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A ROLLING CONTACT MODEL USING GREEN'S FUNCTIONS

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ABSTRACT

Noise from vibrating bodies in rolling contact is an important field within acoustics. E.g. the tyre/road noise, which is the dominating noise source for road traffic noise under normal driving conditions. Previously, a simulation model has been developed to study the sound radiation from rolling tyres. However, the existing contact model has some drawbacks. In this work, a new formulation of the contact problem is given, which uses the local Green's functions of the vibrating body to calculate the contact forces. Additionally, the coupling between adjacent points at the surface of the body is taken into account. Generally, any Green's functions can be used in the model, provided that the local deformation is included. The model is verified for a single degree of freedom (SDOF) mass-spring system and two coupled SDOF mass-spring systems. An example for a tyre is also given.

1 - INTRODUCTION

Today traffic noise is a serious environmental problem [1] and the major noise source at speeds greater than 50 km/h is the car tyres. In order to reduce the radiated sound power from the vehicle/road interaction, a deep understanding of the noise generation mechanisms involved is of major importance.
The vibrations of the tyre structure is considered to be the most dominant noise source in the frequency range up to 1 kHz [2]. These vibrations are caused by the time-varying forces acting in the contact between the tyre and the road surface.
To be able to predict the noise generation, and in that way study the influence of the tyre design on the sound radiation, a tyre noise simulation model is useful, both for research and also for the tyre manufacturers.
When constructing such a rolling simulation model, one of the most important problems, but maybe also the most difficult one, is to calculate the contact forces in an accurate way. If the time record of the contact forces is known, the response of the tyre can be calculated in a straight forward way, provided that an adequate tyre model is available. However, the contact forces depends on the behaviour of the tyre and the road properties, and must in the general case be calculated for each tyre/road combination.

2 - EXISTING MODEL

A complete noise simulation package for tyre/road noise was developed by Kropp [3], concerning a smooth tyre rolling at constant speed over a rough road surface. Since the interaction between the tyre and the road surface is non-linear in character, the model is formulated in the time domain. The response of the tyre is calculated using a convolution technique between the Green's functions (impulse responses) of the tyre and the contact forces acting on the tyre. This is a very general approach, which can be applied to numerous engineering problems, e.g. tyre/road or wheel/rail vibrations, ball bearings, disc breaks and so on.
The Green's functions of the tyre is in the Kropp model calculated using an orthotropic plate under tension on an elastic foundation. The orthotropic plate model show good agreement with measurements in the frequency range up to 2500 Hz.
To calculate the contact forces, a Winkler bedding model is used, which is comprised of uncoupled linear springs in parallel. The use of an elastic bedding model follows from Johnsson [4]. A great deficiency

with this model is how to define the stiffness of the individual springs in the bedding. In order to avoid this problem a slightly different approach is suggested in the following

3 - A CONTACT MODEL USING GREEN'S FUNCTIONS

The idea behind this approach is to include the effect of the local stiffness of the structure directly in the Green's functions to be used in the contact model. In this way, some of the problems related to the bedding model are avoided.
Fig. 1 shows a vibrating structure in contact with a rough road surface

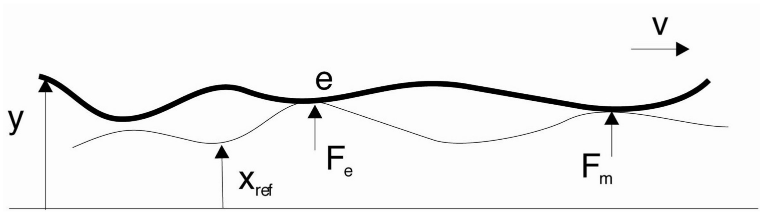

Figure 1: A structure, having the profile y, in contact with a rough surface with profile x_{ref}.

