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A COMPARISON OF THE 2D RING AND 3D ORTHOTROPIC PLATE FOR MODELLING OF RADIAL TIRE VIBRATIONS

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ABSTRACT
Among all acoustical sources of tire-road noise, the radiated sound from radial tire vibrations is of major importance in low and medium frequency ranges. Different mathematical models for tire vibrations modelling have been developed over thirty years: ring on an elastic foundation (2D) [1,2,3], orthotropic plate under tension on an elastic foundation (3D) [4,5], thin shell (3D) coupled with finite element method – FEM – or purely FEM model (3D) [7]. Whereas FEM is a powerful means to model the complexity of tire dynamic behaviour or road-contact interactions, analytical approaches like ring and plate models are still of interest to understand physical phenomena and structure-borne sound [6]. The paper focuses on comparing ring and plate models for evaluating the radial dynamic response of a tire excited by a point force. The tire is considered at rest and unloaded. Theoretical and experimental comparisons are presented for each model. General physical insights are finally discussed for each approach.

1 - INTRODUCTION

Car tires play an important role on the riding comfort. They become the main acoustical source for vehicle speeds above 80 km/h. The vibrations induced on the tire when rolling on a rough road surface are responsible for the tire-road noise in low and medium frequency range. In order to predict tire-road noise emission, it is necessary to know the tire dynamic response. Two different analytical models are studied: the ring model (2D) and the thin plate model (3D). For each one, the tire is supposed at rest (no rotating speed) and unloaded. Only flexural vibrations are considered in this study. The model of damping is hysteretic. The comparison of the models is performed by calculating the FRF mobility of the tire for an harmonic radial point force at the center of the belt. This FRF is obtained by using modal expansion method. The natural frequencies and mode shapes are derived from the ring and plate theory [6]. Measurements have been performed on a smooth (without tread patterns) tire 155/70R13 inflated at 2 bars. Comparison are made between predictions and experimental results.

2 - THE 3D PLATE MODEL

The thin plate model of the tire is shown in Figure 1. Only the normal motion of the plate − the radial motion of the tire − is considered. The equation of motion is, according the Kirchhoff hypothesis of thin plate

$$\left[-T_{ox}\frac{\partial^2}{\partial x^2}-T_{oy}\frac{\partial^2}{\partial y^2}+B_x\frac{\partial^4}{\partial x^4}+2B_{xy}\left(\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}\right)+B_y\frac{\partial^4}{\partial y^4}+s+\rho h\frac{\partial^2}{\partial t^2}\right]w\left(x,y,t\right)=p\left(xo,yo,t\right)$$

where T_{ox} and T_{oy} are membrane tensions produced by the inflated air, B_x and B_y are the longitudinal and transversal bending stiffnesses respectively, B_{xy} the cross stiffness of the belt and s the bedding number of the Winkler foundation. All these quantities are complex when operating in the frequency domain in order to take into account the damping in the process.

The normal displacement $w_{(x,y)}$ represented in terms of Fourier series in which each term is a mode characterizing the structure

$$w\left(x,y\right)=\sum_{r=1}^{N}q_{r}\psi_{r}\left(x,y\right)$$

where q_r is the modal participation factor and y_r the normal mode r of the structure which verifies the boundary conditions - it is the product of two eigenfunctions given by

$$\psi_{r}\left(x,y\right)=\cos\left(\frac{2\pi m}{lx}\right)\sin\left(\frac{\pi n}{ly}\right)$$

where m and n represent the mode numbers in longitudinal x and transversal y direction of the plate. The general expression of the plate FRF mobility obeys

$$Yjk\left(\omega\right)=i\omega\sum_{r=1}^{N}\frac{\phi jr\phi kr}{\omega^{2}r\left(1+i\eta r\right)-\omega^{2}}$$

where ω_r is the undamped natural frequency and η_r the structural damping $-$ loss factor $-$ of mode r. $\phi_{jr}\phi_{kr}$ represents the modal constant of each mode.

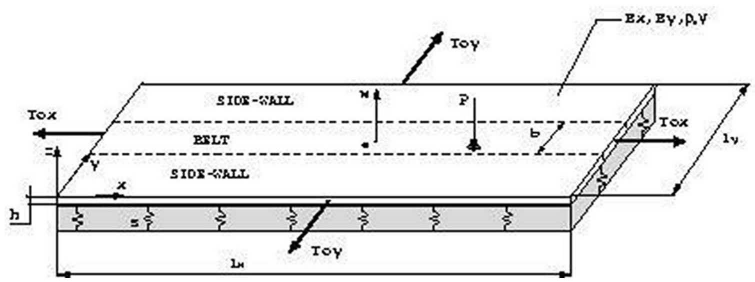

Figure 1: Tire model $-$ orthotropic thin plate under tension on an elastic foundation.

