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IMPEDANCE CONSIDERATION FROM WAVE REFLECTION AT ACOUSTIC/POROUS INTERFACE

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ABSTRACT
The acoustical impedance at ground interface is traditionally derived on the basis of local reaction of the ground to an incident acoustic wave without consideration for interaction between the sound wave and the induced surface waves in the ground. To account for this air-to-ground coupling a two-dimensional model of an acoustic medium over a poro-elastic ground subjected to plane acoustic waves is considered in this paper. From the analytical solution of the coupled wave equations in the two media expressions are derived for the acoustic and seismo-acoustic impedances. The paper presents a parametric study of the effect of frequency and incidence angle on these quantities. In addition, for an instrumented airblast site where noise and vibration measurements were made, the impedance terms are derived from the analytical model and their sensitivity to wave velocity in ground is investigated.

1 - INTRODUCTION
Prediction of long-range, outdoor sound propagation is often based on the assumption that the ground acts as a locally reacting surface represented by its acoustic impedance (e.g. [1]). Commonly used models for impedance calculations include those by Delaney and Bazley [2] and Attenborough [3]. The Delaney-Bazley's single parameter model represents the porous ground by its flow resistivity, whereas that by Attenborough treats the porous ground as a rigid frame with randomly varying pore sizes and is characterised by four parameters. Representation of the energy absorption of the ground by locally-reacting models may fail to realistically account for air-to-ground coupling which develops under the propagation of sound and its interaction with the surface waves in the ground. The present paper aims at addressing this issue by investigating analytically the reflection of acoustic plane waves at the surface of a poro-elastic halfspace. The solution of the coupled wave propagation equations in the two media is used to derive both the acoustic impedance, relating the overpressure and particle velocity in air, and the seismo-acoustic impedance, relating overpressure to particle velocity of ground surface. The paper presents a parametric study of the effect of frequency and incidence angle on impedance. In addition, for an instrumented airblast site, where outdoor noise and vibration measurements were made, the seismo-acoustic impedance is derived analytically and its sensitivity to the surface wave velocity in ground is investigated.

2 - ANALYTICAL MODEL
The model adopted here for the mathematical derivation of impedance is a fluid (acoustic medium) over a poro-elastic half-space subjected to a plane acoustic wave impinging on the interface between the two media. If the vectors $\mathbf{u}$ and $\mathbf{U}$ denote respectively the displacement of solid frame and pore fluid and $\mathbf{w}= \Omega \times (\mathbf{u}-\mathbf{U})$ is the relative pore fluid displacement, with Ω denoting porosity, then Biot's equations of dynamic poro-elasticity can be expressed as [4,5]

$$\left\{\begin{array}{l} \mu\nabla^2\mathbf{u} + (H-\mu)\,\nabla e - c\nabla\zeta = \rho\ddot{\mathbf{u}} - \rho_f\ddot{\mathbf{w}} \\ C\nabla e - M\nabla\zeta = \rho_f\ddot{\mathbf{u}} - m\ddot{\mathbf{w}} - \dfrac{\eta}{k}F\left(\omega\dot{\mathbf{w}}\right) \end{array}\right. \tag{1}$$

where ρ_f = mass density of pore fluid, $\rho = \Omega\rho_f + (1-\Omega)\rho_s$ is the average mass density of material, $m = \alpha\rho_f/\Omega$, in which α represents the tortuosity of pore structure with values ranging from 1 to 3 for most materials [6], $\eta = \rho_f\nu$ is the dynamic viscosity of pore fluid, ν = kinematic viscosity and k = permeability. Furthermore, $e = \nabla\cdot\mathbf{u}$ is the volumetric strain of solid frame, $\zeta = \nabla\cdot\mathbf{w}$ is the increment in fluid content, μ is the shear modulus of skeletal frame, and the deformation moduli C, M, and H are defined by

$$M = \left(\frac{\beta-\Omega}{K_s} + \frac{\Omega}{K_f}\right)^{-1};\ C = \beta M;\ H = \beta M^2 + K + \frac{4}{3}\mu \tag{2}$$

with $\beta = 1 - K/K_s$, where K_s = bulk modulus of solid grains, K_f = bulk modulus of pore fluid, and K = bulk modulus of solid frame which can be calculated from the values of μ and Poisson's ratio, n, according to $K = \mu\left(2/3 + 2n/(1-2n)\right)$. Finally, $F(\omega)$ is a frequency-dependent correction to the viscosity, which is a function of pore structure, permeability and viscosity according to $F(\omega) = \left(1 + \frac{i}{2}\frac{\omega}{\omega_c}N\right)^{0.5}$ where $\omega_c = \dfrac{\eta\Omega}{\rho_f k\alpha}$ and $N \approx 1$ for most porous media [6,7].

