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MODAL AND PROPAGATIVE APPROACH IN STRUCTURAL-ACOUSTIC ANALYSIS BY MODAL DENSIFICATION METHOD

O. Bareille, M. Ichchou, L. Jezequel

Ecole Centrale de Lyon, 36, Avenue Guy de Collongue, 69131, Ecully Cedex, France

Tel.: +33 (0)4 72 18 64 64 / Fax: +33 (0)4 72 18 91 44 / Email: bareille@mecasola.ec-lyon.fr

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ABSTRACT
The prediction of the behavior of complex structures in the medium and high frequency ranges is at stake here. On the basis of primal or dual hybrid models, the contribution of local free and clamped modes are derived through an analytical Modal Densification Method (M.D.M.). The principle of this method is to convert the discrete summations on local modes into continuous ones. Coupling of structures is studied by using of global boundary variables. Simple fully analytical examples are presented.

1 - INTRODUCTION
Even if the last improvements of classical methods such as finite element methods, modal analysis methods and SEA are encouraging enough, a wide-range predictive tool is not yet available. For that purpose, the recent developments of a Modal Densification Method (M.D.M.) are exposed in this paper. For each sub-domain, two models of coupling are defined: the coupling between local and global degrees of freedom, the coupling between global degrees of freedom of neighbouring sub-domains.
Once included in a modal densification scheme, we can draw links with propagative kernel of the corresponding (semi-)infinite structures. The application to the vibration analysis of structural-acoustic systems is straight, whether in 1D or 2D cases. Here are presented one-dimensional examples for better analytical understanding and for comparison with exact or at least existing-method results.

2 - PRIMAL AND DUAL HYBRID MODELS
Each field to be calculated is represented by a set of global and local variables, one of each associated to a solution of a given loading problem.

2.1 - Displacement or force based formulations
We start with the equilibrium equation under the classical assumptions of linearity for an elastic medium. This equation is defined with an elliptical operator $\mathbf{A}$ whose canonical decomposition is $\mathbf{A}$=$\mathbf{T}^*\mathbf{T}$ ($\mathbf{T}^*$ differential operator, $\mathbf{T}$ stress-displacement operator). If harmonic solutions with pulsation ω are sought and Green equation is used, the problem can be written in two dual forms.

$$\begin{array}{c} a\left(u,v\right)-\omega^2\left(\rho u,v\right)=-\left\langle f,\mathbf{D}v\right\rangle \\ b\left(\sigma,\tau\right)-\omega^2 t\left(\sigma,\tau\right)=\omega^2\left\langle d,\mathbf{F}\tau\right\rangle \end{array}$$

$\mathbf{D}$ and $\mathbf{F}$ are boundary operators of displacement and force. Then the boundary conditions along the coupling boundary can be written:

$$\begin{array}{ll} \forall M\in\Gamma & \mathbf{FT}u=-f \\ \forall M\in\Gamma & \mathbf{D}\dfrac{1}{\rho}\mathbf{T}^*\sigma=d \end{array}$$

2.2 - Intermediate problems
Given a set of solutions $\{\psi_i\}_{i=1,k}$ to particular loadings along the boundary Γ, the aim of the intermediate problem is to find the orthogonal complement of this part of the solution in the subspace of general

solutions to homogenous problem $\{\phi_i\}_{i=1,\infty}$. For this purpose, a scalar product is defined: $a(.,.)$ in primal formulation, $b(.,.)$ in dual formulation.
In the primal formulation, the functions ϕ_i are local free modes, whereas in the dual formulation, those are the local clamped modes. By projection on the orthogonal complement of the subspace generated by $\{\psi_i\}_{i=1,k}$, the intermediate problem is built up. Resolution of this problem leads to a resolvent based expression of the solution.
Hence, we can write the coupling of the sub-domain to neighbouring ones only thanks to the global boundary variables, like in classical sub-structuring methods.

