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THIN WALLED STRUCTURES PROPAGATION CHARACTERISATION

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ABSTRACT
The structure passage through hollow frames is of great interest for automotive industry. Indeed those kinds of structures play a dominant role in the energy transfer from engine sources to panels leading to sound radiation and so on. The characterisation of hollow structures is dealt with in this paper by means of a propagation approach. Thus, when hybridized with finite element codes, it provides an effective way to calculate the dispersion curves of realistic thin-walled structures. This propagation approach is shown to be numerically effectual when compared with further attempts proposed in the existing literature. Some numerical results are presented here in order to show the ability of the exposed method to characterise some typical automotive hollow structures.

1 - INTRODUCTION

To analyse the structure vibratory behaviour the classical method is the modal approach that consists in extracting eigenmodes. In view of the limitations of this analysis, Langley [1] proposed another approach called wave approach: the system response is represented as the superposition of an evanescent part and a propagative one. At first approximation it is only the sum of propagation terms. That wave notion can be applied to one-dimension systems like beams and slender structures.

In this point of view, the aim to characterise the hollow frames is to determine the propagation waves and to describe the dispersion curves or the wave number $k = f(\omega)$. The idea is to consider the hollow structures as one-dimension waveguides. So the free propagation in a waveguide whose section is supposed uniform in the plan (y-z) and neutral fibre direction is parallel to the axis x is described by the following displacement field:

$$u_i = U_i e^{-jkx} e^{j\omega t} \tag{1}$$

Firstly, we will expose an analytical calculation of dispersion curves of a cylinder. Then to generalize to all the types of section topology and to display the section modes, we will develop a numerical approach called the propagation method. It will be applied to concrete cases.

2 - CHARACTERISATION BY ANALYTICAL THEORIES: CASE OF A CYLINDER

To understand the propagative phenomena in slender structures, we take an interest in a "perfect" structure, the cylinder and aim to determine its dispersion curves. Three analytical theories have been applied: the Euler-Bernoulli approach, the Donnel-Mushtari one and its extension, the Flugge one.

The figure 1 shows the superposition of the three theories dispersion curves for a hollow steel cylinder whose radius R and thickness h are respectively 50 mm and 2 mm.

Since the cylinder cross-section is considered to have no distortion, the Euler-Bernoulli theory is limited to low frequency range: there is no description of the section modes that appear at higher frequencies. The other two approaches display the setting of section modes as the frequency increases, even if the respective curves are not exactly the same. The disparities are explained by looking at the motion equations and particularly the h/R term, differently neglected in the successive displacement derivatives.

