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CALCULATION OF THE SOUND INTENSITY IN A STRATIFIED ATMOSPHERE

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ABSTRACT
With a new equation the sound intensity is calculated in a horizontal stratified atmosphere. The sound rays are represented with an equation including the starting angle α_0 and the angle α of refraction according Snell's law and a parameter which includes the ratio of the sound speed at a certain height to the sound speed gradient. A new equation to calculate the sound intensity is based on vector calculus of the parameter representation of the sound rays is derived. If the sound speed is linear dependent on the height then the sound rays are circles and the sound intensity can represented with an simple equation dependent on the location and a parameter including the height dependence of the sound speed. With this equation it is possible to estimate the sound level in a simple way for a horizontal stratified atmosphere with a linear height dependence of the sound speed.

1 - INTRODUCTION

Since few years the sound level distribution is calculated most times with numerical methods including the ground effects and the real height dependency of the temperature and the wind profile. Nevertheless seems that it is usefull to know a simple analytical equation for the rapid estimation of the change of the sound level as a result of the height dependent changing sound speed profile.

The calculation of the sound intensity with the arc length between two sound rays is widly used in acoustics for the two dimensional case and the area elements are calculated between the sound rays in the three dimensional case for point sources. The intensity dependent on the starting angle and the angle of refraction according Snell's law is calculated for a one dimensional refractive medium in Ref. [1]. For a two dimensional refractive medium the spreading loss is calculated with a set differential equations [2], and the WKB approximation is used to solve the wave equation [3,4], which are applied to underwater acoustics. If the ground effects are included in outdoor sound propagation numerical calculations have to used for the calculation of the sound level distribution, eg. Ref. [5] and [6]. A german paper [7] presents a combination of analytical and numerical calculation to calculate the excess attenuation of the meteorological influences with respect to the ground impedance. The vector calculus to derive the acoustic intensity in a moving medium is used in two papers of Thompson [8,9] based on the ray equations of an older paper [10]. He calculated the transformation of the area element through a wind and temperature gradient and then the ray equations are used to calculate the intensity.

In this paper equations for the calculation of the sound intensity are derived in chapter 2 with help of vector calculus started from the parameter representation of the sound rays. In chapter 4 the sound level is calculated for the sound field as a result of a line source and the sound intensity of a point source is derived. Both is calculated for a constant height dependent change of the sound speed and with this for the case that the sound rays are circles. To calculate the sound intensity it is necessary in chapter 3 to calculate the sound ray equations dependent on the starting angle α_0, the angle of refraction α and a parameter which includes the meteorology. At least simple equations of the sound intensity as a result of the sound speed gradient is presented for the sound field of a line source and a point source and the results are shown in a figure.

2 - CALCULATION OF THE SOUND INTENSITY

The sound intensity is calculated as a result of the change of the length of the line element (in the vertical direction) for the case of a line source in this chapter. The length of the line element is changed as a result of the increasing or decreasing sound speed in relation to the height above ground. In the case of a point source the change of the length of the line element in the vertical direction is used to calculate the change of the dimension of the area element. The sound speed is changed only in the vertical direction. The increase or decrease of the sound level as a result of the temperature gradient is related to the geometrical sound propagation.
For the calculation of the sound intensity the sound rays has to express dependent on the parameter α which represents the angle between the vertical plane and the tangent to the sound ray at a certain position and the starting angle α_0 of the sound ray concerning the vertical plane.
The sound intensity radiated in the line element $\boldsymbol{dn}$ can calculated as a result of the radiated sound power dP in the direction of the starting angle α_0 of the sound ray as dP/dn. The length —$\boldsymbol{dn}$— of the line element between the vector $\mathbf{d}_\alpha\mathbf{x}$ and $\mathbf{d}_{\alpha_0}\mathbf{x}$ according Fig. 1 can calculated as the result of the vector product

$$\frac{dP}{dn} = \frac{dP}{d\alpha_0} \cdot \frac{d\alpha_0}{dn} \tag{1}$$

$$|d\vec{n}| = \frac{|d_\alpha\vec{x} \times d_{\alpha_0}\vec{x}|}{|d_\alpha x|} \tag{2}$$

The vector $\mathbf{d}_\alpha\mathbf{x}$ and $\mathbf{d}_{\alpha_0}\mathbf{x}$ is calculated as a result of the partial derivative according

$$d_\alpha\vec{x} = \frac{\delta\vec{x}}{\delta\alpha}d\alpha \quad ; \quad d_{\alpha_0}\vec{x} = \frac{\delta\vec{x}}{\delta\alpha_0}d\alpha_0 \tag{3}$$

