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LOWER ATTAINABLE BOUNDS FOR THE ERROR OF RECONSTRUCTING THE VIBRATION FIELD OF A STRUCTURE FROM LIMITED DATA

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ABSTRACT
Considered is the problem of expanding the vibration field measured on one part of an elastic structure to its rest, unmeasured, parts. The problem is important in the cases when distribution and amplitudes of the dynamic stresses and displacements over the entire structure are needed but cannot be measured technically. The main results of the paper are concerned with the principal errors of reconstructing the vibration field and, in particular, with the lower attainable bound for the mean reconstruction error. The results are illustrated by a computer simulation example with a flexurally vibrating rectangular supported thin elastic plate.

1 - INTRODUCTION
Knowledge of the distribution functions and magnitudes of the dynamic displacements and stresses over an engineering structure due to extensive vibration is important for many purposes, e.g., for computing the structural intensity vector field, for choosing the best means of vibration control, for estimating the structure reliability and remaining service time, etc. In practice, however, it is impossible to measure vibration at each point of the structure and available may be only the data measured on some accessible parts of the structure surface. The problem therefore arises which consists in reconstructing the vibration field of the entire structure (including its inaccessible and unmeasured parts) using these limited data.

One such reconstruction problem has been rigorously formulated and studied theoretically and experimentally in references [1,2]. In this formulation, a structure (or an elastic body) with known material and geometric parameters performs harmonic vibrations under the action of unknown forces applied to inaccessible parts. The amplitudes of three vibration displacement components are prescribed on accessible part of the surface, which is assumed to be free of the external forcing, and constitute the input data of the problem. It is required to find, using these data, the vibration everywhere in the structure and, if necessary, the unknown external forces.

It is proved in reference [1] that when the data are known mathematically exactly, i.e., do not contain any noise, the reconstruction problem has an exact and unique, though unstable, solution. But when the measured data are contaminated with a background noise, the problem may have no exact solution and only an approximate solution can be found in this case of practice. As was shown in the previous papers [1,2], the reconstruction error of such approximate solution demonstrates sometime rather strange and unexpected behavior (e.g., it may increase when the accuracy of computations is improved). The objective of the present contribution is to study in detail the sources and peculiarities of the error of reconstruction for a such formulated problem. Two main results of the study are presented below: (1) existence of the optimal vibration field model which minimizes the reconstruction error, and (2) derivation of the lower attainable bound for the error of reconstruction − this new result is obtained on the basis of the Cramer-Rao theorem [3].

2 - PROBLEM FORMULATION, FIELD MODEL
For definiteness, consider a finite thin elastic plate occupying a region $S = S_{ac} \oplus S_{in}$ and performing forced flexural vibrations. Part S_{ac} of the region is accessible for measurement, part S_{in} is inaccessible

for measurement. The plate is subject to unknown harmonic forcing at the boundary of the inaccessible region S_{in} and vibrates flexurally according to the classical equation of Germain-Lagrange. Given is the distribution function $w_{data}(x)$ of displacement at the accessible part of the plate, $x \in S_{ac}$. One needs, using only $w_{data}(x)$, to reconstruct the displacement distribution function $w_{rec}(y)$ in the inaccessible region, $y \in S_{in}$. The input data function $w_{data}(x)$ is assumed to consist of the true displacement function $w_0(x)$ and random noise:

$$w_{data}(x) = w_0(x) + n(x) \qquad (1)$$

The solution to this problem is obtained via the following steps [1]. First, the vibration displacement field of the entire structure is modeled as a truncated series in the plate normal modes:

$$w(z) = \sum_{n=1}^{N} d_n w_n(z) \qquad (2)$$

where modes $w_n(z)$ are known, but their amplitudes d_n are unknown, $z \in S$. Equating the expressions (1) and (2) in the accessible region S_{ac} yields a functional equation and, after discretization, a set of M linear algebraic equations with N unknowns, $M>N$. Solution of the set (SVD is used) gives the vibration field (2) everywhere on the plate and, hence, the sought field $w_{rec}(y)$. Details of the solution can be found in reference [1].

3 - ERROR AS A FUNCTION OF THE FIELD MODEL COMPLEXITY

Figure 1 presents the relative error of reconstruction averaged over S_{in} (solid line) as well as the mean relative error of describing one particular sample of the data (1) by the model function (2) − dashed line, for a rectangular simply supported plate of which 0.6 area is accessible for measurement, the signal-to noise-ratio being equal to SNR=16.5dB.

It is seen that the error in the data diminishes with the number N of modes involved in the model (2): the finer the model, i.e., the greater N, the better approximation of the input data. The error in data tends to zero when the number N tends to infinity.

Quite differently behaves the reconstruction error (solid line): for small N (coarse models) it decreases and for large N (fine models) it increases having a distinct minimum at N_0=18.

It means that there exists the optimal model (2) of the vibration field containing a finite number of the parameters which yields the minimum reconstruction error. The optimal number N_0 of the model parameters depends mostly on SNR: the higher SNR the larger N_0.

