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THE CONVERGENCE CHARACTERISTICS OF AN ADAPTIVE ALGORITHM FOR A MULTICHANNEL ACTIVE NOISE CONTROL SYSTEM

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ABSTRACT
The most widely used adaptive filter algorithms for the feedforward active noise control (ANC) system are the filtered-x LMS algorithm and the multiple error filtered-x (MEFX) LMS algorithm. The convergence characteristics of these algorithms for the ANC system have been studied mostly in the time domain, and it was found that the convergence properties are subject to a distribution of the eigenvalues of the autocorrelation matrix of the filtered reference signal sequence. Analysis in the time domain, however, requires a great deal of computation, and its physical meaning is unclear. This paper presents a method for evaluating the adaptive algorithm for the ANC system for multiple noise sources and multiple control points in the frequency domain. With this method, the convergence properties can be evaluated in the frequency domain with far fewer computations and give us a better understanding of the physical meaning.

1 - REVIEW OF THE MEFX LMS ALGORITHM IN THE TIME DOMAIN
Figure 1 shows a general ANC system for multiple noise sources and multiple control points. Here, I is the number of noise sources, K is the number of reference sensors of the adaptive digital filter array, which is composed of finite impulse response (FIR) filters, M is the number of secondary sources, and L is the number of control points (error sensors). Such a system will be referred to as CASE [I, K, M, L] in this paper. In a CASE [I, K, M, L] ANC system, there are $K \times I$ paths between each noise source and each reference sensor, $M \times L$ different secondary paths between each secondary source and each error sensor, and whose the multiple paths are assumed to be time invariant in this paper. There are $K \times M$ adaptive filters in the controller, which drive the secondary sources by the reference signals.
Referring to Fig. 1, the MEFX LMS algorithm [1-2], which is an extension of the filtered-x LMS algorithm for multiple noise sources and multiple control points ANC system, is summarized as follows:

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{U}(n)\,\mathbf{w}(n) \tag{1}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - 2\mu\mathbf{U}^T(n)\,\mathbf{e}(n) \tag{2}$$

where the superscript T means the transpose, μ is the step-size parameter,

$$\mathbf{e}(n) = \begin{bmatrix} e_1(n) & e_2(n) & \dots & e_L(n) \end{bmatrix}^T, \mathbf{d}(n) = \begin{bmatrix} d_1(n) & d_2(n) & \dots & d_L(n) \end{bmatrix}^T \tag{3}$$

and

$$\mathbf{w}(n) = \begin{bmatrix} \mathbf{w}_{11}(n) & \dots & \mathbf{w}_{M1}(n) & \dots & \dots & \mathbf{w}_{MK}(n) \end{bmatrix}^T \tag{4}$$

The element of $\mathbf{w}(n)$, $\mathbf{w}_{mk}(n)$, is the vector of the adaptive filter weights and may be written as:

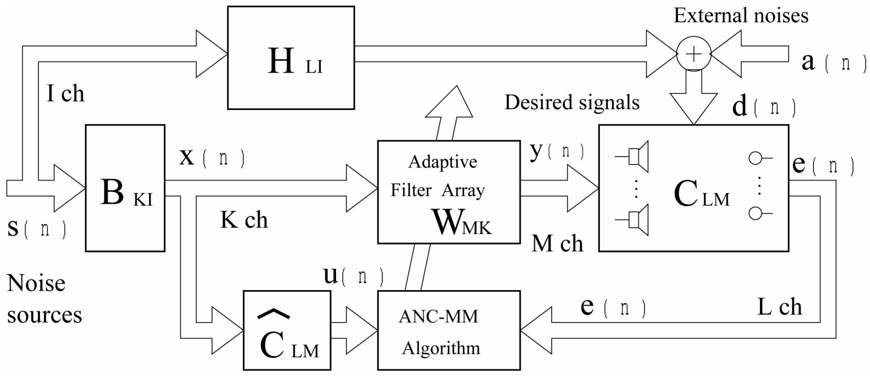

Figure 1: Block diagram of the general multiple channel active noise control system, CASE [I,K,M,L].

