***inter.noise 2000***
***The 29th International Congress and Exhibition on Noise Control Engineering***
***27-30 August 2000, Nice, FRANCE***

**I-INCE Classification:** 7.6

# SIMPLIFIED MODEL OF INTERNAL ACOUSTIC CAVITY IN RECIPROCATING REFRIGERATING COMPRESSORS

**L. Coutaz, L. Gavric**

CETIM, 52 av Felix Louat, 60304, Senlis, France

Tel.: 03 44 67 33 66 / Fax: 03 44 67 36 17 / Email: leon.gavric@cetim.fr

**Keywords:**
COMPRESSOR, CAVITY, VIBROACOUSTIC, MODEL

## ABSTRACT
A simple analytical model of sound radiation of hermetic refrigerant compressor is described in the paper. The model comprises the gas pulsations within the compressor cavity, its interaction with the shell and finally the response of the compressor shell in terms of vibration and radiated sound power. The model is found to be reliable in low frequency domain (up to 1 kHz), for the small and medium size hermetic compressors.

## 1 - INTRODUCTION
Two principal parts of small and medium size hermetic compressors can be distinguished: the compression mechanism linked to the electric drive and hermetically sealed shell. The space between these three principal components, filled with refrigerant fluid, is referred as compressor (or shell) cavity. The most of the sound power of such compressors is radiated by the hermetic shell. The shell radiates sound in all audible range. The low and medium frequencies are particularly important ($0 - 2$ kHz). In this frequency range, compressor shell does not deform significantly when vibrating. Such vibration of the shell can be decomposed in three movements corresponding to the three principal directions ($X$, $Y$ and $Z$). The vibration of the shell can be generated by the gas pulsations within the shell cavity. The gas pulsations are induced by the suction line of compressor. If acoustic modes of the shell cavity coincides with the excitation, the high gas pulsations amplitudes occur. The spatial distribution of pressure is defined by the acoustic response of the cavity. The resulting excitation force generates the vibration of compressor shell and consequently the sound radiation. The vibration response of the shell can be computed using coupled FEM models of cavity and of the shell structure. Once the shell response computed, BEM model can be used to compute sound radiation. This is very laborious and complex task which requires a specialised software and an important computing capacity. A simple analytic model of shell radiation induced by gas pulsation is needed in pre-design phase. Such model permits a simple and accurate evaluation of physical phenomena involved in sound generation.

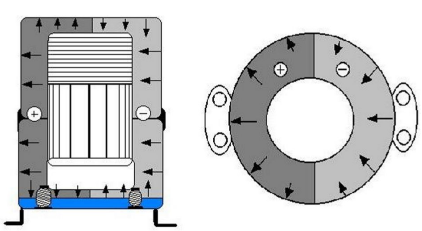

**Figure 1(a):** Lateral translation movement generated by an anti-symmetric mode.

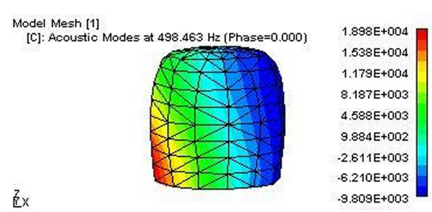


**Figure 1(b):** An acoustic mode of compressor cavity computed using FEM.

## 2 - VIBROACOUSTIC MODEL OF HERMETIC COMPRESSORS
To establish the **acoustic cavity model**, the inner part of compressor shell is assimilated to an annular acoustic cavity. The outer cylinder represents the hermetic shell of the reciprocating compressor and

the inner one approximates the compression/electric drive unit. The acoustic characterisation of the cavity in cylindrical coordinates is based on Bessel equations involving complex asymptotic functions. If modes of the annular cavity are imposed to a rectangular one, the simplified mathematics tools relative to Cartesian coordinates can be used.

