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SELF-SUSTAINED OSCILLATIONS OF GRAZING FLOW ALONG A HELMHOLTZ RESONATOR: A SIMPLIFIED THEORY

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ABSTRACT
Vortex induced self-sustained oscillations of the grazing flow along a Helmholtz(-like) resonator is considered at moderately high acoustical amplitudes. A modification of the single vortex model proposed earlier in the literature is discussed to take more accurately the effect of the geometry of the edges into account. We assume that the vorticity is concentrated into line vortices convected at a constant speed along a straight line from the upstream edge to the downstream edge. We propose a procedure in order to avoid the spurious singularity of the downstream sharp edge. The model predicts qualitatively the critical Strouhal numbers for oscillations and provides an estimation of the pulsation amplitude within a factor two compared to original models and experimental results. In particular the model removes spurious higher order hydrodynamic modes observed in earlier models.

1 - INTRODUCTION

1.1 - Helmholtz-like resonator or acoustical monochromator
Self-sustained oscillations of the grazing flow along a deep cavity are driven by coupling between the instability of the shear layer and the resonant acoustical field in the cavity (Rockwell & Naudasher [1], Nelson [2], Ziada [3], Bruggeman [4], Kriesels [5]).
We consider here the case of a Helmholtz-like resonator: a cavity of depth L_c and cross section S_c connected to the exterior (free field) through a neck of length L_n and cross section S_n (Fig. 1).
The Helmholtz resonance frequency is defined as $\omega_H = c_0\sqrt{\dfrac{S_n}{S_c L_c L_n}}$ where c_0 is the speed of sound in the air. In our case, $\omega_H L_c/c_0$ is not sufficiently small to consider the resonator as compact. However, the configuration retains the property that higher resonance frequencies are not harmonic of the fundamental resonance ω_0. Therefore, the acoustic field will be dominated by a component ω near ω_0 allowing a fair description of the oscillation as a pure harmonic. We can call this an acoustical monochromator (Gilbert [6]).
As we consider low frequencies, the acoustical field inside the resonator can be described in terms of two travelling waves while the acoustical flow in the neck is locally incompressible ($L_n\omega/c_0 \ll 1$).

1.2 - Previous works from literature
Elder [7] and Howe [8] proposed simplified models where the shear layer response is assumed to be linear. Such an approach is limited to very low amplitudes (Bruggeman [4]). Typically, $\hat{u}/U_0$ (where $\hat{u}$ is the average acoustical velocity amplitude across the shear layer and U_0 is the main flow velocity) should be $\mathrm{O}\left(10^{-3}\right)$ or lower.
When $\hat{u}/U_0 = \mathrm{O}\left(1\right)$, the acoustical perturbation induces a break down of the shear layer into discreet vortices and the amount of vorticity shed depends on the acoustical field amplitude (Howe [8]). This high amplitude regime has been studied by Kriesels [5].
Bruggeman [4] introduced the concept of moderate amplitudes $\hat{u}/U_0 = \mathrm{O}\left(10^{-1}\right)$ for which he assumed a break down of the shear layer into discreet vortices but no significant influence of the acoustical field on the amount of vorticity shed. This means that the acoustical field triggers the formation of a discreet vortex

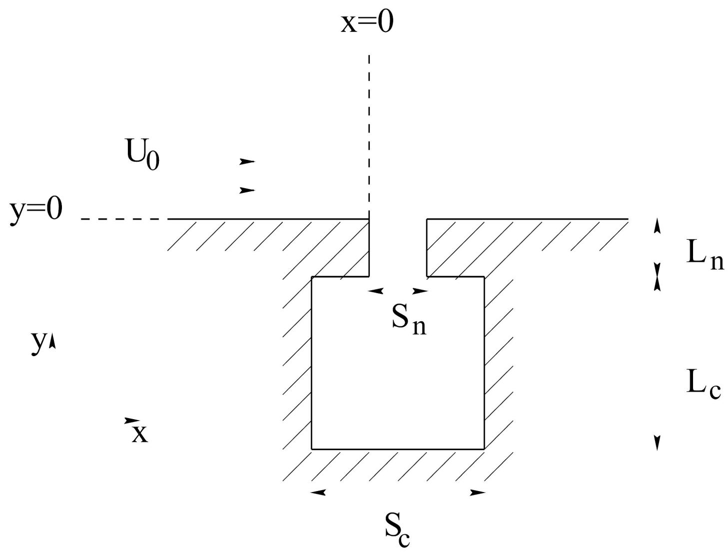

Figure 1: 2D-scheme of the cavity.