$$\mathbf{w}_{mk}(n) = \left[\begin{array}{cccc} w_{mk_1}(n) & w_{mk_2}(n) & \ldots & w_{mk_{N_w}}(n) \end{array}\right]$$

where N_w is the length of the adaptive filter, which should be chosen such that they are longer than the maximum length of the impulse responses of the multiple secondary paths. The filtered reference signal matrix $\mathbf{U}(n)$ is an $L \times MKN_w$ matrix defined by

$$\mathbf{U}(n) = \left[\begin{array}{cccc} \mathbf{u}_{111}^T(n) & \mathbf{u}_{1M1}^T(n) & \ldots & \mathbf{u}_{1MK}^T(n) \\ \mathbf{u}_{211}^T(n) & \mathbf{u}_{2M1}^T(n) & \ldots & \mathbf{u}_{2MK}^T(n) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{u}_{L11}^T(n) & \mathbf{u}_{LM1}^T(n) & \ldots & \mathbf{u}_{LMK}^T(n) \end{array}\right] \tag{6}$$

where the element of $\mathbf{U}(n)$, $\mathbf{u}_{lmk}(n)$, is the vector of the filtered reference signals of the length of N_w, written as

$$\mathbf{u}_{lmk}(n) = \left[\begin{array}{cccc} u_{lmk}(n) & u_{lmk}(n-1) & \ldots & u_{lmk}(n-N_w+1) \end{array}\right]^T \tag{7}$$

where $u_{lmk}(n) = \sum_{j=1}^{N_c} c_{lm_j}(n)\, x_k(n-j+1)$ and N_c is the length of an FIR filter, which is modeled the response of the secondary path, $c_{lm} = \left[\begin{array}{cccc} c_{lm_1} & c_{lm_2} & \ldots & c_{lm_{N_c}} \end{array}\right]$, from the m-th secondary source to the l-th error sensor, where

$$l = 1, \ldots, L \; ; \; m = 1, \ldots, M \; ; \; k = 1, \ldots, K \tag{8}$$

The k-th reference signal, written as

$$x_k(n) = \sum_{i=1}^{I} b_{ki}(n) * s_i(n) = \sum_{i=1}^{I} \sum_{j_b=1}^{N_b} b_{kij_b}(n)\, s_i(n-j_b+1)$$

where $s_i(n)$ is the i-th noise source, $b_{ki}(n)$ is a response from the i-th noise source to the k-th reference sensor. The convergence speed of the MEFX LMS algorithm is subject to the eigenvalue spread for the autocorrelation matrix for the filtered reference signals [1-2], as defined by

$$\mathbf{R} = E\left[\mathbf{U}^T(n)\, \mathbf{U}(n)\right] \tag{9}$$

where $E[\cdot]$ is the statistical expectation operator. Here, assuming that the reference signals are statistically stationary, the time index, n, of the matrix $\mathbf{R}$ drops. To analyze the convergence properties of the MEFX LMS algorithm in the time domain, it is necessary to calculate the autocorrelation matrix $\mathbf{R}$ of the filtered reference signal, whose size is $MKN_w \times MKN_w$.
It is therefore difficult to calculate the eigenvalues of the matrix $\mathbf{R}$, and to investigate the convergence properties of the adaptive filters. Another disadvantage of conventional analysis in the time domain is that the physical meanings of both maximum and minimum eigenvalues of the matrix $\mathbf{R}$ are unclear.

2 - FREQUENCY DOMAIN ANALYSIS OF THE MEFX LMS ALGORITHM

When the convergence speed of the MEFX LMS algorithm is slow, the adaptive filters are considered as time invariant linear filters for a given period, as well as analysis of the CASE [K, M, L] ANC system