2.3 - Calculation of the resolvent
When resolving the intermediate problem, the following expressions of the resolvent are obtained:

$$\mathbf{R}_F = \left(\mathbf{I} - \omega^2 \mathbf{G}_F\right)^{-1} \;\; \mathbf{R}_C = \left(\mathbf{I} - \omega^2 \mathbf{H}_C\right)^{-1}$$

with

$$a(\mathbf{G}_F u, v) = (\rho u, v) \qquad b(\mathbf{H}_C \sigma, \tau) = t(\sigma, \tau)$$

$\mathbf{G}_F$ and $\mathbf{H}_C$ can be expressed with projections on the basis of local free modes $\{\phi_n\}_{n=1,\infty} = \{x_F\}_n$ or local clamped modes $\{\phi_n\}_{n=1,\infty} = \{y_C\}_n$, with respect to scalar products $(\rho.,.)$ and $t(.,.)$. Three methods have been proposed by Jezequel [1] [2] to calculate the resolvent. Estimation of this later is linked to the problem of truncated basis of the two sub-spaces (the local and the global ones). Truncation of this set of eigenfunctions gives a Rayleigh-Ritz scheme whereas fixing the eigenvalues above a given rank behaves like projection scheme. Each scheme bound the real value of resonance frequencies ω_{Fi} or ω_{Ci}. However, simply developing the resolvents with respect to ω^2 allows us to control the quality of estimation of $\mathbf{R}_F$ and $\mathbf{R}_C$.

3 - DENSIFICATION APPROACH
Once chosen the method to estimate the resolvent, the modal densification method can be foreseen. This approach consists in converting all the discrete summations involving the local modal bases $\{x_{Fk}\}_k$ and $\{y_{Ck}\}_k$ into continuous ones.
For instance: with $n(k)$ for the modal density and $f(\omega_k)$ for the function obtained from $x_{Fk}.x_{Fk}$ processed with random phase:

$$\sum_{k=n}^{\infty} \frac{(\rho x_{Fk}, \psi_i)(\rho x_{Fk}, \psi_j)}{\omega_k^2 - \omega^2} \;\; \rightarrow \;\; \int\int_V \rho\psi_i . \rho\psi_j \int_{\omega_n}^{\infty} f(\omega_k) . n(k) . dk . dV dV$$

Following the Modal Densification Method principles exposed in [3], contributions of all the higher range modes are included in wave like impedance or admittance kernels.

4 - ONE DIMENSIONAL EXAMPLES
Three bar problems are treated with MDM on the basis of the dual model. The displacement field is sought for boundary loadings in force. It can be demonstrated that the complete set of functions needed to describe the displacement in bars is:

$$\psi_1(x) = 1 - \frac{x}{l}, \;\; \psi_2(x) = \frac{x}{l}, \;\; \phi_n(x) = x_{Cn}(x) = \sin\left(\frac{n\pi}{l}x\right)$$

When random phase principle is applied and modal densification processed with lower bound of integration $\omega_n = 0$, the main equation to solve is

$$\frac{ES}{l}\left[\left(1 + e^{\frac{i\omega}{c}l} + g(\omega)\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \frac{i\omega}{c}\frac{l}{2}\begin{bmatrix} 1 & -e^{\frac{i\omega}{c}l} \\ -e^{\frac{i\omega}{c}l} & 1 \end{bmatrix}\right]\left\{\begin{matrix} A_1 \\ A_2 \end{matrix}\right\} = \begin{bmatrix} 1 - g(\omega) & e^{\frac{i\omega}{c}l} + g(\omega) \\ e^{\frac{i\omega}{c}l} + g(\omega) & 1 - g(\omega) \end{bmatrix}\left\{\begin{matrix} P_1 \\ P_2 \end{matrix}\right\}$$

with $g(\omega) = \left(1 - e^{\frac{i\omega}{c}l}\right) / \frac{i\omega}{c} l$ and (A_1, A_2, P_1, P_2), boundary variables in displacement and force.
Attention must be paid to projections of boundary loadings on local bases. Once the MDM is applied, the corresponding terms might not be equal to zero anymore (see the second member of the last equation). Exact solutions are obtained with dynamic stiffness methods.