Following [6] and [8], the interaction of a harmonic plane wave with frequency ω, which is incident from the fluid onto the porous solid, can be solved using wave potentials. To this end, the displacement of the fluid medium is represented by a scalar potential and the displacements of the solid and pore fluid are represented by scalar and vector potentials

$$\begin{cases} \mathbf{u} = \nabla\phi_s + \nabla\times\Psi_s \\ \mathbf{w} = \nabla\phi_f + \nabla\times\Psi_f \end{cases} \quad \text{with } \mathbf{\Psi} = (0,\psi,0) \tag{3}$$

An incident acoustic wave is reflected as an acoustic wave in the fluid and refracted as one shear and two pressure waves in the porous solid. For an incidence angle θ from the vertical, the wavenumber in the fluid is $k_f = \omega/c$ and the incident and reflected wave potentials are

$$\begin{cases} \phi_i = A_i \exp\left[i\left(\omega t - k_z z - k_x x\right)\right] \\ \phi_r = A_r \exp\left[i\left(\omega t + k_z z - k_x x\right)\right] \end{cases} \tag{4}$$

where $k_x = k_f \sin\theta$ and $k_z = k_f \cos\theta$. The potentials in the porous medium can be written as

$$\begin{cases} \phi_s = A_1 \exp\left[i\left(\omega t - k_{1z} z - k_x x\right)\right] + A_2 \exp\left[i\left(\omega t - k_{2z} z - k_x x\right)\right] \\ \phi_f = B_1 \exp\left[i\left(\omega t - k_{1z} z - k_x x\right)\right] + B_2 \exp\left[i\left(\omega t - k_{2z} z - k_x x\right)\right] \\ \psi_s = A_3 \exp\left[i\left(\omega t - k_{3z} z - k_x x\right)\right] \\ \psi_f = B_3 \exp\left[i\left(\omega t - k_{3z} z - k_x x\right)\right] \end{cases} \tag{5}$$

where the subscripts 1, 2 and 3 stand for the P_1, P_2 and S waves in the porous medium. The vertical wavenumbers k_{1z}, k_{2z} and k_{3z} can be determined by substituting the above potentials in Eqns (1) and (3) and setting the determinant of the resulting algebraic equations to zero

$$k_{1,2}^2 = \omega^2 \frac{(Hm' + \rho M - 2\rho_f C) \pm \sqrt{(m'H - \rho M)^2 + 4(\rho_f m - m'C)(\rho_f H - \rho C)}}{2(HM - C^2)} \tag{6}$$

$$k_3^2 = \omega^2 \frac{\rho}{\mu}\left(1 - \frac{\rho_f^2}{\rho m'}\right) \tag{7}$$

where $m' = m - i\eta F(\omega)/(\omega k)$ and the definition $k_i^2 = k_x^2 + k_{iz}^2$ is introduced for convenience.

To determine the potential amplitudes for the reflected and refracted waves one has to impose the pertinent boundary conditions at the fluid-porous solid interface. These include:

- continuity of normal fluid displacement
- continuity of normal traction
- continuity of fluid pressure
- vanishing of tangential stress

For a given incidence angle θ and frequency ω, imposition of the these conditions leads to an algebraic equation for the unknown potentials in terms of amplitude of the incident wave A_i. Subsequently, the displacement $\mathbf{u}$ and velocity $\dot{\mathbf{u}} = i\omega\mathbf{u}$ at any point in the poro-elastic medium can be determined with the help of Eq. (3).

The pressure and vertical velocity in fluid at the fluid-solid interface can be calculated from

$$\begin{cases} P_a = -k_f \nabla^2 (\phi_i + \phi_r) = \rho_f \omega^2 (A_i + A_r) \\ V_a = \dfrac{\partial}{\partial t}\left[\dfrac{\partial}{\partial z}(\phi_i + \phi_r)\right]_{z=0} = \omega k_z (A_i - A_r) \end{cases} \tag{8}$$

If V_g denotes the vertical component of particle velocity in solid frame, then the complex acoustic impedance, Z_{aa}, and the seismo-acoustic impedance, Z_{ag}, of the ground are defined as

$$\begin{cases} Z_{aa} = P_a / V_a \\ Z_{ag} = P_a / V_g \end{cases} \tag{9}$$

The wave attenuation in both media can be accounted for by using hysteretic damping. To this end, all modulus terms are represented as complex quantities in the form $K = K_0 (1 + 2i\xi)$ where ξ is the hysteretic damping ratio in the material.

3 - PARAMETRIC STUDY

A number of results are presented here to highlight the influence of some key parameters on the impedance. An attempt is also made to investigate the consequences of treating the ground as a locally reacting surface. The material parameters in these analyses are given in Table 1.