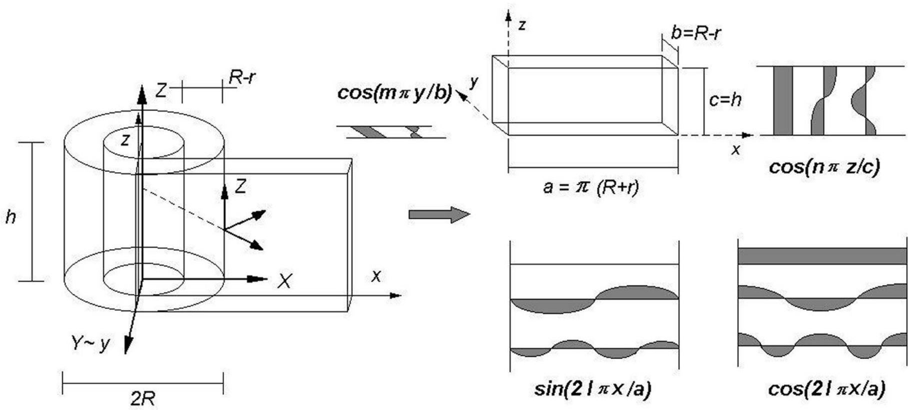


**Figure 2:** Geometry and modes of an equivalent rectangular refrigerant gas cavity.

The acoustic modes of an equivalent rectangular cavity corresponding to those of an annular one are shown on the figure above. To make up for the lack of privileged directivity of the annular cavity modes in the circular direction, corresponding to the $X$-direction, "sinus" modes type are added. So, only even modes are considered. These consistent modes, orthogonal to "cosine" usual ones, respect continuity conditions on $x$=0 and $x = a = \pi\,(R + r)$ surfaces.
The wave equation is employed to compute the acoustic **eigenfrequencies and modes** of the gas cavity. The homogeneous part of the Helmholtz equation leads to the natural frequencies and to two orthogonal mode types of the equivalent rectangular cavity, as follows:

$$\omega_{lmn} = c_0 \sqrt{\frac{4l^2}{(R+r)^2} + \frac{m^2\pi^2}{(R-r)^2} + \frac{n^2\pi^2}{c^2}}$$

$$\begin{cases} p_{(nqm)_1}(x,y,z) = \cos\left(\dfrac{2l}{R+r}x\right)\cos\left(\dfrac{m\pi}{R-r}y\right)\cos\left(\dfrac{n\pi}{h}z\right) \\ p_{(nqm)_2}(x,y,z) = \sin\left(\dfrac{2l}{R+r}x\right)\cos\left(\dfrac{m\pi}{R-r}y\right)\cos\left(\dfrac{n\pi}{h}z\right) \end{cases}$$

The table below shows the accuracy of natural frequencies obtained by three different models.

| **Finite element computation** | | **Annular cavity** | | **Rectangular cavity** | |
|---|---|---|---|---|---|
| N° | $\omega_N$ [Hz] | $(M,N,Q)$ | $\omega_{MNQ}$ [Hz] | $(L,M,N)$ | $\omega_{LMN}$ [Hz] |
| 0 | 0 | (0,0,1) | 0 | (0,0,0) | 0 |
| 1 | 249 | (0,1,1) | 344 | (2,0,0) | 336 |
| 2 | 280 | (1,0,1) | 369 | (0,0,1) | 369 |
| 3 | 391 | (1,1,1) | 506 | (2,0,1) | 500 |
| 4 | 612 | (0,2,1) | 638 | (4,0,0) | 673 |
| 5 | 633 | (1,2,1) | 737 | (4,0,1) | 767 |
| 6 | 746 | (2,0,1) | 740 | (0,0,2) | 740 |
| 7 | 835 | (2,1,1) | 816 | (2,0,2) | 813 |

**Table 1:** Eigenfrequencies of a compressor cavity computed using three different methods.

Gas pulsations generated by the suction muffler inlet could be amplified by the cavity resonances and could create important forces on hermetic shell. Using modal expansion techniques the **steady state pressure distribution** is defined as

$$p(x,y,z,t) = \sum_l \sum_m \sum_n \xi_{lmn}(\omega) \left[ p_{(lmn)_1}(x,y,z) + p_{(lmn)_2}(x,y,z) \right]$$