3 - THE 2D RING MODEL

Although relatively simple, the ring model, used for analyzing the structural behaviour of tires, is also of interest for understanding the tire dynamics. The tire belt is modeled as a ring and the sidewalls are modeled as an elastic foundation with radial and tangential stiffnesses k_r and k_θ respectively. The equations of motion are written as

$$\left\{\begin{array}{l}\frac{D}{a^{4}}\left(u_{3}^{\prime\prime\prime}-u^{\prime\prime}\theta\right)-\frac{K}{a^{2}}\left(u_{3}^{\prime}+u_{\theta}^{\prime\prime}\right)-\frac{p_{o}}{a}\left(2u_{3}^{\prime}+u_{\theta}^{\prime\prime}\right)+\left(\frac{p_{o}}{a}+k_{\theta}\right)u_{\theta}+\rho h\ddot{u}_{\theta}=f_{\theta}\left(\theta,t\right)\\ \frac{D}{a^{4}}\left(u_{3}^{\prime\prime\prime\prime}-u_{\theta}^{\prime\prime\prime}\right)-\frac{K}{a^{2}}\left(u_{3}+u_{\theta}^{\prime}\right)+\frac{p_{o}}{a}\left(2u_{\theta}^{\prime}-u_{3}^{\prime\prime}\right)+\left(\frac{p_{o}}{a}+k_{3}\right)u_{\theta}+\rho h\ddot{u}_{3}=f_{3}\left(\theta,t\right)\end{array}\right.$$

where, with reference to Figure 2, u_3 and u_θ represent the radial - transverse - and tangential - circumferential - displacement, f_3 and f_θ are the external forces per unit area in the radial and tangential direction respectively, $D=Eh^3/12\left(1-\nu^2\right)$ is the bending stiffness, $K=Eh/\left(1-\nu^2\right)$ is the membrane stiffness and a is the mean radius of tire. The dots and primes denote differentiation with respect to t and θ respectively. Although both extensional and inextensional vibrations exist, it was shown that at low frequency, the inextensional - flexural - modes are dominant in the tire response, and so that the resolution of the system is simplified. The forced radial response can be written, according to the modal expansion method

$$u_{3}\left(\theta\right)=\sum_{n=1}^{N}\alpha n\cos\left(n\theta\right)+\beta n\sin\left(n\theta\right)$$

where a_n and b_n are general coordinates to be determined and n is the circumferential wave number. Substituting this equation into the equations of motion and by making use of the orthogonality of the trigonometric functions, yields to each set of generalized coordinates x=a_n,b_n a linear second order differential equations

$$\ddot{x} + G\dot{x} + Kx = Q$$

where G and K are 2×2 matrices with coefficients being function of the model parameters; Q is the generalized force vector.
This system can be solved for time or frequency domain. The response of the ring to an harmonic point force in radial direction is hence derived from these equations giving

$$f_3(\theta, t) = \frac{1}{a}\delta(\theta - \theta^*)\, e^{i\omega t}$$

in which the Dirac δ function is used to represent the point load. θ^* denotes the location of the point load with respect of the ring coordinate ($\theta^* = 3\pi/2$ in this study). Here, viscous damping $-$ G matrix - is introduced due to the time description but hysteritic damping can be used in the frequency domain to be homogeneous with the plate model.

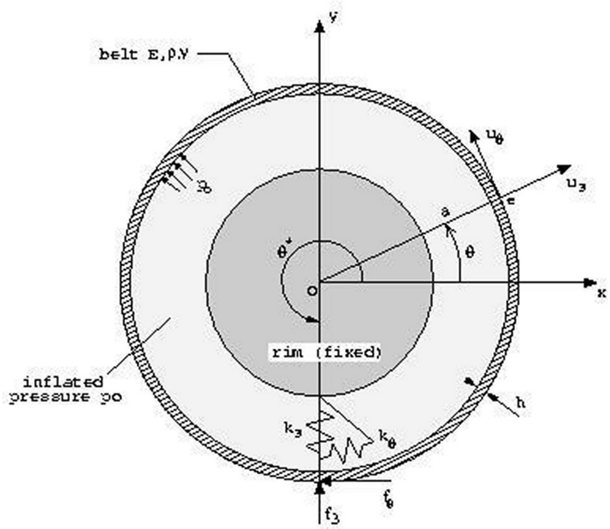

Figure 2: Tire model $-$ ring on an elastic foundation.

4 - NUMERICAL RESULTS AND EXPERIMENTAL COMPARISON

The prediction given by the analytical models are compared to the vibration measurements performed on an inflated car tire. The experimental procedure and the test rig are described in [5]. An experimental modal analysis of the tire is performed at low frequency to identify natural frequencies, damping factors and mode shapes. There is a very good correlation between experimental and theoretical mode shapes for each model. Figure 3 shows the mode shapes calculated from the theory for the first six modes. Table 1 lists the modal parameters for the low frequency flexural modes of the tire. The overall agreement is very satisfactory.

mode n	1	2	3	4	5	6	7
(a) ω_n (Hz)/ η_n (%)	136.5/7.1	163.2/6.3	192.4/7,0	228.0/7.0	264.4/8.0	304.8/7.7	348.5/8.0
(b) ω_n (Hz)/ η_n (%)	143,1/7.0	161,2/7.0	189.0/7,5	222.6/8.3	260.5/9.5	301.9/10.9	346.7/12.5
(c) ω_n (Hz)/ η_n (%)	140,0/6.7	163,6/7.4	193.1/8,0	227.0/8.7	264.7/9.4	306.0/10.2	350.5/10.9

Table 1: Natural frequencies and hysteretic damping for the seven first flexural modes; **(a)**: measurement, **(b)**: ring model, **(c)** plate model.

