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TLM MODEL FOR SOUND PROPAGATION ABOVE GROUND

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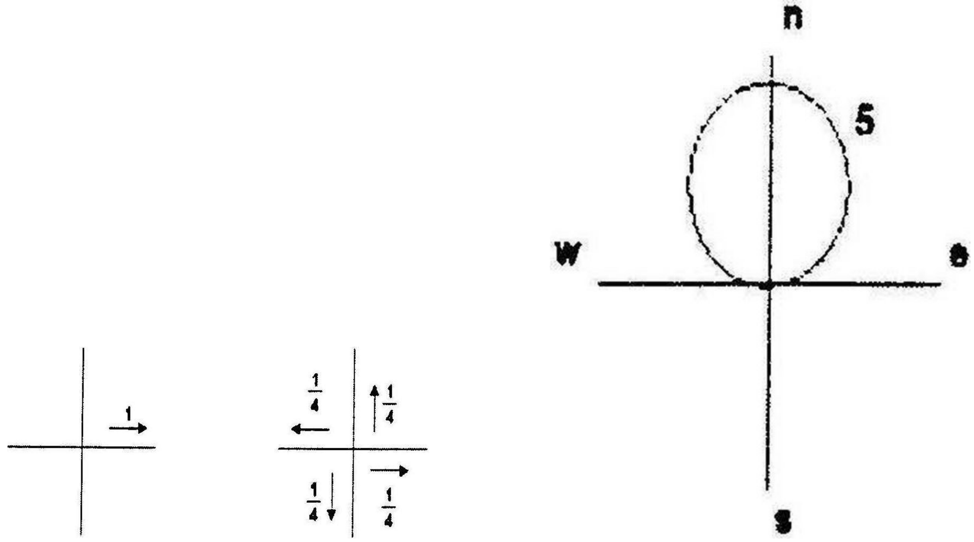
ABSTRACT
The TLM method has been used to study the propagation of a sound pulse from a point source above ground. The ground is modelled as a porous material with rigid pores. The paper discusses some general features of the TLM method and presents calculation results from a 3D axisymmetric realization.

1 - INTRODUCTION

Numerical methods, like the finite difference and finite element methods, receive increased interest in calculations of sound propagation above ground because of the rapid development in computational power. The flexibility given by such techniques allow us to study problems where ground geometry and parameters, as well as the air's density and sound velocity, might have local variations. In addition to the above model types, which are based on the differential equations governing the system, the so-called lattice gas models are undergoing rapid development. In such models, space is discretized by nodes and connecting line segments and the behaviour of masses, energy quanta, or signal pulses moving along these lines are studied under the application of propagation, redistribution, and collision rules. The models are usually implemented in the time domain directly. Locality and parallelism are characteristic features. Locality because updating of nodal values only depend on the nodal values at the node itself and its immediate neighbours at the previous time step, and parallelism because the same rules apply at every node in the system. Parallel computer processing is therefore well suited to this type of modelling. An attractive feature of such modelling is that the nodal updating rules are normally very simple and therefore rapid to compute. Good numerical stability is also a characteristic.
The TLM method, or the Transmission Line Modelling method is one such approach. It represents attempts to model the physical processes more directly, in fact, the TLM method can be said to be a discretized realization of Huygens' principle. Kagawa [1], has shown its accuracy to be equivalent to that of a second order finite difference scheme.

2 - THE MODEL

A signal moving along a line will divide its energy between the intersecting transmission lines, when arriving at a node. Fig. 1 illustrates the situation for a 2D model, we see the energy being equally divided between the 4 directions. For pressure reflection and transmission coefficients, R and T, transmission line theory tells us that these will be -1/2 and +1/2 respectively. A 5'Th line in the form of a loop can be introduced at each node to provide an additional path for the energy. This is of interest when a local reduction in speed of sound or damping of the system is required. See [1] for details. A full 3D model can also be described in a cartesian coordinate system. However, if assumptions can be made of axisymmetry, the calculations can still be done in a modified 2D framework, and considerable computer space and calculation time can be saved.



(a): Constant velocity model. **(b):** Variable velocity model.
Figure 1: A 2D model for TLM calculations.

The matrix equation relating pressure values reflected and transmitted from a node to the incident ones for 2D systems is written as:

$$\begin{bmatrix} p^e_{i,j} \\ p^n_{i,j} \\ p^w_{i,j} \\ p^s_{i,j} \\ p^5_{i,j} \end{bmatrix}_{k+1} = \frac{2}{4+\eta+\zeta} \cdot \begin{bmatrix} 1 & 1 & f_1 & 1 & \eta \\ 1 & 1 & 1 & f_1 & \eta \\ f_1 & 1 & 1 & 1 & \eta \\ 1 & f_1 & 1 & 1 & \eta \\ 1 & 1 & 1 & 1 & f_2 \end{bmatrix} \begin{bmatrix} p^e_{i-1,j} \\ p^n_{i,j-1} \\ p^w_{i+1,j} \\ p^s_{i,j-+1} \\ p^5_{i,j} \end{bmatrix}_k$$

where f_1 is $-1-(\eta+\zeta)/2$, and f_2 is $(\eta-\zeta-4)/2$. η and ζ are related to the reduction in sound speed and damping coefficient of the medium respectively. For 3D axisymmetric systems, the coefficients of the updating matrix are modified by a density defined for each individual line being inversely proportional to the distance from the symmetry axis, for details see [1].