With Eq. (3) the Eq. (2) is transformed into

$$|d\vec{n}| = \frac{\left|\frac{\delta\vec{x}}{\delta\alpha} \times \frac{\delta\vec{x}}{\delta\alpha_0}\right| d\alpha d\alpha_0}{\delta\alpha} \tag{4}$$

$$\left|\frac{d\vec{n}}{d\alpha_0}\right| = \frac{\left|\left(\frac{\delta x}{\delta\alpha}, \frac{\delta z}{\delta\alpha}\right) \times \left(\frac{\delta x}{\delta\alpha_0}, \frac{\delta z}{\delta\alpha_0}\right)\right|}{\delta\alpha} = \frac{\left|\frac{\delta x}{\delta\alpha}\frac{\delta z}{\delta\alpha_0} - \frac{\delta x}{\delta\alpha_0}\frac{\delta z}{\delta\alpha}\right|}{\delta\alpha} \tag{5}$$

With Eq. (5) the derivative of the length of the line element $|\mathbf{dn}/\mathbf{d\alpha_0}|$ can calculated if it is possible to calculate the partial derivative of the sound ray vector ($\mathbf{x}$,$\mathbf{z}$) in the direction α_0 and α, that means if the differentiability of $\delta x/\delta\alpha$, $\delta x/\delta\alpha_0$, $\delta z/\delta\alpha$ and $\delta z/\delta\alpha_0$ exists.
With Eq. (5) the increase or decrease of the sound intensity can calculated with an simple equation. The model is applied later to situations where the sound speed is linear dependent on the height.

3 - ANALYTICAL CALCULATION OF THE SOUND RAY EQUATIONS

The sound ray equations are calculated with a parameter representation. In the present calculation of parameter presentation of the sound ray paths it is assumed that there is a vertical temperature stratification. Basically Snell‘s law of refraction is used to find an expression in which the coordinates x and z are dependent on the starting angle α_0 and the angle of refraction α that means $x = x(\alpha_0, \alpha)$ and $z = z(\alpha_0, \alpha)$, but in this paper geometrical considerations are used to derive the coordinates of the sound ray.
The sound speed $c(z)$=c_0+ Az depends on the sound speed c_0 at the reference height z_0 and the sound speed gradient A with the dimension 1/second. Both parameters are included in the reference length l=c_0/A defined as sound speed divided by sound speed gradient which has the dimension meter. It is assumed that the sound speed is linear dependent on the height and with this the sound rays are circles. The construction scheme is shown in Fig. 1 and it includes the main idea that the center of the circles moves on a horizontal line and the are dependent on the starting angle α_0 of the sound ray. The distance between the horizontal x-axis and the center of the circles depends on the reference length l=c_0/A. With simple geometrical considerations the coordinates of a certain point of the circle can derived as

$$x = \frac{l}{\sin\alpha_0}(\cos\alpha_0 - \cos\alpha) \quad ; \quad z - z_0 = \frac{l}{\sin\alpha_0}(\sin\alpha - \sin\alpha_0) \tag{6}$$

