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TREATMENT OF FREQUENCY DEPENDENT COMPLEX STIFFNESS OF RUBBER BUSHINGS IN TRANSMISSION FORCE ANALYSIS OF A VEHICLE SUSPENSION SYSTEM BY COMMERCIAL MULTI-BODY DYNAMICS PROGRAMS

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ABSTRACT
In order to estimate the forces transmitted onto the cabin in a running vehicle, we need to have proper dynamic models for the vibration isolation components such as rubber bushings. Typically the conventional Voigt model is used in commercial programs. However, this model has limitations in reflection of the frequency dependent characteristics of the vibration isolation components. In this paper, a model called transfer function model here, which can reflect better the frequency dependent complex stiffness and match well with the commercial programs is suggested. Performance of the derived model in dynamic analysis of a vehicle suspension system is illustrated by comparing with the results obtained by taking the Voigt model.

1 - INTRODUCTION
Vibrations in a car induced by the road surface are transmitted to the cabin essentially through vibration isolation components such as coil springs, shock absorbers and rubber bushings. The ride comfort and low frequency noise can be effectively analyzed in terms of such transmission forces. In order to estimate the forces transmitted to the cabin at the design stage, we need to have proper dynamic models for the vibration isolation components. In this paper, a study on the dynamic modeling of rubber bushings is presented.

Commercial dynamic analysis programs such as ADAMS and DADS are equipped with bushing elements, which are so called the Voigt model. The Voigt model is simply a spring with a viscous damper in parallel. While, however, the transmission force by this model is enforced to increase with frequency for constant vibration amplitudes, experimental results on actual rubber bushings do not necessarily support such characteristics. A better representation of the viscoelastic characteristics of rubber bushings is the complex stiffness which is frequency-dependent in a complicated manner. However, the frequency dependent complex stiffness has not been faithfully taken care of in commercial programs in which analysis is done by numerical integration in time domain. In this paper, a method to develop dynamic models of rubber bushings, which can reflect better the frequency dependent complex stiffness and match well with the commercial programs is presented. Performance of the developed model in dynamic analysis of a vehicle suspension system is illustrated by comparing with the results obtained by taking the conventional Voigt model.

2 - COMPLEX STIFFNESS OF RUBBER BUSHINGS
The complex stiffness of a rubber bushing can be obtained by exciting with sinusoidal displacement inputs and measuring the resulting sinusoidal forces. That is, the complex stiffness is defined and measured

in frequency domain. It can be also predicted, e.g. using the finite element method, from material properties, such as complex Young's modulus and density, and geometric shape of the rubber bushing. The latter seems to be more applicable at the initial design stage. Figure 1 shows the complex stiffness of a rubber bushing installed between the trailing arm and the car body in a rear suspension system. While the real part of the complex stiffness of the Voigt model must stay constant, that of the actual rubber bushing increases with frequency. In addition, while the imaginary part of the Voigt model must be proportional to frequency, that is, should be a straight line passing through the origin, that of the actual rubber bushing increases with frequency with an offset at ω=0.

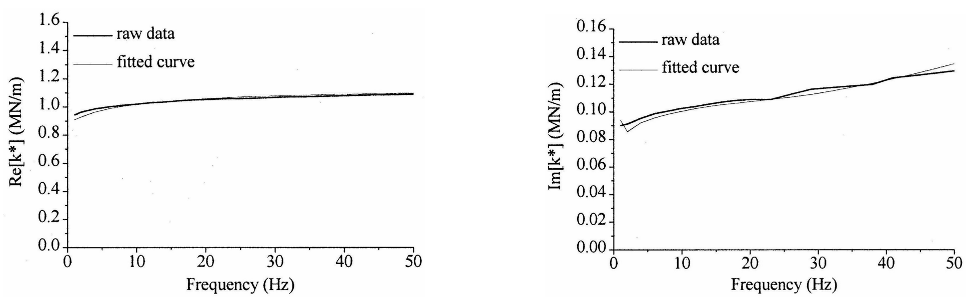
3 - DYNAMIC MODEL OF RUBBER BUSHINGS

As stated above, the Voigt model is quite often used for modeling of rubber bushings. When the number of frequency at which the transmission force peaks is just one and the frequency is known in advance, the Voigt model is acceptable [1]. That is, if coefficient of the viscous damping is selected so that its loss factor may be the same as the actual loss factor at the frequency of concern, the behavior of the model will be sufficiently close to that of the actual bushing. If, however, the number of such frequency is more than one, it is difficult to obtain the satisfactory behavior at all of these frequencies by the simple Voigt model. In addition, these peak frequencies can not be known in advance especially for a complex system such as vehicle.