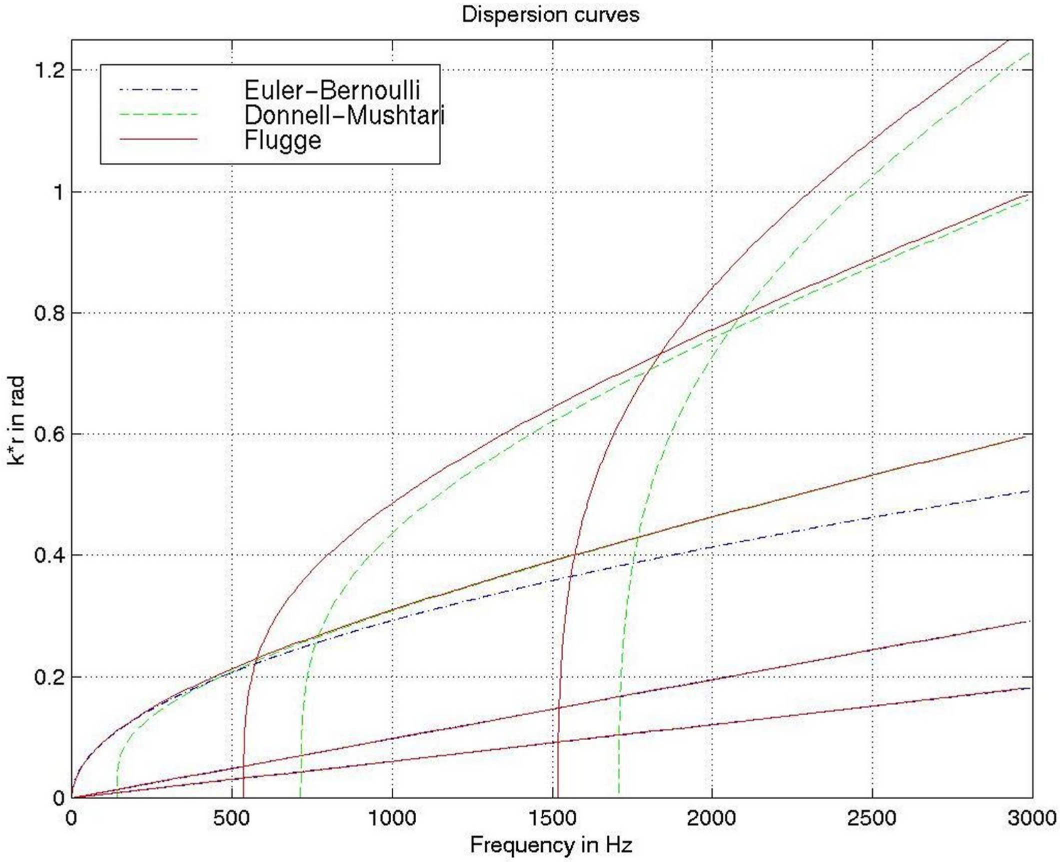

Figure 1: Comparison of three theories dispersion curves.

Accordingly the Flugge theory, that rigorously respects the Kirchhoff hypothesis, is the most appropriate analytical method to determine the dispersion curves of a cylinder.
Those analytical approaches are specific cylinder theories but can not be applied to any hollow structures and particularly those met in an automotive body. Only general numerical methods are suitable.

3 - A NUMERICAL PROPAGATION APPROACH

Many authors took an interest in a numerical determination of dispersion curves for any full section. Gavric's [2,3] and Knothe's [4] works served as the reference of the propagation method. Though they are interesting since they rely on the wave relation, they can not be easily used. Indeed the first one requires the development of a new finite element code with specific elements and interpolation forms and an adapted eigen values extraction method. As the second one is concerned, the structure is designed by an infinite succession of identical hyperelements, so that the cross-sections of each side are the same and the resulting mesh has internal nodes: no variable sections are studied.
In this paper we suggest a new approach whose principal condition is to be quickly planted in any existing finite element code. We try to exclude the explained drawbacks of the other methods. The great principles are following.
The propagation method relies on a two-dimension mesh of a one-dimension waveguide cross-section, that is artificially extended by a little length d for the calculation need: that indirectly amounts to mesh a piece of structure that contains no middle node between the left and right sections (cf Fig. 2).
By convention, the letters G and D respectively refer to left and right. In order to take into account the different motion types in the hollow structures, four degrees of freedom are attached to each node: the three translations and the rotation around the x axis.
The partition of the stiffness and mass matrix according to left nodes and right ones leads to the following motion equations in an finite element point of view:

$$\begin{bmatrix} S_{GG} & S_{GD} \\ S_{DG} & S_{DD} \end{bmatrix} \left\{ \begin{array}{c} U_G \\ U_D \end{array} \right\} = \left\{ \begin{array}{c} F_G \\ F_D \end{array} \right\} \text{ with } S_{ij} = K_{ij} - \omega^2 M_{ij},\ \forall i,j \in \{G,D\} \tag{2}$$

