A SEMI-ANALYTICAL APPROACH TO DESIGN A LOW-FREQUENCY ABSORBER

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ABSTRACT
This paper describes a straightforward technique to design a resonant low-frequency absorber equipped with a transparent facing. The technique is based on in-duct measurements of the acoustical 2-port matrix of the facing using the 2-microphone method. The 2-port of the facing is then combined with a simple analytical model of the partitioned backing volume to enable simulation and optimisation of the wall impedance of the complete absorber. As a practical example the technique has been applied to a micro-slotted aluminium plate well suited for use in a dusty and harsh environment such as for instance the engine compartment of a vehicle. Measurements of the plane wave absorption coefficient have also been performed in order to evaluate the proposed technique.

1 - INTRODUCTION
A well established technique to improve the sound quality in rooms and enclosures is to use porous liners. Typically these absorbers have quite poor low-frequency absorption which may cause problems especially in vehicle applications where strong low-frequency tonal noise is present. The most common way to improve the low-frequency characteristics of an absorber is probably to cover the porous lining with a panel to form a resonant mass spring system. Another related technique is to instead of a structural mass use an acoustically more transparent facing in which the needed mass is given not only by the weight of the facing but also by the contained fluid as well as the acoustic near field. The recent and intense development of such facing materials ranging from sintered aluminium to various kinds of textiles certainly increases the potential of this latter technique. However, not only due to the complexity of these materials but also to the large number of structurally different facings, theoretical models are lacking. In a practical design situation where the engineer is faced with choosing the proper material, this may obviously result in tedious testing. To somewhat reduce these efforts the suggestion here is to, in order to circumvent the lack of models, combine measured data for the facing with a simple model for the backing cavity. These data may be the absorption coefficient as measured in a standing wave tube or, as chosen here, the transfer matrix for the facing obtained using the 2-microphone method. Although the absorption coefficient is perhaps more easily obtained from the manufacturer of the facing material, the transfer matrix is preferable as it is more easily implemented in the complete model for the absorber as will be shown below.

2 - MODELLING OF THE ABSORBER
In the following section a semi-analytical model will be derived for a resonant low-frequency absorber as shown below in Figure 1.

In most applications details of the sound field interacting with the absorber are unknown wherefore the design work must be based on estimations. Usually the frequencies corresponding to the highest sound pressure levels are known and as in most cases also the location of the absorber is fixed, typically for non-acoustical reasons, the design boils down to tuning of the absorber itself. With the target frequencies in the majority of applications being below 500 Hz, the Helmholtz number based upon the dimensions of the absorber is small enough to justify a 1-dimensional analysis as indicated in Figure 1. With the conventional acoustic assumptions the facing may accordingly be represented in the frequency domain.
by a 2’2 transfer matrix, $T_{\text{rel}}$ relating the plane wave pressure and volume velocity at the inlet side to the corresponding quantities at the outlet side of the facing,

$$
\begin{pmatrix}
\hat{p}_i \\
\hat{q}_i
\end{pmatrix} =
\begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
\begin{pmatrix}
\hat{p}_o \\
\hat{q}_o
\end{pmatrix}
$$

($1$)

$t_{nn}$ being the complex matrix elements. As indicated earlier this transfer matrix may be hard to determine analytically wherefore a more pragmatic approach is to determine it by measurements using the 2-microphone method. This technique, which is often used in flow duct acoustics, is well documented in literature, see for instance [1] and will only be briefed here. Thus, in short, a sample of the facing is cut out to fit precisely into a pipe of, most suitably, circular cross-section and area $S_p$. A loudspeaker and two microphones are mounted in the duct on each side of the sample. The transfer functions between the different microphones are then measured twice, using first one and then the other of the two loudspeakers, i.e. using the technique often referred to as the two-source method.