[3]. Thus the MEFX LMS algorithm for the CASE [I, K, M, L] ANC system in the time domain can approximately be expressed in the frequency domain as in the following equations:

$$\mathbf{E}(n,\omega)=\mathbf{D}(n,\omega)+\mathbf{U}(n,\omega)\,\mathbf{W}(n,\omega) \tag{10}$$

$$\mathbf{W}(n+1,\omega)=\mathbf{W}(n,w)-2\mu^{'}\mathbf{U}^{H}(n,\omega)\,\mathbf{E}(n,\omega) \tag{11}$$

where $\mu^{'}=\mu/N_w$, and the superscript H means the Hermitian transpose (complex conjugate and matrix transpose). $\mathbf{E}(n,\omega)$ and $\mathbf{D}(n,\omega)$ are N-point DFT's of $e(n)$ and $d(n)$, respectively, at the n-th iteration, and ω is a frequency (DFT) bin defined by $\omega=0,1,\ldots N/2$. $\mathbf{W}_{mk}(n,\omega)$ is N-point DFT's of $\mathbf{w}_{mk}(n)$. The filtered reference matrix $U(n,\omega)$ is an $L\times MK$ matrix defined by

$$\mathbf{U}(n,\omega)=\begin{bmatrix} U_{111}(n,\omega) & \ldots & U_{1M1}(n,\omega) & \ldots & \ldots & U_{1MK}(n,\omega)\\ U_{211}(n,\omega) & \ldots & U_{2M1}(n,\omega) & \ldots & \ldots & U_{2MK}(n,\omega)\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ U_{L11}(n,\omega) & \ldots & U_{LM1}(n,\omega) & \ldots & \ldots & U_{LMK}(n,\omega)\end{bmatrix} \tag{12}$$

where the element $U_{lmk}(n,\omega)$ is a complex function defined by $U_{lmk}(n,\omega)=C_{lm}(\omega)\,X_k^*(n,\omega)$, $C_{lm}(\omega)$ is the transfer function from the m-th secondary path to the l-th error sensor, $X_k(n,\omega)$ is the k-th noise source defined by $X_k(n,\omega)=\sum_{i=1}^{I}B_{ki}(\omega)\,S_i(n,\omega)$, where $B_{ki}(\omega)$ is the transfer function from the i-th noise source to the k-th reference sensor.

The behavior of the MEFX LMS algorithm can approximately be analyzed at each separate frequency bin ω. In other words, the convergence properties of the MEFX LMS algorithm in the time domain are approximately determined by $(N/2+1)$, using the classical method of evaluating the convergence properties of the conventional LMS algorithm in the time domain, the convergence properties of the MEFX LMS algorithm are determined by with the eigenvalues of the power spectrum matrix $\mathbf{R}(\omega)$ defined by

$$\mathbf{R}(\omega)=E\left[\mathbf{U}^{H}(n,\omega)\,\mathbf{U}(n,\omega)\right] \tag{13}$$

Since the size of the power spectrum matrix $\mathbf{R}(\omega)$ is $MK\times MK$ at each frequency bin ω, fewer computations are required to find the eigenvalues of the matrix $\mathbf{R}(\omega)$ in the frequency domain in comparison with those in the time domain. As well as the analysis in the time domain, the longest time constant, $\tau_{\max}(\omega)$ at a frequency bin ω and $\tau_{\max}$ over the whole frequency range is obtained as

$$\tau_{\max}(\omega)>\frac{\max_\omega\{\lambda_{\max}(\omega)\}}{2\lambda_{\min}(\omega)} \quad , \quad \tau_{\max}>\frac{\max_\omega\{\lambda_{\max}(\omega)\}}{2\min_\omega\{\lambda_{\min}(\omega)\}} \tag{14}$$

where $\lambda_{\max}(\omega)$ and $\lambda_{\min}(\omega)$ are the largest and the smallest eigenvalues of the matrix $\mathbf{R}(\omega)$ at a frequency bin, and $\max_\omega\{\}$ and $\min_\omega\{\}$ denote the maximum and the minimum eigenvalues over the whole frequency range. It is clear that the convergence speeds of the MEFX LMS algorithm at different frequency bins are comparative, and the longest time constant is subject to the ratio of the maximum eigenvalue to the minimum eigenvalue of the matrix $\mathbf{R}(\omega)$ over the whole frequency range.