The structure is discretised in the space domain, which means that the response, as well as the contact forces, are only calculated at discrete points along the surface.
The displacement of the structure's contact points can be written as the convolution between the contact forces and the Green's functions according to equation 1.

$$y_e(t) = \sum_m F_m(t) * g_{m,e}(t) = \sum_m \int_0^t F_m(t)\, g_{m,e}(t-\tau)\, d\tau \tag{1}$$

where $F_m(t)$ is the contact forces and $g_{m,e}(t)$ is the Green's function from point m to point e.
The non-holonome boundary condition at the interface state that the contact point has to be above or on the reference surface. This condition is non-linear in nature and make it appropriate to solve the contact problem in the time domain. Additionally, the contact force can only be positive, i.e. no adhesive effects are considered in the model.
In the general case the displacement, y, and the force, F, are vectors, and the Green's function, g, is a matrix.
For discrete time steps, the convolution in equation 1 can be written according to equation 2.

$$y_e(N\Delta t) = \sum_m F_m(N\Delta t)\, g_{m,e}(0)\, \Delta t + \sum_m \sum_{n=0}^{N-1} F_m(n\Delta t)\, g_{m,e}((N-n)\Delta t)\, \Delta t \tag{2}$$

The first term in equation 2 corresponds to the current force at time $N\Delta t$ acting on the system, while the second term (the double summation) corresponds to forces acting in the past, which can be considered as a known constant.
To find the solution, an iterative method is used, which is briefly described in the following:

- The displacement at time step $N\Delta t$ is first calculated assuming no contact, i.e. the current contact forces are zero. The response depends only on old forces, according to equation 3.
- If the surface of the structure is below or on the reference surface, a contact force will act in opposition to the displacement. At the points in contact equation 4 give the necessary condition to be fulfilled.
- The contact forces, F_m, needed to create this displacement can be calculated by solving the linear system of equations given in equation 5. Note that equation 5 holds for each point in contact.
- The procedure is repeated for the next time step.

$$y_e^0(N\Delta t) = \sum_m \sum_{n=0}^{N-1} F_m(n\Delta t)\, g_{m,e}((N-n)\Delta t)\, \Delta t \tag{3}$$

$$\Delta y_e(N\Delta t) = x_{e,ref}(N\Delta t) - y_e^0(N\Delta t) \tag{4}$$

$$\Delta y_e (N\Delta t) = \sum_m F_m (N\Delta t)\, g_{m,e}(0)\, \Delta t \tag{5}$$

The main difficulty in the procedure is to find which points on the structure that are in contact with the reference surface. When the correct contact points are found, the problem is simply to solve a system of linear equations, which is straight forward. If the number of contact points are incorrect, some of the contact forces may become negative, or the boundary condition may not be fulfilled. At each time step in the procedure the solution must be checked for the correct contact points.
In this approach, in opposite to the Winkler bedding model, the coupling between points on the surface of the structure is taken into account. The matrix $g(0)$ may be seen as a general bedding, where the shearing between surface points is taken into account by the elements in the matrix. In the case when $g(0)$ is a diagonal matrix, it would represent the Winkler bedding model used in the existing tyre noise simulation model.

4 - VALIDATION OF THE MODEL

To illustrate and to validate the approach, some simple examples are given. First a single degree of freedom (SDOF) mass-spring system is used and afterwards, the approach is applied to two coupled SDOF systems. The two models are shown in figure 2. Linear contact springs are used to include the effect of the local deformation in the Green's functions of the systems.

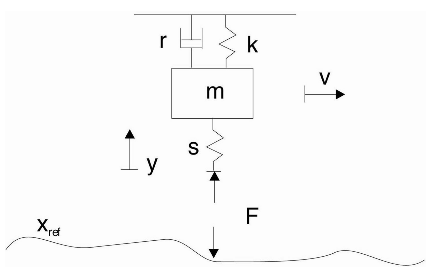

Figure 2(a): SDOF mass-spring system on a rough surface.

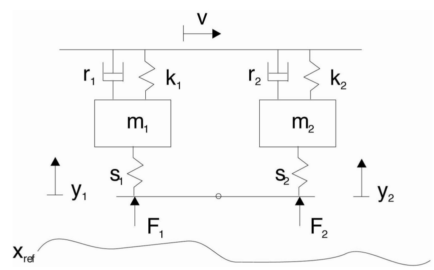

Figure 2(b): Two coupled SDOF mass-spring systems.

The two systems were tested for two different kind of roughness excitation. In the first case the roughness is a sine function, and in the second case a normal distributed random roughness profile is used for the excitation.