which absorbs the vorticity shed at the upstream edge of the opening of the cavity. This corresponds to the model of Nelson [2] for the resonator and of Holger [9] for the edge-tone. A further simplification is to assume that the vorticity is concentrated into line vortices convected at a constant speed U_Γ along a line joining the upstream to the downstream edge of the cavity. We use for U_Γ the value $0.4 \times U_0$ proposed by Bruggeman [4]. Using this model, Bruggeman [4] and Howe [8] obtained predictions of critical Strouhal numbers for strong oscillations for sharp edged cavity (Fig. 1). The predicted amplitudes are however much larger than the observed amplitudes (at least a factor three).
A major problem is that in the model the vortex path hits the singularity of the downstream edge. This does not occur in real life (Bruggeman [4], Howe [8]). This problem can be reduced by considering a cavity with rounded rather than sharp edges. This removes the singularity. For such a cavity, Hirschberg [10] proposed to assume a uniform acoustical velocity at the shear layer. The comparison with experiments (Bruggeman [4]) and numerical simulations (Kriesels [5], [11]) shows that this model predicts the acoustical source power within a factor two for Strouhal numbers $S_r = \dfrac{\omega W}{2\pi U_0} \approx 0.5$ based on the width W of the cavity opening in the streamwise direction.
We propose an alternative model for a sharp edged cavity.

2 - SIMPLIFIED MODEL FOR A SHARP EDGED CAVITY

The key idea of our model is that we assume that the upstream singularity of the acoustical field is essential while the effect of the downstream one should be reduced by the finite size of the vortex core and departure from the imposed vortex path (Bruggeman [4], Howe [8]).

2.1 - Determination of the acoustical velocity in the neck of the resonator

As proposed by Bruggeman [4], the locally incompressible acoustical velocity field at the shear layer position is calculated by conformal mapping and potential theory. The conformal mapping used is based on the Schwarz-Christoffel transformation (Milne-Thompson [12]) of a physical polygone into a complex plane. In order to avoid the spurious singularity of the downstream sharp edge, we assume a geometry which is not the actual opening geometry (Fig. 2).
This assumption enables us also to obtain an explicit relation for the conformal transformation of the physical plane (z-plane) to the complex plane (τ-plane) (Fig. 3).
By means of the potential theory and using a Newton method for integration, we can determine the acoustical velocity in terms of the vortex position. Near the singularity (upstream sharp edge of the cavity), for simplicity's sake, we determine the acoustical velocity by using the fact that the acoustical flow velocity behaves locally as $z^{-1/3}$ as a function of its position z relative to the sharp edge (Prandtl [13]). The results are summarized in a least square fit formula.

2.2 - Modelisation of the cavity

The system is described as a forced linear acoustical system:

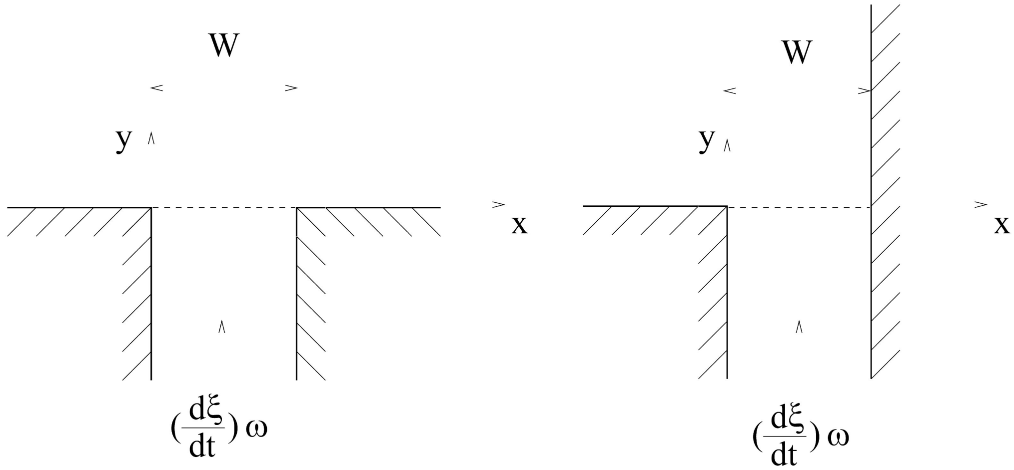

Figure 2: Actual geometry of opening (left) and geometry (right) used to calculate the acoustical field distribution by means of a conformal mapping.