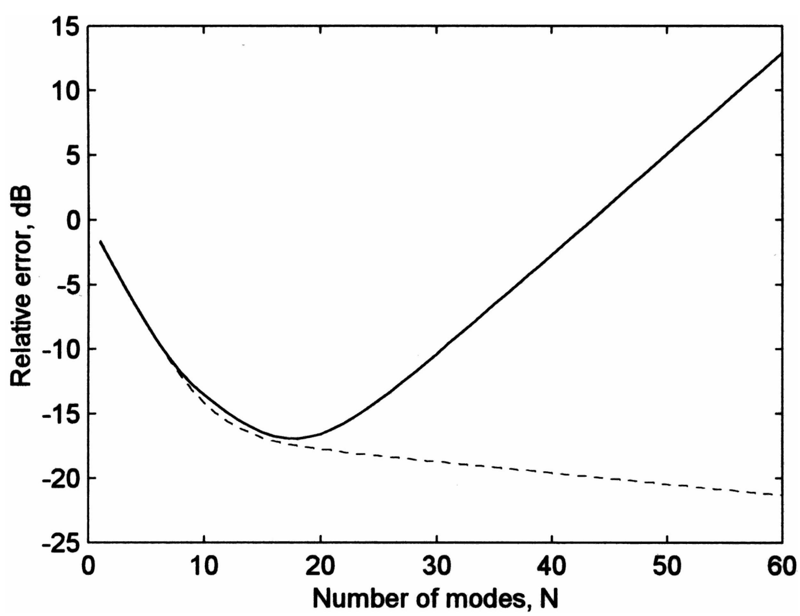

Figure 1: Reconstruction error (solid line) and error in the input data (dashed line) versus model complexity.

The fact that the finer models ($N>N_0$) may give larger reconstruction error seems at variance with common sense ("More exact models must always give better description of reality"). Its physical explanation is the following. Finer models containing higher spatial harmonics ($N>N_0$) describe mostly the noise components instead of the signal. Therefore they give better description of the input data but have nothing to do with the actual vibration signal when expanded to unmeasured parts of the structure.
The existence of the optimal model which gives minimum reconstruction error is a general result valid probably for all linear structures and models of the type (1), (2). In practice, where available are only input data, finding the optimal model is a problem. One of approaches to solve it is proposed and validated experimentally in reference [2].

4 - LOWER ATTAINABLE BOUND ON THE RECONSTRUCTION ERROR

A very useful parameter of the reconstruction problem is the lower attainable bound for the variance of the reconstructed field. It gives the bound on the mean reconstruction error which may be achieved only under very special conditions and against which different reconstruction algorithms can be compared and evaluated. The derivation of the bound is based on calculation of the so called Fisher information matrix and applying the Cramer-Rao theorem concerning estimation of non random parameters from random noise [3]. The bound strongly depends on the model of random noise in the input data (1). We assume here that the noise is an instrumentation noise, i.e., a spatially delta-correlated complex Gaussian random field in region S_{ac} with standard deviation independent of the space coordinates. Omitting the details, Figure 2 depicts the lower bound on the relative mean reconstruction error averaged over the reconstruction region as a function of the dimension of the measurement region, S_{ac}/S, for the flexurally vibrating rectangular simply supported plate.

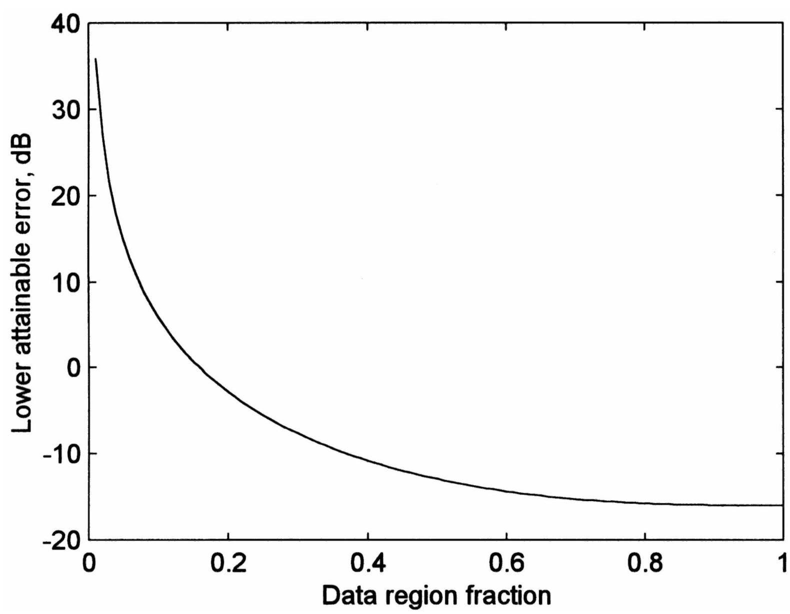

Figure 2: Lower attainable bound for the mean reconstruction error vs dimension of the measurement region.

When the data are collected from the full structure surface the lower bound is equal to signal-to-noise-ratio with opposite sign (-16.5 dB). When dimension of the measurement region diminishes the bound monotonically increases, so that when vibrations are measured only on a small part of the structure surface, $S_{ac}<0.2S$, the reconstruction error may be very large. If, for example, the data are measured on 15% of the structure, the mean reconstruction error can not be smaller than 100%.

REFERENCES

1. **Yu.I.Bobrovnitskii**, The problem of the field reconstruction in structural intensimetry: statement, properties, and numerical aspects, *Acoustical Physics*, Vol. 40(3), pp. 331-339, 1994

2. **Yu.I.Bobrovnitskii**, The problem of expanding the vibration field from the measurement surface to the body of an elastic structure, In *6th International Conference on Recent Advances in Structural Dynamics, Southampton, UK*, pp. 1719-1731, 1997

3. **C.R.Rao**, *Linear Statistical Inferences and Applications*, New York: John Wiley & Sons, pp. 548, 1965