Boundary conditions are applied by matching individual transmission line impedances to load impedances. This yields a set of reflection and transmission coefficients that can be used for waves travelling from one ρc medium to another. In the present case, where the ground material is considered as a porous material having a rigid frame, the transmission into the material is characterized by a strong damping, having the character of diffusion more than propagation. It is equivalent to the "second compressional wave" in the Biot theory for wave propagation in porous materials.

One dimensional harmonic wave propagation in such a material has a damping coefficient close to $\gamma = 2\sqrt{\omega\phi/\rho' c'}$ for highly damped materials, where ϕ designates the flow resistance, and primed quantities ground material densities and velocities adjusted for porosity and tortuosity With k_s and Ω, designating the structure factor and porosity of the material respectively, $\rho' = \rho k_s/\Omega$ and $c' = c/\sqrt{k_s}$ [3].

Reflection coefficients for such materials can, again for cases of highly resistive grounds, be approximated by

$$R = \frac{\phi^2 + \left(\omega\rho'\right)^2 - (\gamma\rho c)^2}{\phi^2 + (\omega\rho' + \gamma\rho c)^2}$$

where unprimed quantities are the ones of the medium above the ground. The TLM model is a time domain one. Source signals must therefore be defined which are narrow enough in frequency content to justify use of the above harmonic expressions. In the present study, we have made use of Ricker type wavelet functions.

3 - RESULTS

Figure 2 shows that the system responds by differentiating the signal. As the field parameter of the model is acoustic pressure, the source signal therefor corresponds to a velocity source input.

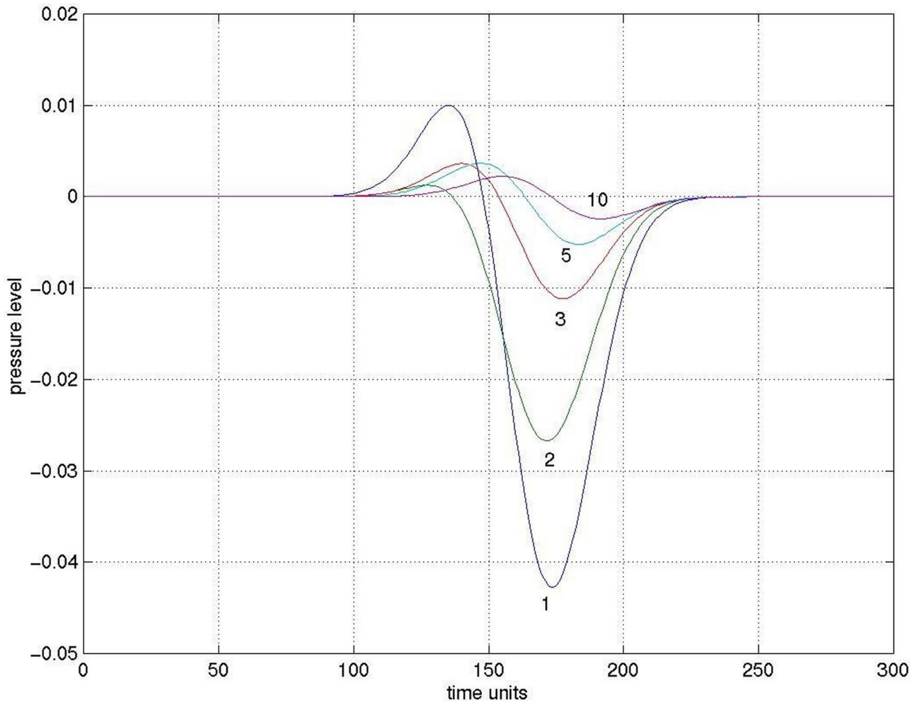


Figure 2: The response at indexed cell lengths directly in front of the source to a point source at the axis of symmetry; the input signal was a Gaussian function $f\left(t\right)=e^{-\pi^2\left(f_0 t-1\right)^2}$, where f_0 is the centre frequency of the signal.