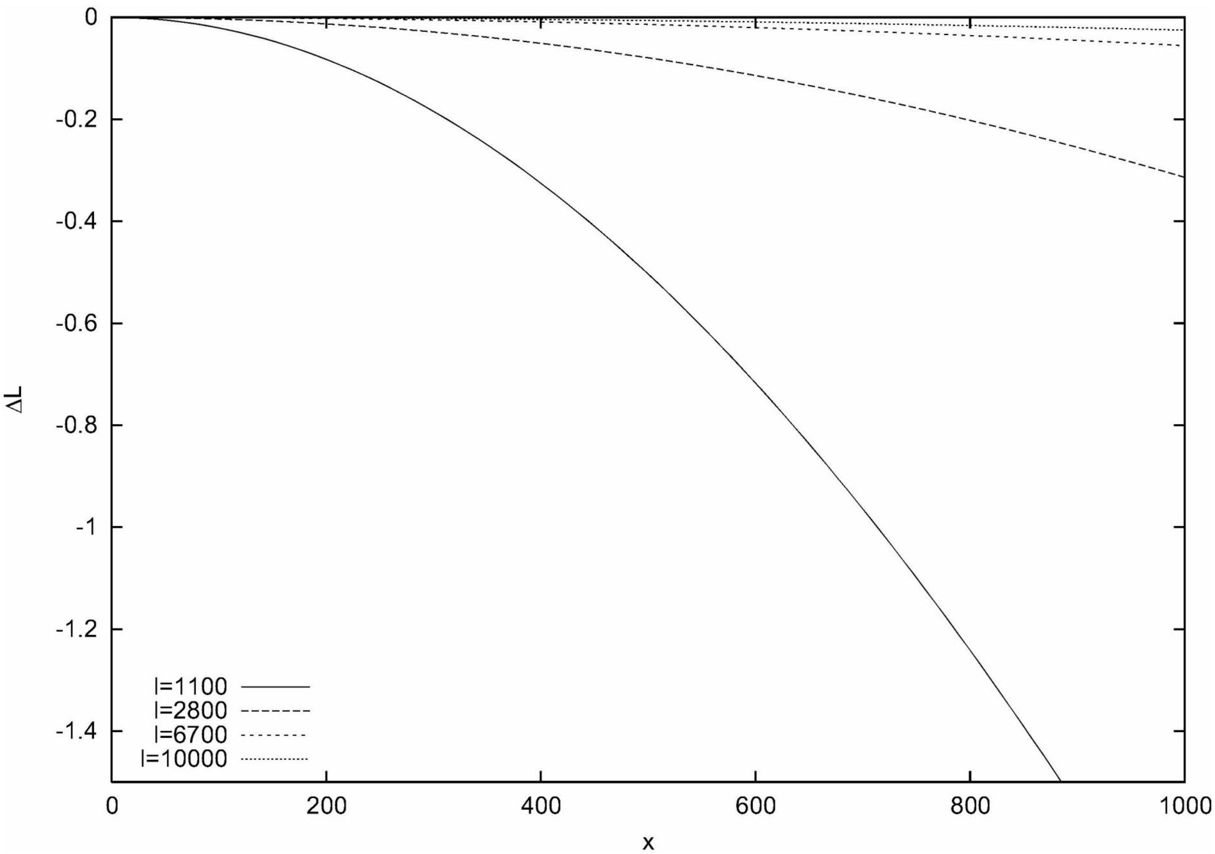

Figure 1: Construction of the sound rays and length of the line element *dn*.

The Eq. (6) can also derived from Snell's law of refraction but with a more extensive calculation. With Snell's law of refraction sound ray equations can calculated for the cases that the sound speed $c(z)$ is dependent on the height with $c(z)=(c_0+Az)^n$ where $n=1/2$, $-1/2$ and -1 that means the sound rays are cycloids, parabolas or catenoids. The conventional use of Snell's Law of refraction in application for a moving medium is discussed in [11]. It is possible to calculate the sound field respectively the intensity with a simple equation (dependent on the location x, y, z and the gradient of the sound speed) if the sound rays are circles.

4 - CALCULATION OF THE SOUND ON THE CASE OF A LINEAR HEIGHT DEPENDENT SOUND SPEED PROFILE

In the following the change of the sound field of a line source in respect to the undisturbed sound field and the increase or decrease of the sound level of a point source is presented. From Eq. (5) it is clear that a more extensive calculation is necessary to derive started from Eq. (6) the partial differentials and at least the sound field which is not possible to present it in this paper. The effective sound speed is used by Pierce [12] defined as the sum of the thermal sound speed and the velocity of the wind in the direction of propagation. In the following parts one has to look very carefully to the boundaries of the model and if it is possible to use the effective sound speed. In general the thermal sound speed as a result of the temperature can used in all cases of the following calculation.

4.1 - Sound field of a line source as a result of a height dependent change of the sound speed

The increase or decrease of the sound level as a result of the height dependent change of the sound speed $c(z)$ is calculated for the case of a linear height dependent sound speed $c(z)=c_0(1+z/l)$ profile. The change of the sound level is calculated as a result of the sound ray parameter including starting angle α_0 and the angle of refraction α which describes a certain position of the sound ray. At least the differential $d\alpha_0/dn$ which is used to calculate the sound level is represented as a result of the location (x,z) and the reference length $l=c_0/A$ which describes the height dependent change of the sound speed.
The change of the sound level ΔL_{line} of a line source as a result of the change of the sound speed (I_r intensity as a result of the sound refraction) as increase or decrease of the undisturbed sound intensity

I_{geom} as a result of the geometrical sound propagation is

$$\Delta L_{line} = 10\log\frac{I_r}{I_{geom}} = 10\log\left|\frac{2l}{\sqrt{4l\left(l+(z-z_0)\right)+x^2+(z-z_0)^2}}\right| \quad (7)$$

It is shown in Fig. 1 that as a result of the temperature stratification the line element (respectively area element) is shifted and rotated in comparison with the undisturbed sound propagation of a line source.

4.2 - Change of the sound intensity of a point source as a result of the sound speed profile
The intensity dP/dA for the case of a omnidirectional point source is calculated dependent on the sound power dP radiated in the area element dA within the room angle $d\Omega$ ($dP/dA = (dP/d\Omega)\,(d\Omega/dA)$). The increase or decrease of the sound level with respect to the undisturbed sound field is calculated as

$$\Delta L_{point} = 10\log\left|\frac{1}{\left(1+\frac{z-z_0}{2l}\right)^2+\left(\frac{x}{2l}\right)^2+\left(\frac{y}{2l}\right)^2}\right| \quad (8)$$

The change of the sound level of a point source as the result of the change of the sound speed with height is two times the change of the sound field in case of a line source.