ρ_f (kg/m^3)	ρ (kg/m^3)	Ω	α	n	ξ	ν (m^{2}s^{-1})	K_f (Pa)	K_s (Pa)	K (m^2)
1.2	1800	0.35	1.25	0.40	0.01	1.45 $\times 10^{-5}$	1.3 $\times 10^5$	3.6 $\times 10^{10}$	2.3 $\times 10^{-10}$

Table 1: Material parameters in impedance calculations.

In addition, the sound speed is assumed equal to 330 m/s. The shear-wave velocity of ground, $V_s = (\mu/\rho)^{0.5}$, however, is taken as a variable. It is shown in the following that the ratio between these two velocities is a major factor affecting the impedance.

Figure 1 displays the variation with incident angle, θ, of the absolute values of Z_{aa} and Z_{ag} for $V_s =$ 300 m/s and for frequencies ranging from 10 to 100 Hz. The figure demonstrates that, whereas Z_{aa} is strongly dependent on frequency, Z_{ag} is fairly insensitive to frequency but on the other hand it is strongly dependent on θ. However, in the range of practical interest between 45 to 90 degrees, one may discount any θ-dependence.

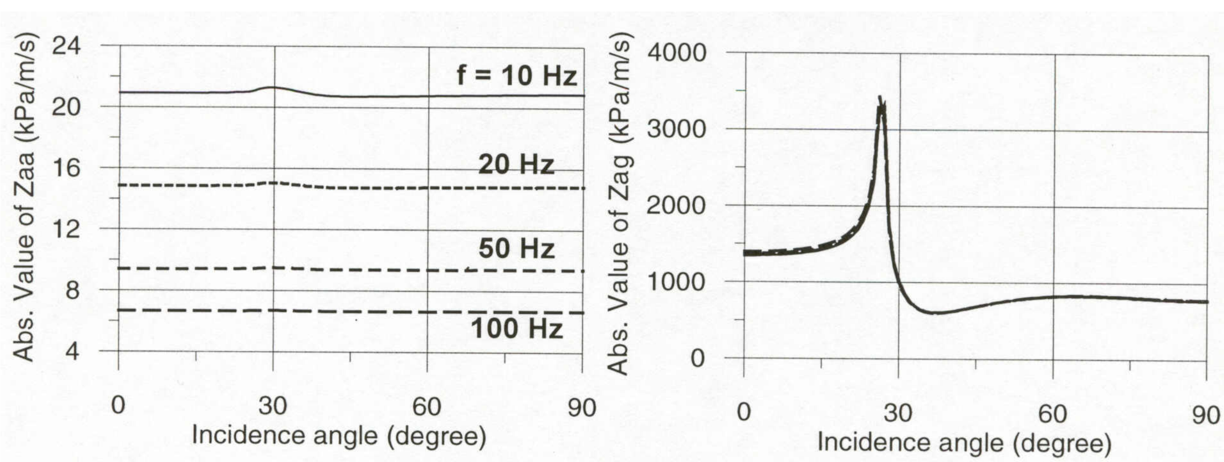

Figure 1: Variation with incident angle of absolute values of Z_{aa} and Z_{ag} for $V_s = 300$ m/s.

Sensitivity studies of seismo-acoustic impedance have largely focused on the effect of frequency and such material parameters as permeability and porosity. Whereas these parameters have a key role on sound propagation and interaction of acoustic waves with the ground, it appears that the shear wave velocity of the ground may have an equally important influence on impedance. The airblast experiments in the forests of Norway during the summer and spring of 1994 and 1995 provided a valuable opportunity to evaluate this assertion. The experiments were performed at two sites in the forests of Norway with the intention of, among others, collecting overpressure and ground response data to calibrate the existing low-frequency sound propagation models or advance a new model [9]. Over the measurement stretch, which ranged from 200 m to about 15 km, the overpressure data were consistently smaller than their predicted counterparts with the measured data dropping to 1/10-1/100 of the theoretical values at larger distances. This difference is larger than can be explained by simple consideration of wave attenuation in the ground. Other justifications for this deviation, such as the acoustic-to-seismic energy transfer through the second pressure wave in the porous ground [10], can hardly be sufficient either. A new explanation, based on the interaction of sound wave and surface Rayleigh wave in the ground, has been pointed out recently as a more likely mechanism [11]. The fact that a sound wave propagating at about the same speed as that of the Rayleigh wave in the ground induces large ground response is believed to be a determining factor in wave propagation in a transeismic sound propagation regime. The analytical model developed in Sec. 2 provides a simple tool to test the significance of this condition. Figure 2 shows the variations of the real and imaginary parts of Z_{aa} and Z_{ag} with V_s for the porous material in Table 1 and for a frequency $f = 10$ Hz. The impedances correspond to incident sound waves at grazing angle. Figure 3 displays a 3-D plot of the variation with both frequency and shear-wave velocity of the absolute value of Z_{ag}. The remarkable drop in the value of Z_{ag} at $c/V_s \approx 1$ is the most important feature in these plots. Indeed a gross estimation of the seismo-acoustic impedance calculated using the field data from the Norwegian blast tests exhibited some relatively small values at large distances [11]. Measurement of the surface wave dispersion curve at one of the test sites has indicated that for a typical frequency of overpressure signal of 8 Hz at large distances one should expect a surface wave velocity of about 300 to 350 m/s.