4.1 - Single degree of freedom system

The mass-spring system is moving at a constant velocity, v, over the surface.
The Green's function of the mass-spring system including the contact spring, can be calculated according to equation 6.

$$g(t) = \frac{1}{s}\delta(t) + \frac{e^{-\frac{r}{2m}t}}{2m}\sin\left(\sqrt{\left(\frac{k}{m}\right)^2 - \left(\frac{r}{2m}\right)^2}\, t\right) H(t) \tag{6}$$

where $\delta(t)$ is the delta function and $H(t)$ is the Heaviside step function.
As expected, the stiffness of the contact spring determines the first value of the Green's function.
Table 1 shows the parameters used for the model in the simulation.

m [kg]	r [Ns/m]	k [N]	s [N]	v [m/s]
1	50	500	1×10^5	1

Table 1: Parameters used in the simulation.

In figure 3 the results for the displacement for the two different roughness cases are shown.
The displacement of the contact point is always above or on the reference surface, which is consistent with the theory.

4.2 - Two coupled mass-spring systems

To be able to study the influence of the coupling between two contact points, a model consisting of two SDOF mass-spring systems is used. The coupling is in this case a lever without mass. Depending on how the lever is mounted, different kind of couplings may be obtained.

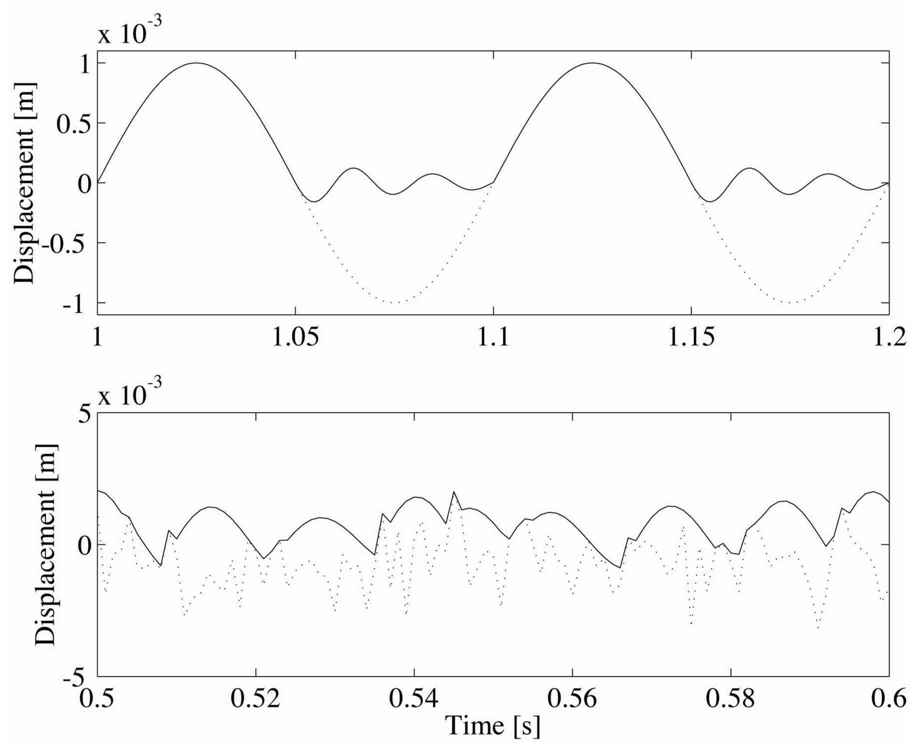

Figure 3: Displacement response of SDOF mass-spring system on two different surfaces; − contact point ×× reference surface.

The parameters for the two mass-spring systems are chosen to be identical and the same as for the single mass-spring system studied in section 4.1. Table 1 shows the parameters used in the simulation. The coupling factor was in this case c=0.4.
The Green's function matrix for the systems is given in equation 7.

$$g(t) = \begin{bmatrix} g_{1,1}(t) & g_{1,2}(t) \\ g_{2,1}(t) & g_{2,2}(t) \end{bmatrix} \tag{7}$$

where

$$g_{1,1}(t) = g_{2,2}(t) = \frac{1}{s}\delta(t) + \frac{e^{-\frac{r}{2m}t}}{2m}\sin\left(\sqrt{\left(\frac{k}{m}\right)^2 - \left(\frac{r}{2m}\right)^2}t\right)H(t)$$
$$g_{1,2}(t) = g_{2,1}(t) = cg_{1,1}(t)$$

Figure 4 show the results from the simulations.
It can be seen that the coupling between the points in the contact is significant and must be taken into account. The time difference between the two contact points is 0.01 s at this speed.