The commercial programs offer so called system elements expressed by a transfer function in the Laplace domain as follows:

$$G(s) = \frac{F(s)}{X(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}{s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0} \tag{1}$$

where $n \leq m$. This form of the transfer function can be directly used for rubber bushings. Under the assumption that $G(s)$ is an analytic function on the right half plane containing the $j\omega$ axis, the complex stiffness can be obtained from $G(s)$ with $s = j\omega$ [2]. In order for the transfer function model to be used for rubber bushings, coefficients of $G(s)$ can be selected by curve fitting of the estimated or measured complex stiffness. Figure 1 shows raw data of the complex stiffness of the rubber bushing stated above together with the fitted curves. In this case study, the highest order of both numerator and denominator of $G(s)$ were 4 for the satisfactory fitting.



(a): Real part. **(b):** Imaginary part.

Figure 1: Complex stiffness of rubber bushing and fitting results.

4 - DYNAMIC ANALYSIS OF A VEHICLE SUSPENSION SYSTEM

To illustrate the performance of the proposed transfer function model in transmission force analysis, a vehicle suspension system shown in figure 2 was taken for analysis, where the ball represents the center of the sprung mass.

The forces transmitted onto the sprung mass through the rubber bushings installed on the trailing arms in the rear suspension system were calculated when the vehicle model was driven on a road profile as shown in figure 3 at a speed of 72km/h. The transfer function model was employed just in the trailing arm bushings because it was considered that excitation by the selected road profile would be influential mostly on the trailing arms which are installed to insulate vibrations in the fore-and-aft direction.

Figure 4 shows the transmission forces calculated with the proposed transfer function model and the Voigt models tuned at three frequencies. While the magnitude of the larger peak at the higher frequency was much dependent on the employed model, that of the smaller peak at the lower frequency was not. The latter phenomena can be explained by the fact that the behavior near the first peak is related to the global

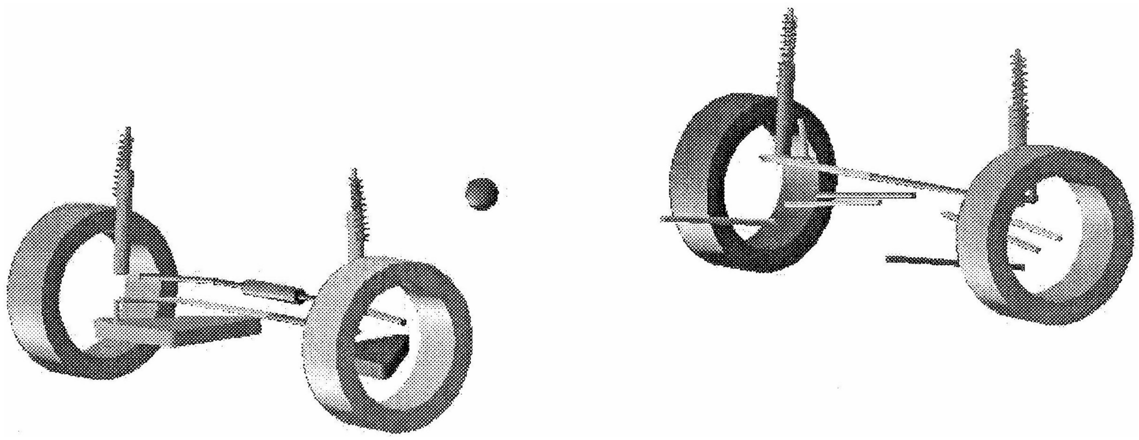


Figure 2: Vehicle model.

