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ACOUSTIC FIELD ATTENUATED BY FINITE BARRIERS BY MODIFIED HELMHOLTZ INTEGRAL EQUATION

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ABSTRACT
The vehicle traffic creates high levels of noise. The most immediate protection is the placement of an acoustic barrier. One of the problems in the design of the barrier is the calculation of the modified acoustic field in the illuminated and shadow zones, due to radiation an scattering from barrier. The boundary integral equation method has been used extensively as a numerical technique to solve full space problem. A half-space formulation to account for the existence of an earthly plan, with an acoustic barrier has been given. A so called half-space Green's function within a modified version of the Helmholtz integral equation. The Helmholtz integral equation coefficient can be evaluated by a closed boundary. Experimental checking of the numerical results from barriers of several different shapes supported on a infinite surface plane are accomplished to verify this formulation.

1 - INTRODUCTION
The different works on the modelization of the acoustic behaviour of the barriers could be classified by the employed method. In this way, Redfearn, Fher, Kurze, Maekawa, Kawai, generalize the different geometries of the barriers and express the attenuation as a function of dimensionless variable determined by their geometry. The investigations carried out by, Kawai, Fujiwara, Yuzawa, studying the attenuation starting from the absorbent materials of the barrier. Pierce and Fujiwara approaching the study of the diffraction for different obstacles, as thick barriers. Acousticiens like, Scholes, Isei, Jonasson and Thomanson, study the presence of surfaces out from the barrier and the interference due to the waves reflected by these. Finally, in this work, the use of the boundary elements technique was used. A Helmholtz integral equation for a half-space formulation was used to remove the contribution due to the infinity plane. The calculation results are compared with model experiments for several kinds of barriers. The experimental and calculated results are in good agreement.

2 - THE MATHEMATICAL DESCRIPTION OF THE PROBLEM
The velocity potential field ϕ will described in terms of the Helmholtz equation $\nabla^2\phi + k^2\phi = 0$ in a acoustic domain B, in three dimensions, with appropriate boundary conditions, and the Sommerfeld radiation conditions in the far-field. The Helmholtz integral equation of the first kind derived from Helmholtz-Kirchhoff's formula in B, with boundary surface S, see Fig. 1, a following expression is obtained neglecting the time factor $e^{j\omega t}$, (time dependence convection has is been used, due to a time-harmonic point source),

$$C(P)\,\phi(P) = 4\pi\phi_i(P) + \int\int_S \left(G_k(P,Q)\frac{\partial\phi(Q)}{\partial n_Q^+} - \frac{\partial G_k(P,Q)}{\partial n_Q^+}\phi(Q) \right) dS(Q) \tag{1}$$

Here the integration runs over the boundary S, of the acoustic field domain considered, the ground and the surface barrier. The contribution from the far-field boundary of the acoustic domain, the closure provided by the sphere at infinity, has been removed analytically by invoking the Sommerfeld radiation condition. Where, $\phi(Q)$ is the velocity potential field satisfying Helmholtz equation and boundary conditions. $\phi_i(P) = [exp(-jkd)]\, d$ direct incident wave potential field in the absence of the barrier; $G_k(P,Q)$ is the Green's function for the $k = \omega/c$ wavenumber; r is the distance between any two points P and Q. The free space Green's function is the symmetric factor $G_k = \exp(-jkr)/r$ is at an edge or corner, $C(P)$ can be evaluated by

$$C(P) = 4\pi - \int\int_S \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS \tag{2}$$

where, $C(P) = 4\pi$ for $P \in B$, $C(P) = 2\pi$ for $P \in S$ and an only tangent plane exists; $C(P) = 0$ for $P \in B'$.
You will proceed to decompose the space $B \cup B'$ in three enclosures, the acoustic enclosure B, the enclosure formed by the half space B_H, and the enclosure to the acoustic barrier B_b, so that $B' = B_H \cup B_b$. The acoustic enclosure will be bounded now by the surface S_H, not common with the barrier and the surface barrier S_b, the barrier will be bounded by the surface $S_b \cup S_c$, where the superficial boundary S_c is the common boundary among the barrier and the infinite surface S_H, that it defines the half space. The normal n^+ is taken pointing outwards from the domain of the interest B, (see Fig. 1).

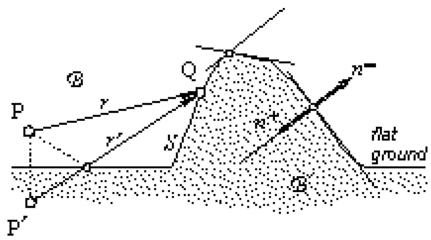

Figure 1(a): Barrier and surrounding ground.

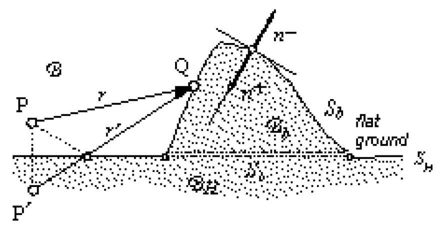

Figure 1(b): Nomenclature for the half space problem (P' being symmetric to P with respect to the ground surface).

