateral extent multilayers excited by a normal incidence plane wave, reducing the validation in the case of 1-D motion. Secondly, preliminary mechanical characterisation of the porous material is not made. Only Vigran and al. [6] show satisfactory results for a sandwich plate with a foam core, assuming the porous material being isotropic. The present paper deals with validation in comparison with a real 3-D motion of the porous material with skeleton motion and acoustic coupling: it concerns the impedance measurement of a resonant porous sample in a duct with lateral air gap. Because determination of specific porous parameters (porosity, air flow resistivity, tortuosity, viscous and thermal characteristic lengths) has been widely presented [7-8], focus is first made on the determination of the skeleton mechanical properties.

2 - MECHANICAL PARAMETERS DETERMINATION
Anisotropy is currently observed on foams [9] and fibrous materials. In order to investigate both isotropic and axisymmetrical viscoelastic skeletons, a quasistatic measurement method has been developed [10]. It is based on a small amplitude sinusoidal compression of a cubic sample of 50 mm edge size between two planes for which applied force, longitudinal and lateral displacements are monitored. The ratio $K$ of force over longitudinal displacement is homogeneous to a stiffness, and the ratios $\frac{R}{R'}$ between the two lateral and the longitudinal displacements are homogeneous to a Poisson ratio. Positioning the sample successively according to its three axes gives three triplets. If all triplets are equal and $R=R'$, then the
material is isotropic: a numerical inversion is processed to determine the Young modulus $E$ and the Poisson ratio $\nu$. If only two triplets are equal, the material is axisymmetrical. Then five independent complex components of stress-strain tensor are determined: moduli $E_L$, $E_T$, $G_{LT}$ and ratios $\nu_{LT}$, $\nu_{TT'}$, where subscripts $L$ and $(TT')$ are respectively related to longitudinal and transverse directions. The quasistatic frequency range is restricted to frequencies which are well below the resonance of the sample, typically from 0.1 Hz up to 100 Hz. In this low frequency range, inertial and viscous coupling with the air can usually be neglected.

The material used for the validation experiment is a plastic foam. Measurement of triplets $(K, R, R')$ shows that this material is not isotropic. However isotropic inversion can be made for each triplet: this gives three couples of pseudoYoung modulus and Poisson ratio according to each direction $L$, $T$ and $T'$. These values do not characterise the viscoelastic tensor of the material. An axisymmetrical inversion has been also made, using direction $L$ for symmetry. Table 1 gives the corresponding values. The other parameters are: $\phi=0.97$, $\sigma=165500 \text{ Nm}^{-2}$s, $\alpha_{\infty}=1.8$, $\Lambda=60 \mu m$, $\Lambda'=180 \mu m$, $\rho_{\text{skeleton}}=39.5 \text{ kgm}^{-3}$, $\eta_{\text{skeleton}}=0.11$.

<table>
<thead>
<tr>
<th>Isotropic inversion</th>
<th>$L$</th>
<th>$T$</th>
<th>$T'$</th>
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<tbody>
<tr>
<td>$E$</td>
<td>206 kPa</td>
<td>136 kPa</td>
<td>100 kPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.45</td>
<td>0.31</td>
<td>0.26</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Axisymmetrical inversion</th>
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<tbody>
<tr>
<td>$E_L$</td>
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<td>$E_T$</td>
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<td>$G_{LT}$</td>
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<tr>
<td>$\nu_{LT}$</td>
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<td>$\nu_{TT'}$</td>
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Table 1: Parameters determined from isotropic and axisymmetrical inversions.

3 - EXPERIMENTAL VALIDATION
The validation experiment concerns the impedance measurement of a porous sample in a duct of section $10 \text{ cm} \times 10 \text{ cm}$ (Figure 1). The porous sample and the boundary conditions have been chosen so that one resonance of the skeleton can be observed in the frequency range of 50 Hz to 500 Hz. Its thickness is 10 cm. The rear surface of the sample is bonded to the rigid end of the duct. Lateral surfaces are separated from the duct by an air gap of either 1.8 mm or 5 mm width. The measurement is achieved using Sybert ans Ross technique [11].

Simulations have been done with a 3-D finite element code, Phenix, coupling poroelastic, fluid and elastic elements [5]. One quarter of the geometry has been meshed by $5 \times 5 \times 10$ linear hexaedral elements for the porous sample. A compatible mesh is used for the air domain, with only one element along the air gap width. The excitation is simulated by a flat piston at a distance from the sample. This ensures flatness of the acoustic field where pressure is taken for the calculation of the impedance. Both isotropic and axisymmetrical laws are used to model the skeleton motion. The values corresponding to the longitudinal direction are used for the isotropic model: this direction corresponds to the axis of the tube. A calculation using equivalent fluid element (= motionless skeleton) is also made for comparison purpose.

Figure 2 presents the comparison between measured and simulated absorption coefficients. The general shape of both curves is well predicted by the equivalent fluid model. By comparison with the measured quantities, it emphasizes the skeleton resonance around 150 Hz (Fig. 2a) and 200 Hz (Fig. 2b).
agreement with Biot models is very good, specially when isotropic law is used: the resonance is well estimated and the influence of the skeleton motion is clearly noticeable.

![Graphs showing absorption coefficients for different air gaps](image)

(a): Lateral air gaps of width 1.8 mm.  
(b): Lateral air gaps of width 5 mm.

Figure 2: Comparison of simulated and measured absorption coefficients of a porous sample in a duct.

However results are not so satisfactory when axisymmetrical law is used. This shows that axisymmetrical law is not applicable to this material. Two reasons can explain this fact. In the first instance, the stiffness in the two lateral directions are not strictly equal (Table 1), and that distorts the inversion. Secondly the symmetry axis may not coincide exactly with the longitudinal direction.

In conclusion, it appears that the determination of skeleton mechanical parameters for simulation purpose has to be made carefully because of anisotropy. The use of isotropic law is efficient when mechanical parameters are related to the kind of deformation undergone by the material. In these conditions, it is shown that the use of 3-D poroelastic elements based on Biot theory is a powerful way to model real structures including porous media.

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REFERENCES