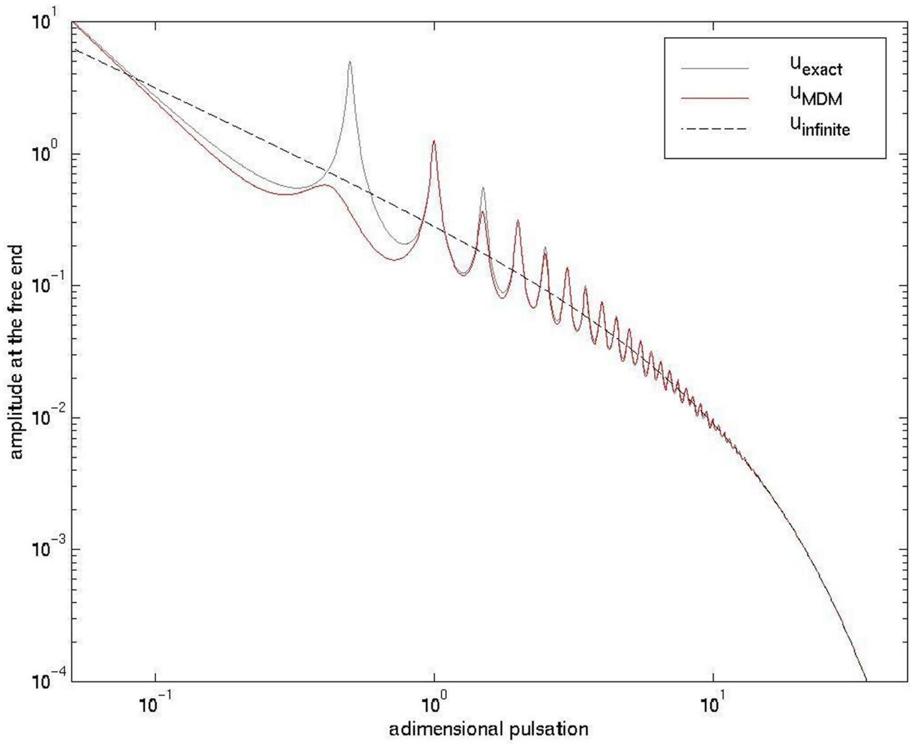

Figure 1: Transfer response of a free-free bar.

4.1 - Free-free bar

This equation can be solved in a straightforward manner in the case of a bar with free ends. The conditions in forces are $P_1 = F_0$ and $P_2 = 0$.

The responses to static loadings give the exact static response, whereas the medium and high frequency range responses are obtained thanks to the modal densification results on local modes.

4.2 - Free-clamped bar

The boundary conditions imposed are $P_1 = F_0$ and $A_2 = 0$. The response in displacement (amplitude and phase) is observed at the point where the force is applied.

4.3 - Free coupled bars

Two bars are assembled end to end. They have different mechanical characteristics. The assembly is excited at the uncoupled end of the first bar whereas the displacement is observed at the free end of the second bar. Say boundary variables are $\left(A_1^{'}, A_2^{'}, P_1^{'}, P_2^{'}\right)$ for bar I and $\left(A_1^{''}, A_2^{''}, P_1^{''}, P_2^{''}\right)$ for bar II. The conditions of continuous displacements and forces between the two bars are stated as:

$$A_2^{'} = A_1^{''},\ P_2^{'} = P_1^{''}$$

5 - CONCLUSION

An overview of the last improvements of the Modal Densification Method have been illustrated by examples on bars. The coupling conditions are easy to describe and to implement.

Moreover, results in amplitudes and phase are in good agreement with those obtained by the dynamic stiffness method.