The next figures 3 are from a calculation of sound propagation above a flat ground with a trench. The porous material was described with k_s=10, Ω=0.1, ϕ=100000 Pa s/m^2, while the fluid parameters are the ones of air. The figures 3a, 3b, and 3c show snapshots of the wave in the vicinity of the trench. Characteristic features are the direct wave, the ground reflection and the interaction of the waves with the trench geometry. The latter shows that edge diffraction is an important phenomenon. The calculation domain was defined as a 800 $\times$800 cell system. Each transmission line, or cell length, is 0.0962m. The porous material is 80 cells deep, and the trench is positioned between 400 and 600 cell-lengths in the horizontal direction. The central frequency, f_0 was 100Hz for this calculation.
The outer boundaries of the domain are given a ρc load impedance modified by a directional term with respect to the source position. Matching the individual impedance line impedances to such a load impedance give reflection coefficients $R = 1 - \sqrt{2}\cos\theta/1 + \sqrt{2}\cos\theta$, where θ is the angle between the surface normal and the line of sight to the source. This absorption condition is not perfect and will give some reflection. We are presently testing other boundary absorption schemes.
In a further test we modelled the air / porous material interface in a 1D simulation. Using the same material parameters as for the figures 3. Figure 4 shows time histories at three different positions. The interface between the media is at 2.5m. The upper plot shows the incident and reflected signal at a position well away from the interface. The middle and lower plots show pressure signals a little into the porous material. The rapid decay is evident.

4 - CONCLUSION

We have shown that the TLM method, like the finite difference and finite element methods, can be used to model sound transmission over grounds having big changes in the geometry. The method is relatively easy to program, and from the point of view of numerical stability it has advantages. Very large parameter changes over an interface can be handled.

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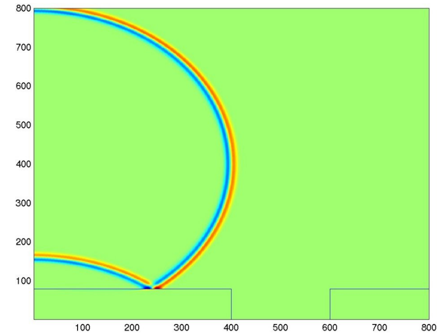

Figure 3(a): The sound field at 170ms.

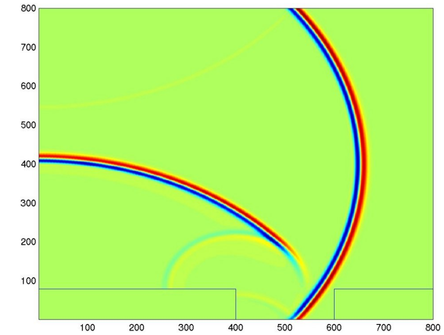

Figure 3(b): The sound field at 272ms.

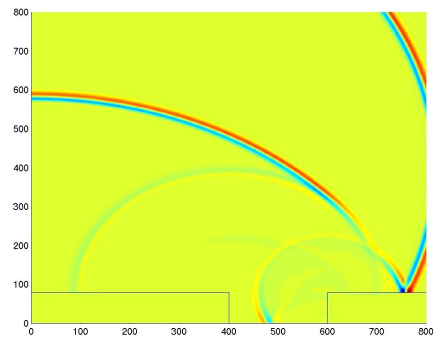

Figure 3(c): The sound field at 340ms.

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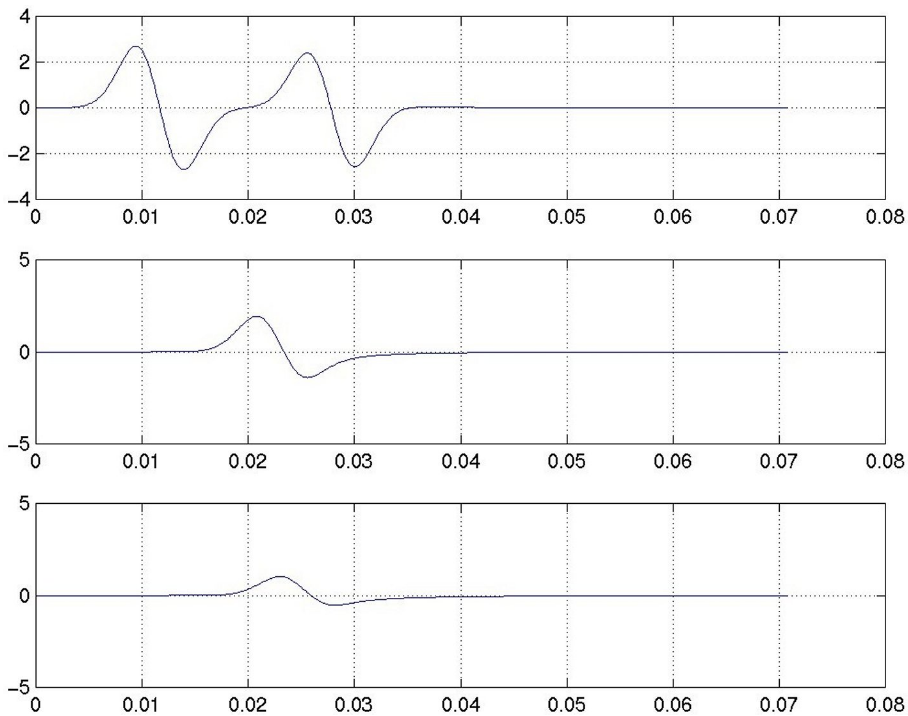

Figure 4: Time histories at positions x=0.48, 2.65, and 2.79m.