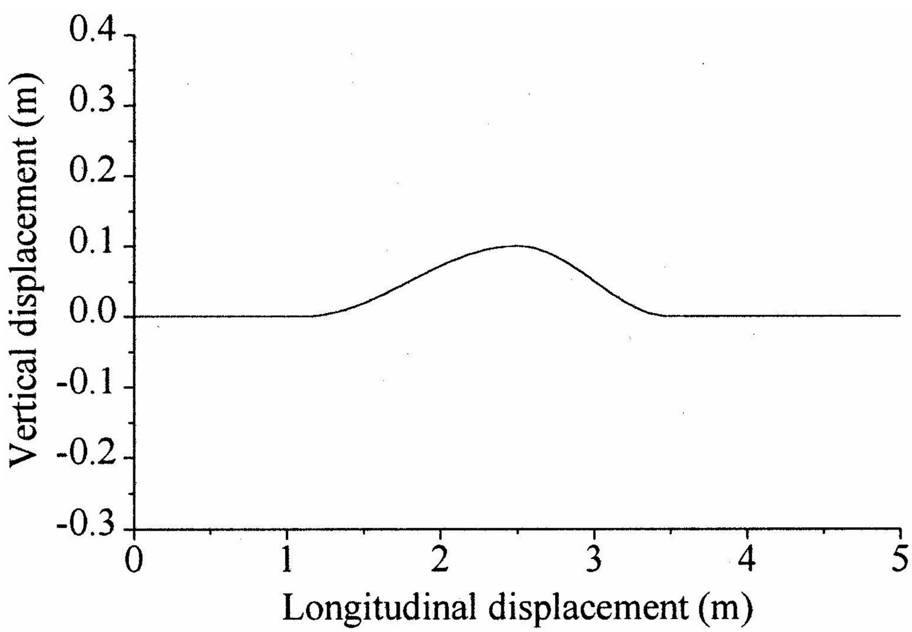


Figure 3: Road profile.

dynamics of the vehicle which is predominated by the coil springs and shock absorbers rather than rubber bushings. In this example, the number of the frequency at which transmission force is critical is just one for the chosen excitation so that the Voigt model as well can be effectively used in the transmission force analysis. However, the number of such frequency is not known in general in advance. Even if the number is known *a priori*, location of the peak frequency cannot be estimated in a straightforward manner. Figure 4 shows that the Voigt model significantly mal-estimates the transmission force at the second peak frequency when the model is tuned at inadequate frequencies.

5 - CONCLUSIONS

In this paper, limitations of the conventional Voigt model when used for rubber bushings were pointed out and use of transfer function model of the polynomial fractional form was proposed as a solution. While the Voigt model can be useful at only a pre-selected single frequency, the proposed model can reflect well the frequency dependent complex stiffness of the rubber bushings over the frequency range of interest. Thus the suggested technique can be used successfully when the frequency points where the transmission forces are critical are not known in advance.

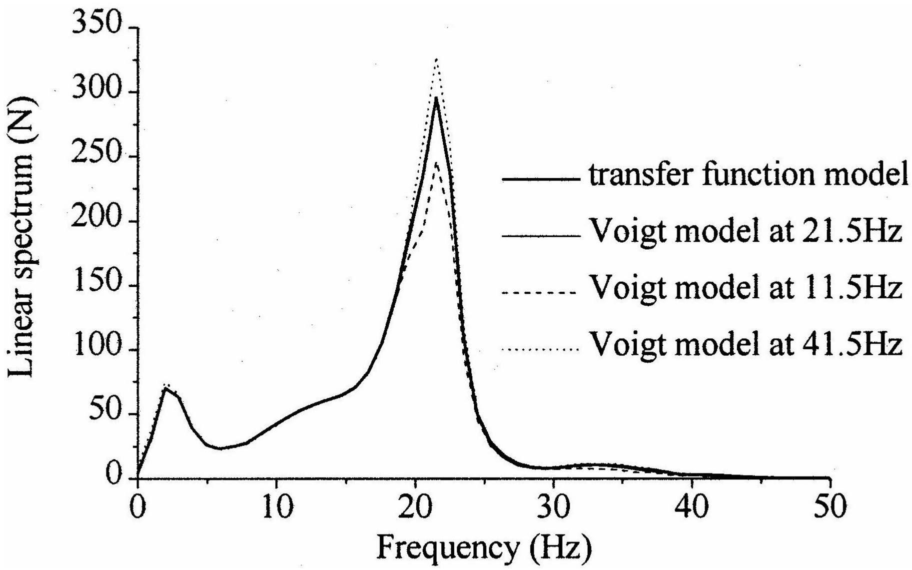


Figure 4: Forces transmitted to cabin through rubber bushings installed on trailing arms in rear suspension system.

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