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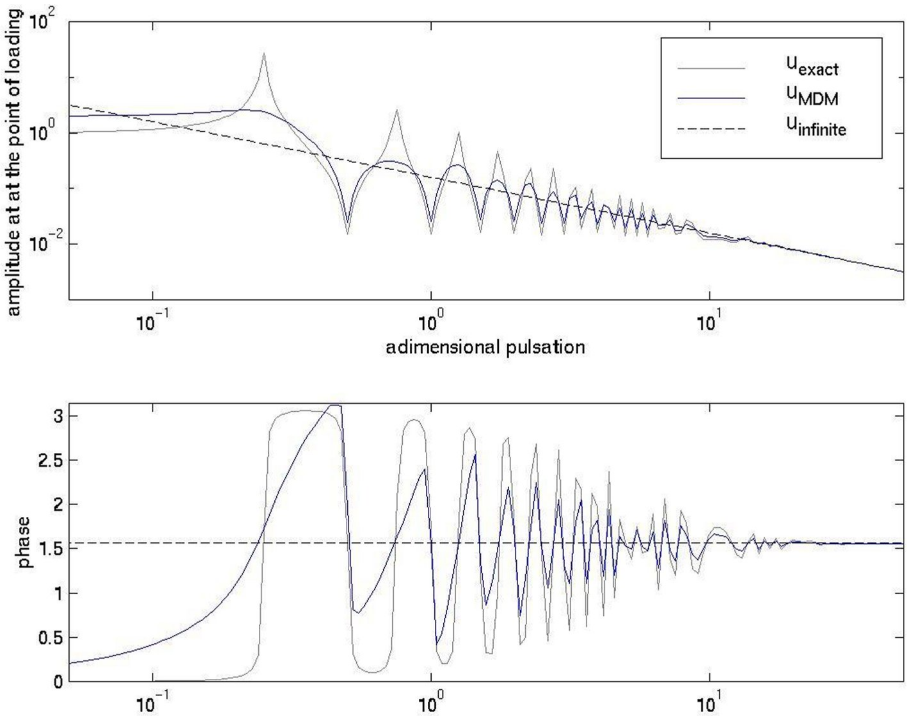

Figure 2: Displacement and phase at the point of loading of a free-clamped bar.

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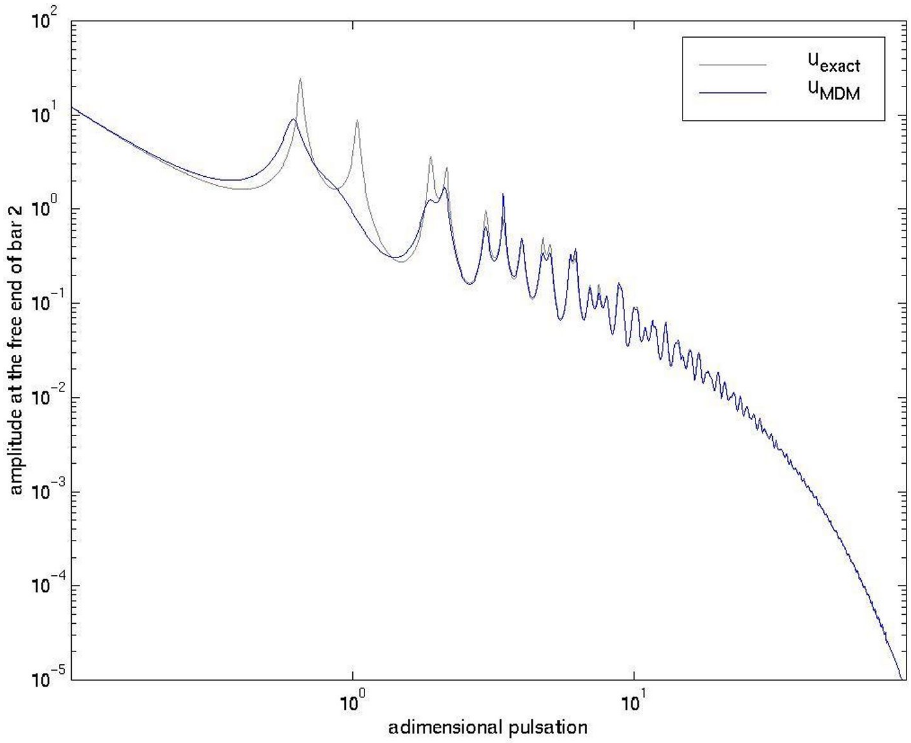

Figure 3: Displacement at the end of an assembly of two different bars.