5 - IMPLEMENTATION OF A TYRE MODEL

The purpose with the presented contact model is to implement it in a tyre noise simulation model. To show the applicability of the method for this problem, an example is given where the contact forces and the corresponding radial displacements is calculated using this approach.
A two-layer tyre model based on the general field equations is used to calculate the Green's function of the tyre [5]. Since the assumptions behind this model are quite general, all wavetypes are included and the reaction to concentrated impulsive loadings can be calculated. This means also that the local deformation is included in the model.
The tyre is modelled as a plate, which is a good approximation above the ring frequency of the tyre, which approximately is 400 Hz. The length of the plate corresponds to the circumference of the tyre. The tyre is discretised into 512 points along the circumferential direction.
Figure 5a shows the Green's functions of the two-layer plate model when excited by a pulse in the middle of the plate.

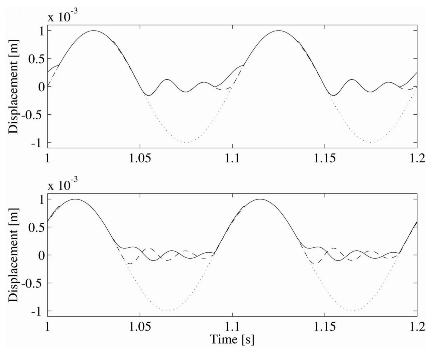

Figure 4(a): Displacements of the contact points on a sinusoidal shaped surface; – coupled springs, – uncoupled springs, ×× reference surface.

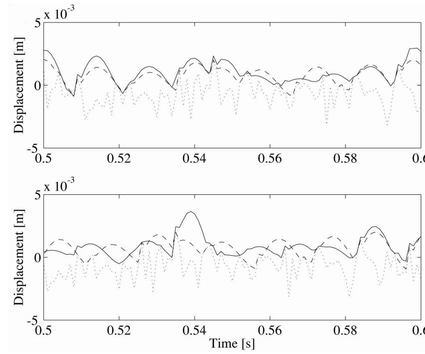

Figure 4(b): Displacement of the contact points on a random shaped surface; top figures: point 1; bottom figures point 2.

Figure 5b shows the calculated response of the tyre for one revolution. Since the tyre is circular in reality, the edges of the plate model will actually represent the same points. This effect can be seen in the response in the beginning and in the end when the contact area reach the edges.

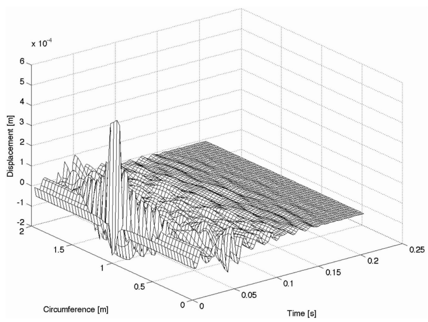

Figure 5(a): The Green's functions along the tyre circumference.

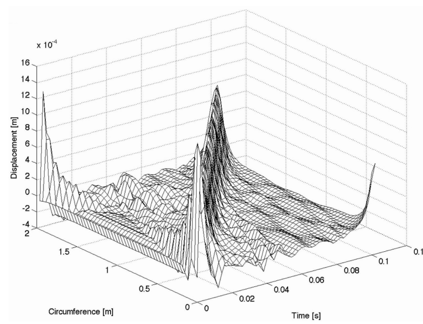

Figure 5(b): Displacement response of the tyre for one revolution.

6 - CONCLUSIONS

A slightly different approach of calculating the interface forces in a rolling tyre simulation model has been presented. The model shows results consistent with theory when applied to a SDOF mass-spring system.

With this approach it is possible to include the coupling between adjacent points on a structure's surface, which e.g. can represent a shearing effect.

It is possible to use the contact model to study the response of a tyre rolling on a rough road surface if the local deformation is included in the Green's functions. Here the coupling between the contact points is important to include.

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