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VIBRATION OF SPECIALLY TAPERED BEAMS AND PLATES

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ABSTRACT
It is shown in this paper that propagation time of the wave running along a finite beam or a circular plate tapered in such a way that their thickness decreases rather smoothly may turn to be infinite. As the wave never reaches the outlet of the beam or plate, it will not reflect from it. Inlet impedance relative to the bending vibration of this beam or plate is equal to that of an infinite uniform beam or plate. The preliminary experimental results of application of such beams and plates for structural damping are also discussed in this paper.

1 - THEORY

Wavenumber of the bending wave propagating along the plate is related to its thickness h by the following equation:

$$k = \left(\frac{12\rho\omega^2}{Eh^2}\right)^{1/4} \tag{1}$$

Here ρ is the density of the plate, E is its Young's module, ω is the frequency. If the thickness of the plate changes rather smoothly, the bending wave travels through it without reflection. The criterion of smoothness

$$\frac{dk}{dx}\frac{1}{k} \ll k \tag{2}$$

means that the change of the thickness of the plate on the wavelength is rather small. This condition coincides with that of applicability of WKBJ. It may be shown that this condition fulfils uniformly on x only when $h(x)$ is given by

$$h\left(x\right) = \varepsilon x^2 \tag{3}$$

The cross-section of the plate with such a dependence of thickness is depicted on fig. 1.

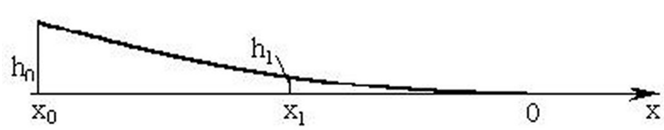

Figure 1: Parabolically tapered plate.

Substituting (3) into (1) we find that wavenumber of the bending wave tends to infinity as x approaches zero:

$$k\left(x\right) = \left(\frac{12\rho\omega^2}{E}\right)^{1/4}\frac{1}{\varepsilon^{1/2}}\frac{1}{x} \tag{4}$$

Phase and group velocities

$$c_{ph} = \frac{\omega}{k} = \varepsilon^{1/2} \left(\frac{E\omega^2}{12\rho} \right)^{1/4} x \tag{5}$$

$$c_{gr} = \frac{\partial \omega}{\partial k} = 2\varepsilon^{1/2} \left(\frac{E\omega^2}{12\rho} \right)^{1/4} x \tag{6}$$

tend to zero as the wave approaches the tapered edge of the plate x=0. Propagation time of the wave packet with central frequency ω from a certain cross-section x_0 to cross-section x_1 equals:

$$T = \int_{x_0}^{x_1} \frac{dx}{c_{gr}} = \frac{1}{2\varepsilon^{1/2}} \left(\frac{12\rho}{E\omega^2} \right)^{1/4} \left| \ln \frac{x_1}{x_0} \right| \tag{7}$$

From (7) one can see that propagation time of the wave tends to infinity as $x_1 \to -0$. It means that the wave coming from the thick part of the plate will never reach the tapered edge and consequently will not reflect from it. Thus the plate of a finite length made of a non-absorbing material can completely absorb the incident bending wave. The absorption in this case is due not to the transition of the energy of vibration to the heat, but to its accumulation in the vicinity of the tapered border of the plate. One can refer to such a plate as a "vibrational black hole". The similar behavior of the waves is possible in the quantum mechanics (for the wave of probability: see [1]) as well as for the internal waves in the stratified atmosphere [2]. For acoustic waves a model of the "acoustical black hole" is discussed in [3].
It appears that in the case of the dependence $h(x)$ given by (3) the exact solution of the wave equation of the transversal vibration of the beam with rectangular cross-section

$$\rho S(x) \frac{\partial^2 \zeta}{\partial t^2} = -\frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 \zeta}{\partial x^2} \right] \tag{8}$$

can be found analytically. In (8) ζ is the displacement of the particles in the transversal wave, $S(x)$ is the area of a cross-section of the beam with the width q_0:

$$S(x) = q_0 \varepsilon x^2 \tag{9}$$

$I(x)$ is the moment of inertia of the cross-section:

$$I(x) = \frac{1}{12} q_0 \varepsilon^3 x^6 \tag{10}$$

Substituting (9) and (10) into (8) and taking into account harmonic time dependence $\zeta(t)$, we obtain:

$$Ax^2 \zeta = \left(x^6 \zeta'' \right)'' \tag{11}$$

where primed symbols denote the derivative with respect to x, and

$$A = 12 \frac{\rho \omega^2}{\varepsilon^2 E} \tag{12}$$

Equation (11) has an exact analytical solution in the form of the power functions. Let us substitute $\zeta(x) = x^\alpha$ into (11). We obtain an equation of the fourth degree for α:

$$\alpha(\alpha + 4)(\alpha + 3)(\alpha - 1) = A \tag{13}$$

This equation can be reduced to biquadratic by substituting $\alpha = \mu - 3/2$. We find μ from the biquadratic equation

$$\mu^4 - \frac{17}{2}\mu^2 + \frac{225}{16} - A = 0 \tag{14}$$

and derive the expression for α. Thus, the wave equation for the bending wave propagating along the beam with the thickness decreasing according to the parabolic law possesses a solution in the form of a linear combination of four power functions:

$$\zeta(x) = \sum_1^4 C_i x^{\alpha_i} \tag{15}$$

$$\alpha_{1..4} = -\frac{3}{2} \pm \sqrt{\frac{17}{4} \pm \sqrt{4+A}} \tag{16}$$

where A is given by (12).
The similar theoretical results may be obtained for the axisymmetric bending wave in the circular plate with the thickness decreasing parabolically to the center or to the periphery. In the case of decreasing of the thickness to the center there also exist exact solutions of the wave equation (see [4]), similar to (15), (16).

2 - EXPERIMENT

Making experimental samples one can not realize the ideal tapering of the beams (plates) in the vicinity of the edge, for it is impossible in practice to reduce the thickness of the beam or plate to zero in strict accordance with the parabolic law. So the actual sample will be terminated at some finite thickness. In the absence of absorption the wave must be totally reflected from this outer border. Nonetheless, even the small absorber placed on the edge will provide an effective absorption because of the high concentration of the wave energy on the tapered part of the sample.
This is demonstrated in fig. 2 representing the results of the damping of the aluminum beam. The graphs show time dependence of the displacement of the particles in the wave. The first graph corresponds to the simple rectangular beam of size 960 $\times$40 $\times$9 mm. The second $-$ to the similar beam with the last 100 mm. tapered parabolically and tapered border coated with plasticine. In this case the ratio of the damping part of the beam (i. e. the last 100 mm.) to the uniform part is denoted $\delta M/M$.

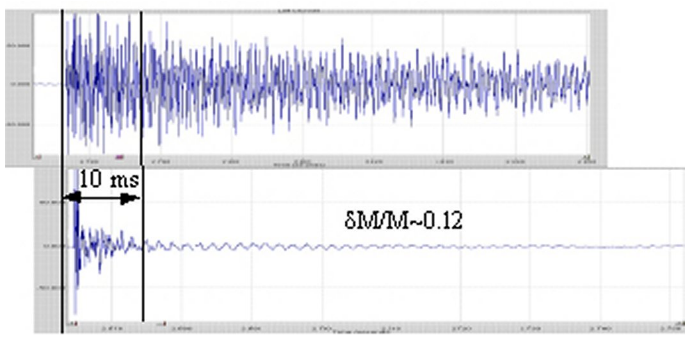

Figure 2: Damping of the aluminum beam (upper graph $-$ non-damped beam, lower $-$ damped beam).

The spectra of the vibration without damping (line 1) and with the damping (line 2) are shown in fig. 3.
Fig. 4 illustrates the schema of the experiment when the big steel plate is damped by a small aluminum disk parabolically tapered from some radius to the periphery. The outer border of this disk is coated with plasticine (fig. 5).
The spectra of the vibration of the steel plate without and with the damping are depicted in fig. 6.