The magnitude of the radial point FRF mobilities at the center of the belt are plotted in Figure 3. About 50 modes in the circumferential direction have been taken into account for the computation.

Structural parameters of each tire models have been updated to correctly fit to the measured curve. At low frequency − [0-400] Hz − there is an acceptable agreement between theory and measurement for both models. Above 400 Hz, the ring model fails as expected [3]; the mobility of the tire becomes flat and smooth.

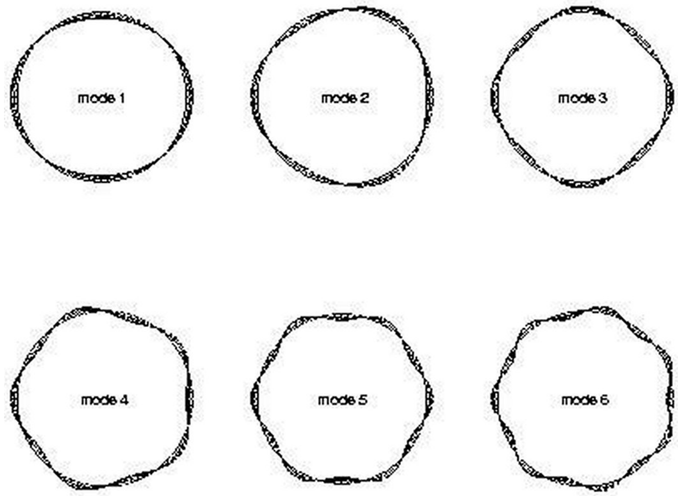

Figure 3: First sixth flexural mode shapes of the tire.

At high frequency, the mobility tends to the behaviour of an infinite plate, which is real and independent of frequency [5,6]. The reason of this difference is the fact that the ring model is a one-dimensional waveguide whereas for frequencies higher than about 400 Hz, wave propagation in axial direction is possible. This means that the tire is a two dimensional waveguide in this frequency range.

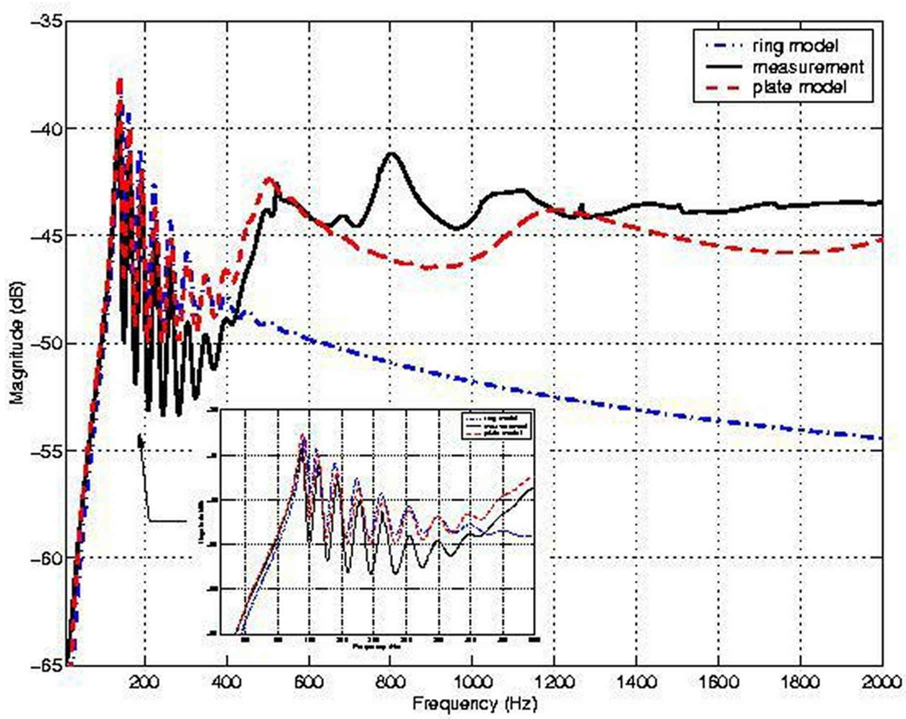

Figure 4: Comparison between calculated and measured point FRF mobility ($\theta = 3\pi/2, x_o = l_x/4$).

5 - CONCLUSIONS

The analytical models introduced in this paper provide an effective description in predicting the dynamics of tires in the low and medium frequency range. It has been shown that the 2D ring model is valid at low frequencies up to about 400 Hz while the 3D plate model is valid over the whole frequency range [0-2000] Hz. At low frequencies, the 2D circular ring model gives a correct description of the one dimensional wave propagation in the tire. At higher frequencies (when the wavelength is close to the tread width of the tire), the tire becomes a two dimensional waveguide and the plate model has to be used.

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