The filtered reference signal $u(n)$ in the time domain is expressed by the convolution of $x(n)$ and c, while $U(n,\omega)$ in the frequency domain is expressed by a simple multiplication. From Eq. (13), the determinant of the matrix $\mathbf{R}(\omega)$ can be expressed by the following equation:

$$\mathbf{R}(\omega)=|S_1(\omega)|^2\ldots|S_I(\omega)|^2\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|^{K}\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right| \tag{15}$$

It is clear that the determinant of the power spectrum matrix may be expressed in terms of multiplication of the input power spectra, $S_1^2(\omega),\ldots S_I^2(\omega)$, the determinant of matrix $\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)$, and the k-th power of the determinant of matrix $\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)$. It is well known that the determinant of the matrix is equal to the product of its eigenvalues. The eigenvalues of the matrix $\mathbf{R}(\omega)$ are influenced, therefore, by $\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|$ and $\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right|$. In general, if the determinant of the matrix is small, the smallest eigenvalue of the matrix is small, and the smallest eigenvalue of the matrix $\mathbf{R}(\omega)$ is also small. Therefore, we can evaluate the convergence speed with the ratio $\rho(\omega)$ defined by the following equations instead of Eq. (14):

$$\rho_C(\omega)=\frac{\max_\omega\left\{\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|\right\}}{\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|} \quad , \quad \rho_C=\frac{\max_\omega\left\{\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|\right\}}{\min_\omega\left\{\left|\mathbf{C}^{H}(\omega)\,\mathbf{C}(\omega)\right|\right\}} \tag{16}$$

$$\rho_B(\omega)=\frac{\max_\omega\left\{\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right|\right\}}{\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right|} \quad , \quad \rho_B=\frac{\max_\omega\left\{\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right|\right\}}{\min_\omega\left\{\left|\mathbf{B}(\omega)\,\mathbf{B}^{H}(\omega)\right|\right\}} \tag{17}$$

Since the transfer function of the secondary paths, $\mathbf{C}(\omega)$, should be measured prior to the ANC processing, the influence of the secondary paths on the convergence characteristics can be evaluated in advance [3]. The physical meaning of the matrix $\left|\mathbf{C}^H(\omega)\,\mathbf{C}(\omega)\right|$ was discussed in the literature [3], and some solutions to achieve the faster convergence speed and large noise cancellation by increasing the value of $\left|\mathbf{C}^H(\omega)\,\mathbf{C}(\omega)\right|$ at frequencies were also discussed there when $\left|\mathbf{C}^H(\omega)\,\mathbf{C}(\omega)\right|$ is small.
However, since the transfer matrix $\mathbf{B}(\omega)$ cannot generally be measured prior to ANC cancellation, a possible way to achieve good performance is to reduce the correlations among the output of the reference sensors [4].

3 - CONCLUSIONS

This paper described an evaluation method of the convergence characteristics of the adaptive algorithm for a general multiple noise sources and multiple control points ANC system in the frequency domain. The convergence speed can be evaluated separately at each frequency bin by $\left|\mathbf{C}^H(\omega)\,\mathbf{C}(\omega)\right|$ and $\left|\mathbf{B}(\omega)\,\mathbf{B}^H(\omega)\right|$. At a certain frequency bin, the small value of the determinant, results in a slow convergence speed, a large computation error and less cancellation of noise. In other words, when the value of the determinant is flat throughout the frequency range, the convergence speed is fast. A possible way to achieve good performance is to adjust the position of the reference sensors, loudspeakers and error microphones.

REFERENCES

1. **S.J. Elliott, I.M. Stothers and P.A. Nelson**, A multiple error LMS algorithm and its application to active control of sound and vibration, *IEEE Trans.*, Vol. ASSP-35, pp. 1423-1434, 1987

2. **S.J. Elliott and P.A. Nelson**, Active Noise Control, *IEEE signal processing magazine*, Vol. October, pp. 12-35, 1993

3. **G.Chen, M.Abe and T.Sone**, Effects of multiple secondary paths on convergence properties in active noise control systems with LMS algorithm, *Journal of Sound and Vibration*, Vol. 195, pp. 217-228, 1996

4. **G.Chen, M.Abe and T.Sone**, Improvement of convergence speed in the filtered-x LMS algorithm with correlated reference signals, *Electronics and Communications in Japan*, Vol. Part 3, Vol. 80, pp. 20-28, 1997