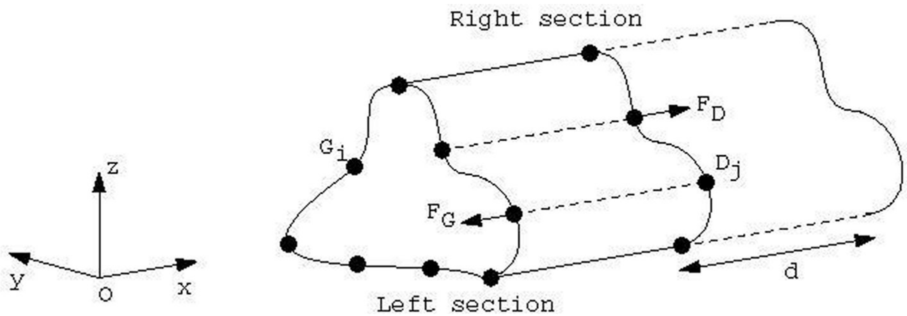

Figure 2: Cross-section of a one-dimension waveguide.

The system is classical in finite element method, but does not consider the waveguide hypothesis. It is introduced by the definition of the term $\lambda = e^{-jkd}$. The influence on the displacement and strain vectors is:

$$U_D = \lambda U_G \tag{3}$$

$$F_D = -\lambda F_G \tag{4}$$

By combining equations (2), (3) and (4), the new eigen values system is:

$$\left(S_{DG} + \lambda\left(S_{GG} + S_{DD}\right) + \lambda^2 S_{GD}\right) U_G = 0 \tag{5}$$

The equation (5) is sufficient to extract and post-treat the eigen values λ. Since this operation is made at each frequency step, it is an implicit system to determine the dispersion curves of any hollow structure.

4 - EXAMPLES

Firstly, as a numerical validation, the propagation method is compared to the Flugge theory for the cylinder. The figure 3 shows the superposition of the dispersion curves. More over, thanks to the propagation method, the successive propagative "modes" may be animated (cf Fig 4).
On figure 3 the wave numbers of bending, traction-compression and torsion waves are superposed: for the waves propagating as early as 0 Hz, the propagation method validity is shown. As the waves associated with distortion section "modes" are concerned, there is an excellent correlation on the cut-on frequencies but a foreseeable difference on the wave number amplitude as the frequency increases: it is the evidence of the analytical theory limitations. The terms, that are neglected in the low frequency range, have to be taken into account in the medium and high frequency range, but unfortunately the analytical expressions would become too complicated. On this example the propagation method numerically gives a satisfying description of the dispersion curves.
The second example is extracted from a typical automotive structure: we study a piece of rail, whose length and thickness are respectively 55 cm and 1,76 mm. The material is HLE275D steel. The figure 5 presents the mesh that is cut by three plans orthogonal to the neutral fibre generating three cross-sections. Two sections, called section 1 and section 2, have a very close topology. As the section 3 is concerned, the topology is completely different. The aim of this case is to determine the dispersion curves for two different cross-sections and display the influence of geometry on the propagative results (cf Fig. 6).
The figure 6 shows the influence of a variable cross-section. Indeed some waves are propagating for a given cross-section, whereas they vanish for another type. To characterise such hollow structures, it is essential to consider the breaking of impedances, i.e. the principal cross-sections with a strong geometric variation. It is physically fundamental, because it contains the notion of a wave trap. So by modifying the geometry of a frame, it becomes possible to exclude the waves that carries most of energy and then generates most of noise.

5 - CONCLUSION

In this paper, we presented the propagation method specially developed to characterise the dispersion curves of any hollow structure. It is based on a finite element approach by only considering the left and right nodes of a cross-section. After a comparison with several analytical theories on a hollow cylinder, it has been applied to a realistic automotive frame and has shown off the waves trap notion.