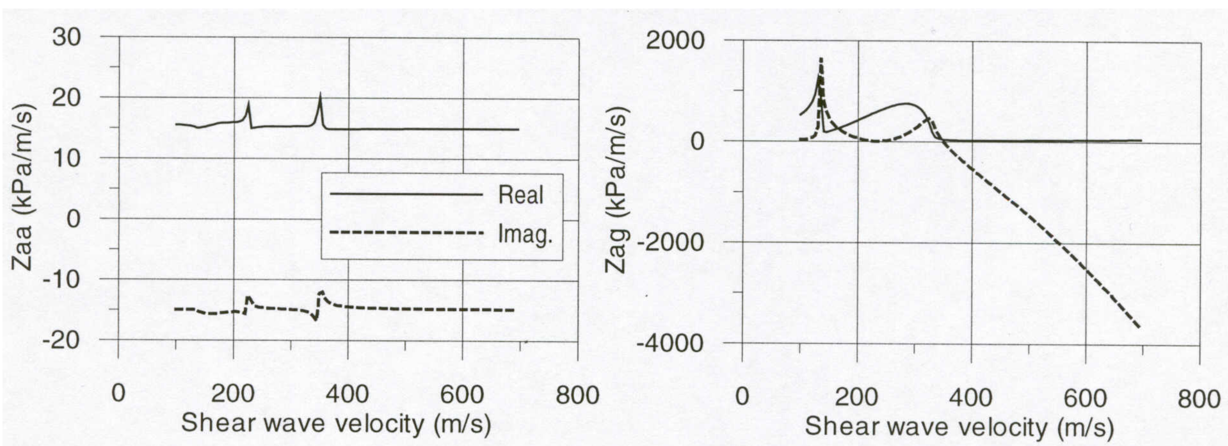

Figure 2: Variations of real and imaginary parts of Z_{aa} and Z_{ag} with V_s for $f = 10$ Hz.

These observations suggest that the use of a locally reacting model may fail to capture an important element in the coupling of airborne acoustic energy to seismic energy for transeismic conditions. Moreover, determination of impedances using methods that rely on a locally-reacting concept may provide inadequate results.

4 - SUMMARY AND CONCLUSION

The paper presented a theoretical model for the reflection/refraction of acoustic plane waves at the interface with a poro-elastic halfspace. The model was used to determine the complex acoustic and seismo-acoustic impedances at the interface and examine their sensitivities to angle of wave incidence and shear-wave velocity of ground. The results showed a remarkable dependence of the seismo-acoustic impedance on c/V_s, with the impedance attaining a minimum value at $c/V_s \approx 1$. This suggests that the use of a locally reacting model that ignores the interaction of sound wave with surface wave in ground may fail to capture the acoustic-to-seismic energy coupling and result in wrong predictions of overpressure and ground vibration.

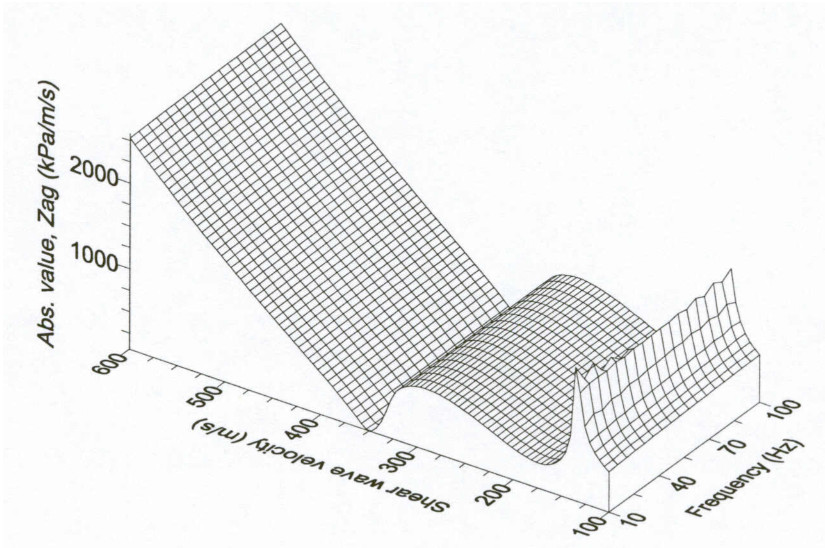

Figure 3: Variation with frequency and shear-wave velocity of absolute value of Z_{ag}.

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