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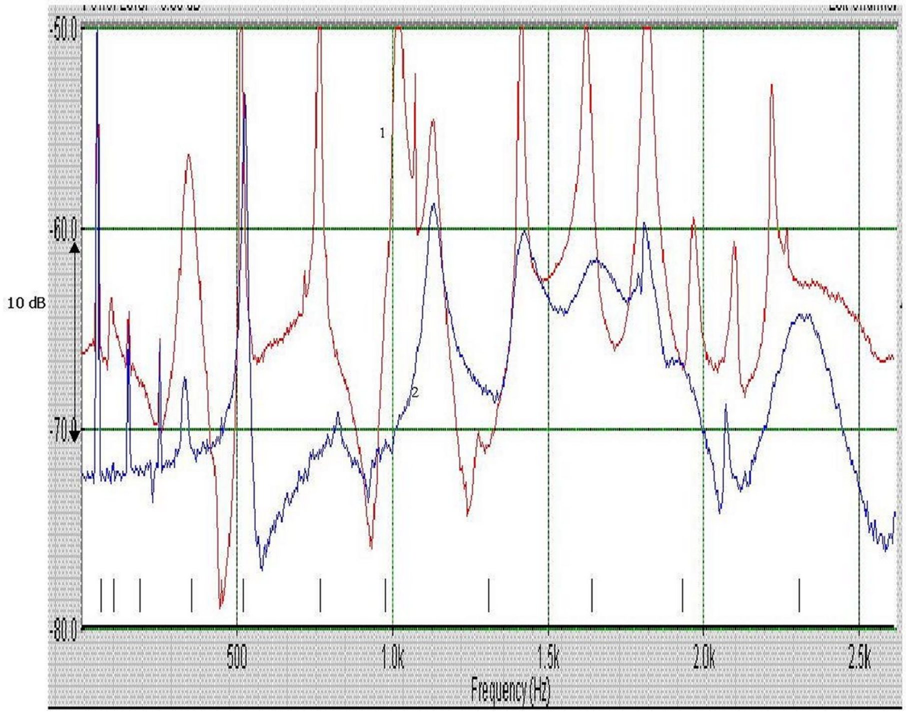

Figure 3: Damping of the aluminum beam; spectrum of vibration.

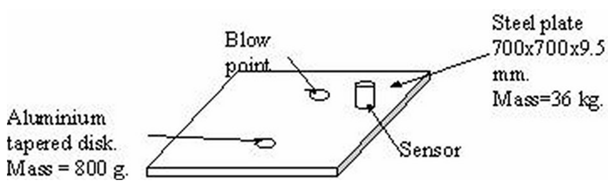

Figure 4: Experimental scheme of the damping of the steel plate by the aluminum parabolically tapered disk.

Figure 5: Tapered disk (the right disk is coated with the plasticine around the border).

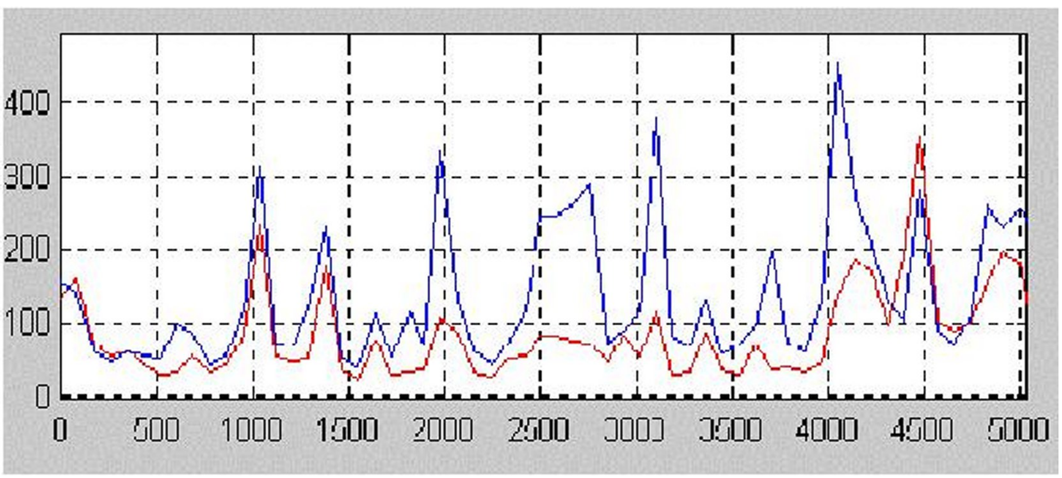

Figure 6: Spectrum of the vibration of the steel plate damped by the tapered disk (the upper line without damping, the lower with the damping).