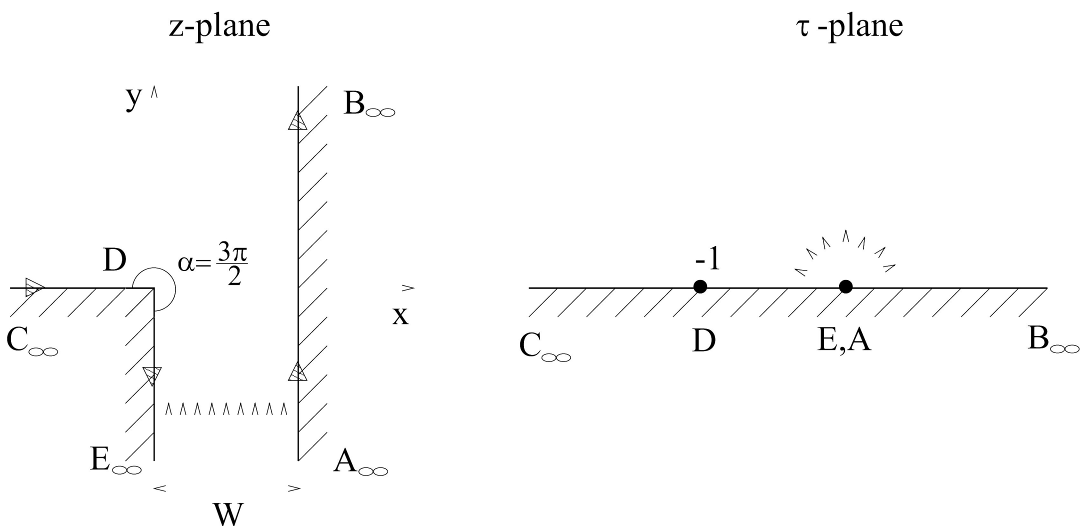

Figure 3: Conformal mapping.

$$M\frac{d^2\xi}{dt^2}+R\frac{d\xi}{dt}+K\xi=F\left(t\right) \tag{1}$$

where x is the acoustical displacement, $\xi=\int\hat{u}dt$ corresponding to the average of the acoustical displacement in the neck. The damping coefficient R was calculated from the measured quality factor ($Q\approx 30$) of the resonator in absence of main flow. The right-hand side of the equation of motion (eq. 1) represents the sum of the 'external' forces (turbulent losses and Coriolis force) which act on the acoustical mass-spring system.

Vortex shedding at the inner side of the neck and the corresponding turbulent dissipation is described by means of a quasi-stationary model (Hirschberg [10]) so that we find using the Powell/Howe analogy:

$$F\left(t\right)=\int_{V_n}\underbrace{-\rho_0\left(\vec{\omega}\times\vec{v}\right)}_{\text{Coriolis force}}\cdot\underbrace{\frac{\vec{u}'}{\left|d\xi/dt\right|}}_{\text{Geometric factor}}dV-\underbrace{\frac{1}{2}S_n\rho_0\left|\frac{d\xi}{dt}\right|\frac{d\xi}{dt}}_{\text{Turbulent losses}} \tag{2}$$

The Coriolis force $-\rho_0\left(\vec{\omega}\times\vec{v}\right)$ can be written in terms of the circulation $\Gamma_n\left(t\right)$ (Nelson's model [9]).

Finally, assuming that the displacement of the acoustical particles is harmonic: $\xi=\hat{\xi}\cos\left(\omega t\right)$, we obtain a set of two equations for $\hat{\xi}$ and ω by multiplying equation 2 by respectively $\cos\left(\omega t\right)$ and $\sin\left(\omega t\right)$ and by integrating over a period of oscillation. The equations are explicit and we get a system of two equations of the second degree for $\hat{\xi}$ and ω which is easy to solve numerically.