In order to obtain a sufficiently low eigenfrequency of the absorber considering the, in most cases, quite strongly restricted thickness, $h$ of the device, the resonator has to be rather of Helmholtz than quarter wave type. Thus the area of the inlet orifice, $S_h$ has to be smaller than the cross-sectional area, $S_c$ of the backing volume. As a consequence of this, the acoustics of the facing mounted on the absorber will be influenced by a near field that are not present in the 2-microphone set-up. To account for this, the Karal end-correction [2] is written in transfer matrix form, further below denoted $M$,

$$
\begin{pmatrix}
\hat{p}_i \\
\hat{q}_i
\end{pmatrix} =
\begin{bmatrix}
1 & ipck\Delta/S_h \\
0 & 1
\end{bmatrix}
\begin{pmatrix}
\hat{p}_o \\
\hat{q}_o
\end{pmatrix}
$$

($2$)

where

$$
\Delta \approx 0.48\sqrt{S_h}
$$

($3$)

for the current circular geometry. $k$ denotes the wavenumber and $\rho c$ the characteristic impedance. For the end-correction inwards the backing cavity, the area $S$ equals $S_c$ whereas it for the outer end correction tends towards infinity thus representing a baffled orifice. The complete transfer matrix for the porous facing may, first correcting the measured matrix to account for the difference in area between the measurement cross-section and inlet orifice,

$$
T' =
\begin{bmatrix}
t_{11} & S_p/S_h t_{12} \\
S_h/S_p t_{21} & t_{22}
\end{bmatrix}
$$

thus be written,

$$
C = M_o T' M_i
$$

($4$)

relating the incident field, $\hat{p}_{in}$, $\hat{q}_{in}$ to the field in the backing cavity, $\hat{p}_c$, $\hat{q}_c$. As these two latter quantities are related by the impedance of the cavity,

$$
Z_c = -i\frac{\rho c}{S_c} \cot (kh)
$$

($5$)

the impedance of the low-frequency absorber, $Z_{\text{abs}} = \hat{p}_{in}/\hat{q}_{in}$ may now readily be formulated.

$$
Z_{\text{abs}} = \frac{c_{11}Z_c + c_{12}}{c_{21}Z_c + c_{22}}
$$

($6$)

and from this the ratio between incident and absorbed power, i.e. the absorption coefficient,
\[ \alpha = 1 - \frac{\left| Z_{abs} - \rho c / S \right|}{\left| Z_{abs} + \rho c / S \right|} \tag{7} \]

3 - AN APPLICATION

The technique suggested above has been applied to a resonant absorber using a micro-slotted and slightly corrugated aluminium plate as facing. The plate, shown in Figure 2 below, is stamped from a 1mm thick plate resulting in a porosity of approximately 1%.

![Figure 2: Detailed structure of micro-slotted facing.](image)

The absorber to be tested has a circular backing cavity with a diameter and depth of 97 and 100 mm respectively. The dimensions are chosen to precisely fit the standard B&K 4206 impedance tube used for the evaluation. The facing is formed by the micro-slotted plate being covered with a 1 mm thick flat and solid aluminium plate in which a centric circular hole of diameter 50 mm has been turned. The resulting constriction of the acoustic flow yields the fluid mass, which together with the fluid spring of the backing cavity constitutes the desired resonant system. A well-known problem in the practical design of resonators is leakage, i.e. the backing cavity is not airtight and accordingly the losses are increased whereas the stiffness of the acoustic spring as well as the efficiency of the device is strongly reduced. To minimise this problem, the cavity has been turned from a piece of nylon into a can upon which the facing has been glued.

![Figure 3: Comparison between a) simulated and b) measured absorption factor.](image)

As seen from the figure the agreement between measurements and the semi-analytical model is reasonable. Maximum absorption is obtained around 400 Hz, i.e. somewhat higher than was predicted but considerably lower than the peak at 670 Hz that would have been expected from a conventional lumped model of the absorber fully neglecting the slotted plate. The most probable cause for the discrepancies is the fact that the transfer matrix for the facing was determined using a much smaller cross-section than the one encountered by the sound wave in the standing wave tube. A fact, which in combination with
the solid orifice plate, may alter the structural behaviour of the facing as captured in the transfer matrix. Another possible explanation for the discrepancies is non-linearities associated with high acoustic velocities through the slots of the facing.

4 - CONCLUSIONS
An engineering method for the design of a resonant low-frequency absorber has been presented. The technique, being based upon the 2-microphone method, offers the possibility to include ever so complex materials for the porous facing already in the initial stages of design. Consequently, the ”trial and error” part of the development efforts is greatly reduced. As a practical example the technique has successfully been applied to Helmholtz absorber using a micro-slotted aluminium facing giving reasonable agreement between modelling and practice and clearly illustrating the potential of the proposed method.

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REFERENCES