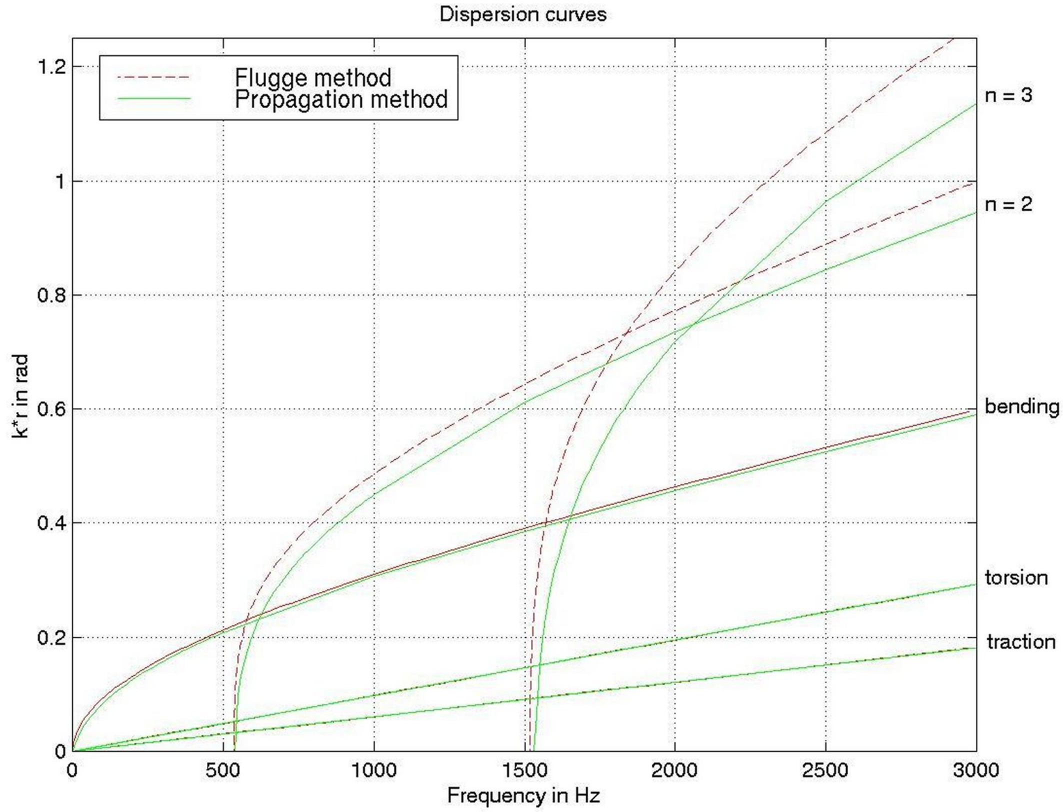

Figure 3: Comparison of dispersion curves: Flugge theory and propagation method.

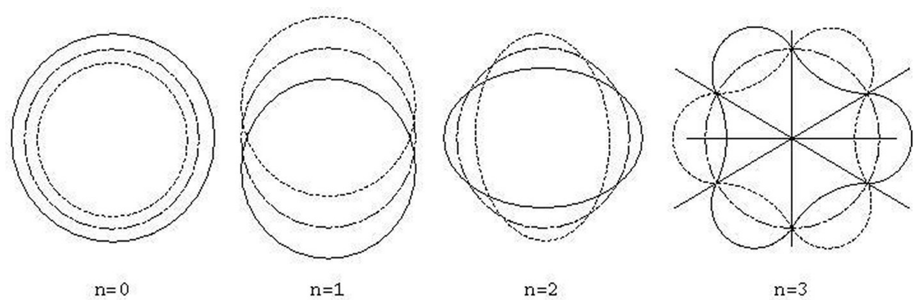

Figure 4: The first four modes of the cylinder.