3 - RESULTS

The method proposed in the previous section provides an analytical solution of the acoustical particle displacement $\hat{\xi}$ in the neck of the resonator. This method is applied to the original models of Hirschberg [10] and Bruggeman [4] (section 1). Therefore, the results present the ratio $|u'|/U_0$ between the acoustical velocity amplitude $|u'| = \hat{\xi}\omega$ and the main flow velocity U_0 in terms of the Strouhal number $S_r = \dfrac{\omega_0 W_{eff}}{2\pi U_0}$ (where W_{eff} is the effective width of the neck).
The results obtained for two configurations are shown in figure 5a. The main characteristics of the different models are summarized in a scheme (Fig. 4).

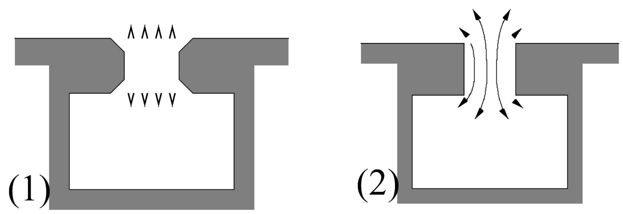

Figure 4: Configurations studied: (1) model 1: uniform acoustical velocity and turbulent losses during half an oscillation period (W_{eff}=0.03); (2) model 2: non-uniform acoustical velocity and turbulent losses during an entire period (W_{eff}=0.03).

The original model of Hirschberg [10] overestimates higher order hydrodynamic modes while the proposed model predicts only two hydrodynamic modes. The difference in the pulsation amplitude predicted for the two different configurations can be qualitatively explained by means of the Vortex Sound theory (Howe [8]). For the configuration 1 (Fig. 4), the initial power absorption is lower due to the rounded downstream outer edge. Furthermore, the sound absorption is also low at the rounded inner edges.

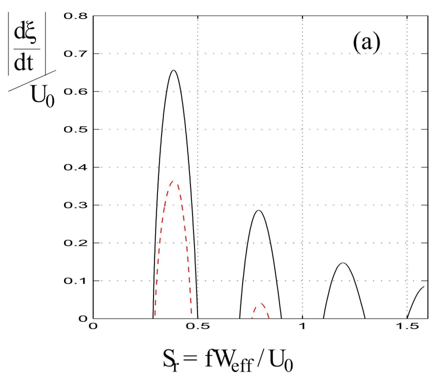

Figure 5(a): Predicted pulsation amplitudes $\left|\dfrac{d\xi}{dt}\right|/U_0$ in terms of the Strouhal number fW_{eff}/U_0 for model 1 ($--$) and model 2 ($-\ -\ -$).

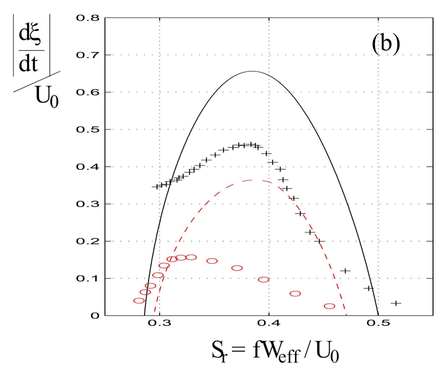

Figure 5(b): Predicted and measured pulsation amplitudes in terms of the Strouhal number for model 1 ($--$; +) and model 2 ($-\ -\ -$; o).

Compared to experimental results (Fig. 5b), the original model of Hirschberg [10] provides a prediction of the pulsation amplitude within a factor two for the first hydrodynamic mode. The simple model proposed for a sharp edged cavity predicts the pulsation amplitude within a factor two which is an improvement compared to the original model of Bruggeman [4] and Howe [8]. Moreover, the proposed model does not predict spurious higher order hydrodynamic modes. The high value of the observed Strouhal number compared to the measured Strouhal number is expected to be due to the deviation from a straight vortex path at high acoustical amplitudes. This hypothesis has been discussed by Kriesels [5].

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