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INVERSE METHODS FOR SOURCE STRENGTH RECONSTRUCTION OF COMPLEX STRUCTURES

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ABSTRACT
Inverse methods for source strength reconstruction may be required when the source is not accessible for direct vibration measurement for any reason, e.g. when the source is rotating, too hot, too complex to access etc. A common feature of these methods is that the relationship between source vibration and the radiated field is expressed in terms of a transfer function matrix, generated in many different ways such as experimental, analytical or numerical methods. Provided that a detailed sound pressure field mapping around the source to be identified is feasible, the unknown vector of the vibration input of the system can be determined by matrix inversion. The paper gives a short overview of various inverse techniques. Special attention is paid to a newly emerging and promising numerical technique, the inverse boundary element method. Various mathematical procedures such as Singular Value Decomposition and Thikhonov regularization is outlined, and a physical interpretation of these highly abstract mathematical tools and the obtained data are discussed. The applicability and typical results of the procedure is demonstrated on real-life, mainly automotive structures.

1 - INTRODUCTION
The reconstruction of source velocity distribution plays an important rule in noise reduction: its knowledge makes possible to understand noise generation mechanisms and shows sometimes directly the possibilities to reduce the noise. Various methods promise to able to reconstruct the source velocity distribution, but sometimes the solution is not as good as expected.
This paper gives a short review of various methods, a brief description of their theory and features. The right choice of inverse method depends on the nature of the problem, and this paper does not aim at ranking the described methods. The more so, since they are further in continuous development. Their capabilities are also limited because of their computational complexity, which however can frequently overcome by new computer technologies.

2 - COMMONLY USED SYMBOLS

- p_h: pressure along the hologram surface containing sensing points
- v_s: normal surface velocity containing source points

3 - NEARFIELD ACOUSTICAL HOLOGRAPHY
The base idea for this method is that the Helmholtz's integral equation [1] can be transformed into a two-dimensional spatial Fourier-transform if the measuring and the sensing surfaces are both plane and can be handled as infinite baffled:

$$F_{xy}\left\{p\left(x,y,z=z_h\right)\right\}=j\omega\rho F_{xy}\left\{v_n\left(x,y,z=z_s\right)\right\}^{*}F_{xy}\left\{G\left(x,y,z=z_h-z_s\right)\right\} \qquad (1)$$

where G, the half space Green's function, is: $G=\exp\left(-jkz\right)/2\pi z$. The above mentioned restrictions lead to a simple way of surface velocity reconstruction and the obtained solution is very stable; noise

effects and geometry inaccuracies are well suppressed. This means that the source geometry have to be only approximately plane. The infinite extension prescribes the pressure along the edges of the hologram surface to go to zero. This seems to be a very hard restriction but the use of two-dimensional window functions − similar to the one-dimensional ones used in the audio technology − can help in the most of cases.
The traditional Acoustical Holography (AH) did not give any prescriptions for the distance of the hologram plane to the source plane. In the special case when this distance is smaller than the critical wavelength, the attainable surface resolution is significantly higher. The reason is that the evanescent waves are also taken into account during the pressure measurements which information content is significant. Because it's importance it got a new name: Nearfield Acoustical Holography (NAH).
Note that the transformation of the Helmholtz's equation can also be done if the shape of the source is in some kind of coordinate system plane, not only in Cartesian. Such derivations for cylindrical and spherical objects can be found in [2].

4 - INVERSE FRF METHODS

In order to overcome the geometrical limits of NAH different methods were developed. Their common property is that the connection of the source points and the measuring points is described in a frequency dependent transfer matrix. A general matrix equation can be written:

$$\{p_h\} = [\mathrm{c}]\{\ \mathrm{v_s}\} \tag{2}$$

The elements of the acoustic transfer matrix can directly be measured or numerically calculated. To achieve mathematically stable solutions two things are to be satisfied: the number of rows in $[c]$ usually, but not necessarily, has to be bigger or equal to the number of columns and the condition number of the transfer matrix should not be large. This second leads to the same stipulation as by NAH, namely that the distance between the source and sensing surfaces should be kept low. In addition, the microphone positioning and the choice of source points should be matched, which however is not too difficult, but should not be forgotten. Further descriptions can be found in [3].
The most exact way is to perform frequency response function measurements, hence the name of the method. In the case of high source and sensing point numbers the number of FRF's to measure grow extremely high and the process tend to be too time- or resource-consuming.

4.1 - Airborne source quantification

A special variation of the inverse FRF methods has originally been developed for automotive applications, where the total emitted sound radiated from very complex sources should be decomposed into a number of partial sources. The overall sound is measured in a specific point under well-defined operating conditions, and the aim of the test is to quantify the contributions of a number of predefined source areas. The method is therefore called acoustic source quantification, ASQ [4].
The method is essentially a two step extension of the inverse FRF technique, by introducing a power-based source substitution technique. The source is divided into a reasonable number (N_p) of partial surfaces or patches. It is assumed that a number of ideal point sources are distributed along the source surface, and the sound power radiated from any patch is generated by a couple of these sources per patch. If the sources are uncorrelated and of equal source strength for any patch, the squared sound pressure in a faraway point R can be calculated by

$$p\left(R\right)^2 = \sum_{\mathrm{p}=1}^{\mathrm{N_p}} \overline{\left|\mathrm{H}\left(\mathrm{p},\mathrm{R}\right)\right|}^2 q\left(p\right)\ ^2 \tag{3}$$

where $\overline{\left|H\left(p,\mathrm{R}\right)\right|}^2$ is the squared, averaged acoustic transfer function from the point sources of patch p to point R, and $q(p)$ is the squared, summed source strength for patch p. The transfer functions are measured directly or by using a reciprocal technique under laboratory conditions, the source strengths are derived from an inverse procedure.
To do so, a number of "indicator microphones" are placed in the vicinity of the source and the sound pressures are measured under real operating conditions. Using Eq. (3) for the indicator sound pressures, the source strengths of the relevant patches can be calculated, usually performed by means of a least square procedure or singular value decomposition.
The ASQ method is well suited for moving or rotating sources, where the farfield microphone R usually measures the total noise under operating conditions according to a standard procedure. The breakdown of the overall sound pressure level into partial contributions from various parts of the source (e.g. engine,

exhaust, intake, cooler etc. in case of a car or truck) can provide valuable information for the noise control engineer in the optimisation process of the product under investigation.