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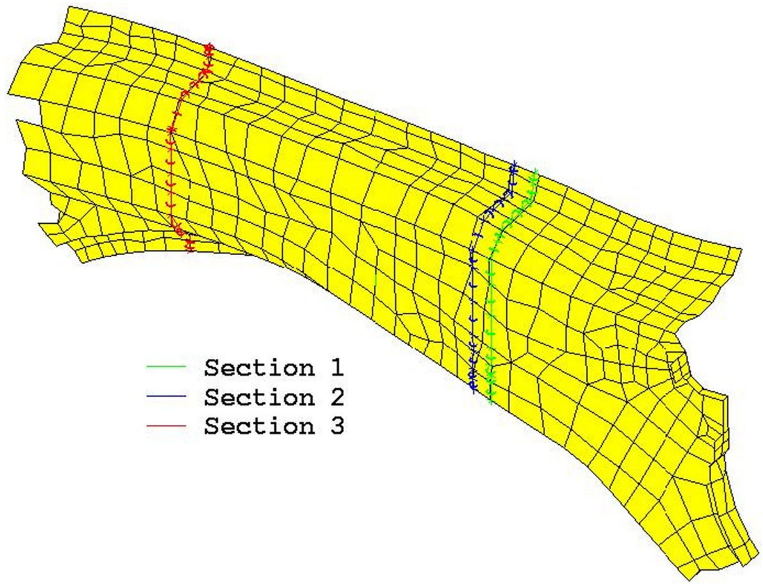

Figure 5: Mesh and cross-sections of a rail.

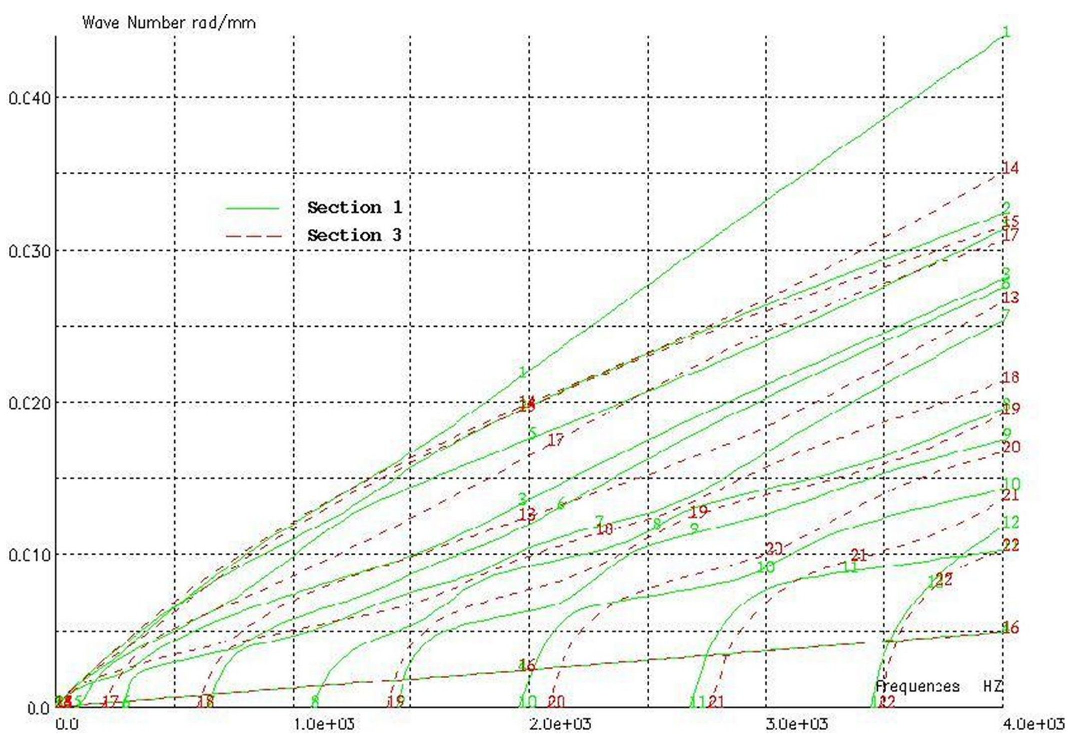

Figure 6: Comparison of dispersion curves for two cross-sections of the rail.