where $\xi_{lmn}(\omega)$ represents the modal amplification factor. This pressure field expression allows us to calculate the created forces on hermetic shell and the corresponding radiated sound field [1]. The vibration of the shell is defined by this pressure distribution and by shell local stiffness. To reduce mathematical complexity, the vibration can be represented as summed of shell "breathing" and three "rigid body" vibration [2]. The shell "breathing" is generated by the spatial average of inner pressure. Great part of reciprocating compressors, because of their important stiffness, have their shell natural frequencies higher than the cavity ones. In this case, the deformations induced by $p_0(\omega)$ could be computed by "static" corresponding formulas on each shell sides. The resulting radiated sound field can be described by acoustic monopole model. Although, gas pulsation could develop "rigid body" vibration which are induced by anti-symmetric modes (Fig. 1). The amplitudes of the three resultant force components $(F_X(\omega), F_Y(\omega), F_Z(\omega))$ which generate "rigid body" vibration, are computed from pressure distribution integration on sides of the shell. The induced radiated sound field can be described by three dipoles. By unrolling the annular cavity around its centre axis, the computation of $p_0(\omega)$ and $(F_X(\omega), F_Y(\omega), F_Z(\omega))$ is realised using modal expansion in the equivalent rectangular coordinates system. These components are indicated in the following expressions:

$$p_0(\omega) = \frac{1}{a(2b+c)} \left[ \int_{x=0}^{a} \int_{z=0}^{c} p(\omega,x,b,z)\,dxdz + \int_{x=0}^{a} \int_{y=0}^{b} p(\omega,x,y,0)\,dxdy + \int_{x=0}^{a} \int_{y=0}^{b} p(\omega,x,y,c)\,dxdy \right]$$

$$F_X(\omega) = \int_{x=0}^{a} \int_{z=0}^{c} p(\omega,x,b,z) \sin\left(\frac{2\pi x}{a}\right) dxdz$$
$$F_Y(\omega) = \int_{x=0}^{a} \int_{z=0}^{c} p(\omega,x,b,z) \cos\left(\frac{2\pi x}{a}\right) dxdz$$
$$F_Z(\omega) = \int_{x=0}^{a} \int_{y=0}^{b} p(\omega,x,y,c)\,dxdy - \int_{x=0}^{a} \int_{y=0}^{b} p(\omega,x,y,0)\,dxdy$$

**3 - COMPARISON OF COMPUTED AND MEASUREMENT DATA**

The developed analytical model has been applied on different compressors. Accurate computation results have been obtained for vibrations, gas pulsations and the radiated sound power. Experimentally obtained sound power level is compared with corresponding computation results in figure 3. The example corresponds to small hermetic compressor (0.12 kW of cooling capacity). The amplitudes around 500 Hz, are due to rigid body vibrations excited by the anti-symmetric modes of the cavity (see Fig. 1).

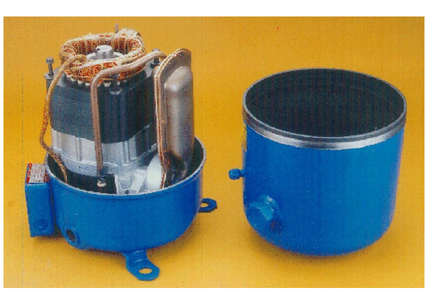

**Figure 3(a):** View of open medium size hermetic compressor.

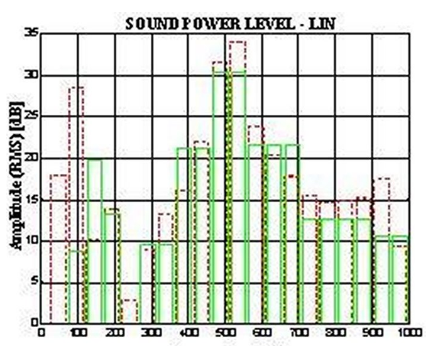


**Figure 3(b):** Comparison of computed and measured sound power levels; – dotted line: computed data / solid line: measured data.

## 4 - CONCLUDING REMARKS

A simple analytical model of an internal cavity of reciprocating compressors has been developed. The successive geometric simplifications and "static" response approximations used in the model, provide accurate results for small and medium size compressor in low frequency domain ($0 - 1$ kHz). Considering the difficulties of establishing FEM models, the use of such analytical technique seems to be appropriate to evaluate relative influence of design variables on the sound power level.

## REFERENCES

1. **Pan and Jones**, Gas Path Sound Transmission In Spherically-Shaped Reciprocating Compressor: Theory and Experiment, *Transaction of the ASME*, Vol. 121, pp. 8-17, 1999

2. **Koopmann and Fahnline**, Designing Quiet Structure - A sound Power Minimization Approach, *Academic Press Inc, San Diego, California, USA*, pp. 23-57, 1997