5 - EXAMPLES OF THE CHANGE OF THE SOUND LEVEL AS A RESULT OF THE CHANGE OF THE SOUND SPEED

Here the measurements of Klug [13] are used which measured the temperature and wind speed up to a height of 65 m. He used the "Monin − Obukhov" similarity theory to describe the refraction profile for stable conditions as a result of measurements above ground. The "Monin − Obukhov" similarity theory used a logarithmic linear profile to describe the temperature and the wind profile in the boundary layer. With this Klug calculated sound speed profile which is logarithmic linear dependent on the height z.
In the following two measurements of Klug are used to calculate a linear approximation of the temperature, wind speed and sound speed profile. It is clear that a linear approximation of a logarithmic linear temperature and wind speed profile is height dependent and so the parameters of the linear approximation are calculated dependent on the height z.
For sound measurements at a height of e.g. 10 m up to a distance of 1000 m only the very flat sound rays e.g with a maximum height of 30 m are of interest which depends on the sound speed gradient. That is the reason that the sound speed and the wind speed gradient near the ground (until a height of 20 m) are of interest. Under near neutral conditions the sound speed is $c\ (z)$= 340,1+0,31z up to a height of 10 m or $c(z)$=364,5+0,13z at unstable conditions. With this the reference length l defined as sound speed divided by sound speed gradient is in the range of 1100 m under near neutral conditions or 2800 m at unstable conditions. The increase of the sound level of a point source as a result of the change of the sound speed is shown in Fig. 2. Temperature measurements form Ref. [14] are also shown in the figure. For example an the change of the sound level is shown for a decrease of the temperature of −8,4°C/100m (l=6700m) and an increase of 5,5°C/100m (l=10000 m).
In Fig. 2 the decrease of the sound level (as a result of the sound speed gradient) **in relation to the geometrical sound propagation** of a point source is shown. The decrease of e.g. −1 dB means that "a measurement" leads to 1 dB less that you want to expect as a result of the geometrical sound propagation.

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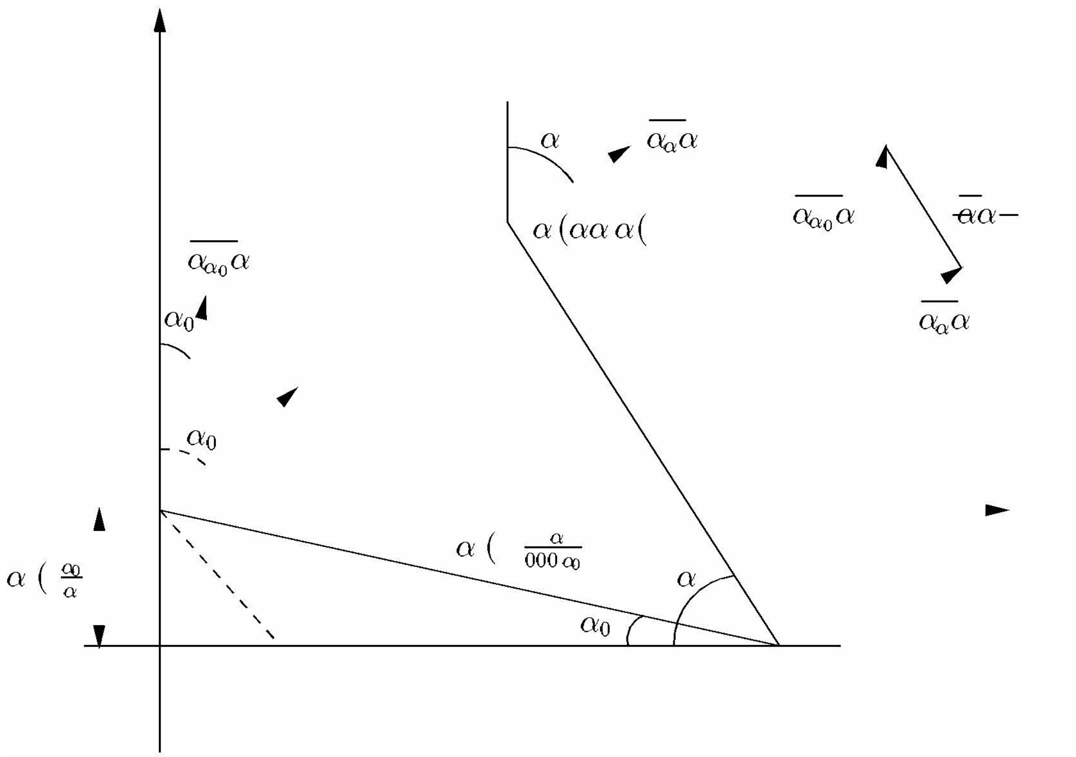

Figure 2: Decrease of the sound level **in relation to the geometrical sound propagation** as a result of the change of the sound speed.

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