Carried out the decomposition where the acoustic barrier is in contact with the infinite plane surface, the previously mentioned formulation should be modified in the problem of the half space. For the half space bounded by the infinite plane B_H, (see Fig. 1), the half-space Green's function, denoted by $G_k^H(P,Q)$, is a properly defined function everywhere in the upper half-space except at the location of the source where it has a singularity of a known form; it satisfies the following set of equations,

$$\left(\nabla^2 + k^2\right) G_k^H(P,Q) = \delta(P - Q) \tag{3}$$

the boundary condition on the ground, the Sommerfeld condition at infinity. It will be similar to the overlapping of the direct wave and the diffuse wave from the plane S_H and have the form:

$$G_k^H(P,Q) = \exp(-jkr)/r + R_H \exp(-jkr')/r' \quad (P,Q) \in R^3 \tag{4}$$

where, R_H is the reflection coefficient ($-1 \le R_H \le 1$); r' is the distance between two points P' and Q; R_H=1 for rigid plane ($\partial\phi_t/\partial n = 0$), and R_H=-1 for soft plane ($\phi_t = 0$). If the ground is not perfectly reflecting, the "image-source" should be multiplied by a weighting function depending of the model of ground considered. For locally reacting grounds or porous stratified media it is possible to have a Green's function expressed either in a closed form or in series or asymptotic expansion, $(\partial/\partial n + j\omega\rho_0/Z_n)\,\phi_t$, for a dispersive surface. When P is in S_H, as r=r', the Green's function for the half-space decreases to:

$$G_k^H = (1 + R_H)\, e^{-jkr}/r \tag{5}$$

The above equation is not valid for soft plane, since R_H=-1 and the Green's function G_k^H reduces to zero. Nevertheless, for a practical problem these extreme cases are not given. The boundary integral of the half-space problem becomes:

$$C(P)\,\phi(P) = 4\pi\phi_t(P) + \int\int_S \left(G_k^H(P,Q)\,\frac{\partial\phi(Q)}{\partial n_Q} - \frac{\partial G_k^H(P,Q)}{\partial n_Q}\phi(Q)\right) dS(Q) \tag{6}$$

where, $\phi_t = \phi_i + \phi_d$ is the incident velocity potential for the half space; $\phi_d(P)$ diffuse wave potential for the reflecting wave. At infinity a Sommerfeld radiation condition is enforced on the diffracted part of the field ϕ_d, ensuring that there are only outgoing waves at infinity.

$$\lim_{r\to\infty} r^w \left(\partial\phi_d/\partial r - jk\phi_d\right) = 0 \,(w = 1, \text{ to 3D}) \tag{7}$$

The boundary of the acoustic barrier should be divided in two parts. The first S_b exposed to the acoustic means, the second S_c are that it is in contact with S_H. Then, the boundary integral equation for the radiation and acoustic dispersion will become in:

$$C(P)\,\phi(P) = 4\pi\phi_t(P) + \int\!\!\int_{S_b} \left(G_k^H(P,Q)\frac{\partial\phi(Q)}{\partial n_Q} - \frac{\partial G_k^H(P,Q)}{\partial n_Q}\phi(Q)\right) dS(Q) \tag{8}$$

$C(P) = 4\pi$ for P interior to B, $C(P) = 0$ for P exterior to B. For $P \in S_b$ and $P \notin S_H$,

$$C(P) = 4\pi - \int\!\!\int_{S_b+S_c} \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS \;\; P \in S_b \text{ and } P \notin S_H \tag{9}$$

For $P \in S_b \cap S_H$, applying the second Green's identity to ϕ and G_k^H on the acoustic domain B that excludes the singular point P and to the source point, (see Figs. 2 and 3), it is,

$$C(P) = (1 + R_H)\left[2\pi - \int\!\!\int_{S_b+S_c} \frac{\partial}{\partial n^+}\left(\frac{1}{r}\right) dS\right] \tag{10}$$

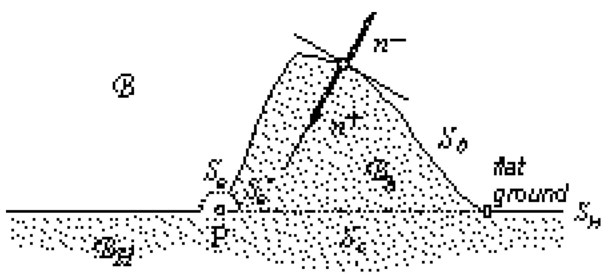

Figure 2: The point P on $S_b \cap S_H$.

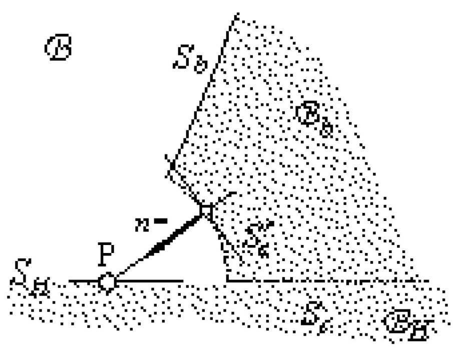

Figure 3: The point P on $S_b \cap S_H$.

The surface S_c it does not contribute to equation (8), and it should be discretized to calculate $C(P)$ in the equations (9) and (10). The dummy elements of S_c are used only to integrate these last equations, but there is not acoustic variable associated with these elements. Furthermore, the size of the mesh of the dummy elements is not problem while it is enough good one to integrate the one derived normal in the equations (9) and (10).

3 - CONCLUSIONS

The method is almost completely numerical and does not include atmospheric absorption due to wind and temperature gradients. The directivity of the source is not considered. A comparison between numeric results for different types of barriers with analytic solution and experimental results are in good agreement. In these results, when the source is relatively far away from the screen, the shape of the screen does not have a very significant influence on the excess attenuation. By a modified version of the Helmholtz integral equation can be solved the acoustic problems associated with acoustic barriers on an infinite plane surface. The numeric test has been taken out for the radiation and dispersion of barriers with different shape.

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