4.2 - Inverse boundary element method

This method makes use of analytically generated transfer functions. The boundary element method itself is a standard radiation prediction method [5]. It transforms the Helmholtz's integral equation to a matrix equation for arbitrary shaped sources. It generates matrices analytically that describe the relationship between surface pressure and normal velocity (Eq. 4) and transfer matrices from source points to measurement points located everywhere in the free-field (Eq. 5):

$$[A]\,\{p_s\} = [\;\mathrm{B}]\,\{\mathrm{v_s}\} \tag{4}$$

$$[a]\,\{p_s\} + [b]\,\{v_s\} = \{p_h\} \tag{5}$$

The acoustic transfer matrix is not difficult to deduce:

$$[c] = [a]\,[A]^{-1}\,[B] + [b] \tag{6}$$

Because of its complexity, the method cannot deal with sound reflections from arbitrary shaped objects in the vicinity of the source. Therefore the measuring conditions have to be comparable to those in the free-space. Experiments showed nevertheless that the idealistic treatment of the environment does not result in real problems.

To demonstrate the capabilities of this method an example will be shown now. An automotive tyre has been studied. In the first series of measurements the tyre was run on a laboratory test drum facility in a semianechoic chamber (Figure 1).

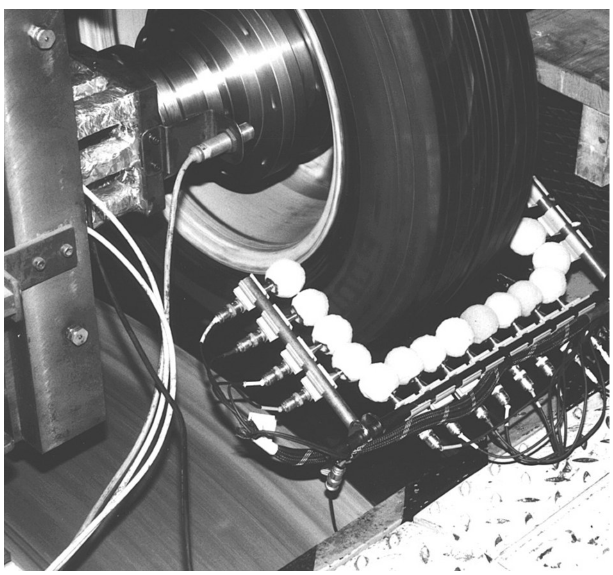

Figure 1: The tyre on the test bench.

In the second series the tyre was mounted in the middle of a trailer [6] that was town on highways (Figure 2). In all two cases, the near pressure-field around the tyre was scanned by 16 microphones, which were mounted on a rotating microphone boom.

The laboratory measurements could be verified because of a third series of measurement. During real-life conditions, where the behaviour of the tyre could not change a lot in respect to the test ones, the source velocity distribution could also be reconstructed, and the results are quiet good. The source velocity distributions of the two measurements on a mid-range frequency (512 Hz) can be seen on Figures 3 and 4.

5 - CONCLUSIONS

As shown above all of the methods have restrictions in their formulations. The right choice of the most suitable method to a selected problem is not obvious. The engineer has to decide which method's restrictions would be the most appropriate, or which restriction's violation would have the most influence

Figure 2: The tyre on the trailer.

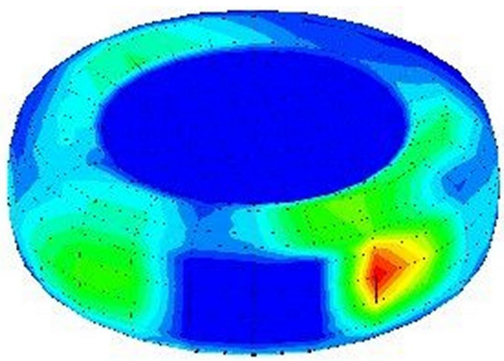

Figure 3: Velocity distribution from test bench measurements.

on the obtained solution. If computers of high computational power are available, analytical methods can save a lot of time. But only the nature "knows perfectly the physics", therefore direct measurement are always the most accurate and informative.

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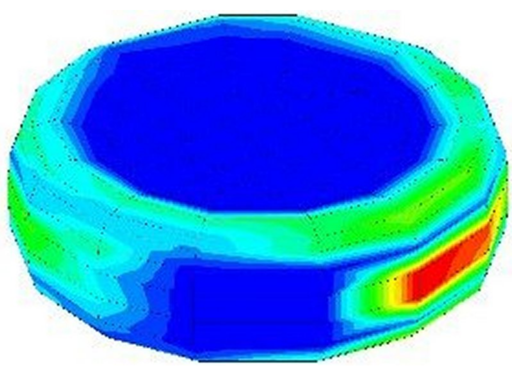

Figure 4: Velocity distribution from trailer measurements.