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## MODAL ANALYSIS OF STRUCTURES USING ACOUSTICAL EXCITATION

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### ABSTRACT

Conventional modal analysis techniques that include shaker or hammer testing are not always suitable for the characterization on light plate shaped structures as Printed Circuit Board (PCB). This paper presents a non-contact modal analysis technique using an acoustical source as excitation. In previous studies, literature shows that this type of excitation was used for the determination of modal parameters but few studies have used this method for the identification of mode shapes. The acoustical technique allows the determination of mode shapes with the identification of modal parameters of the structure by the measurement of the Frequency Response Function (FRF) between the pressure in the near field of the structure measured by microphones and the response the structure measured by accelerometers. Literature shows that this type of excitation is very often considered as a local source of excitation instead of a pressure source applied at all locations of the structure. These pressure sources are usually correlated. The extraction of mode shapes for the acoustical excitation is modeled by different types of system including SIMO (Single Input Multiple Outputs), MISO (Multiple Inputs Single Output) and MIMO (Multiple Inputs Multiple Outputs). The effect of the coherence of the acoustical loading at different discretized locations of the structure is investigated. The results are compared with some experimental results to show the validity of the model. The experimental comparison of the conventional techniques and the acoustical excitation are presented for PCB and industrial structures. Conclusion is drawn on the type of acoustical excitation and on the modal model to use with this technique. Discussion on application field of the acoustical modal excitation technique is also presented.

### 1 - INTRODUCTION

Acoustic excitation is a very attractive, non-contact method for excitation of flexible structures. Unfortunately, the source does not produce a localized force on the structure and the excitation must be considered as a correlated multiple input. This paper presents an original non-contact modal analysis technique, based on a MISO model (Multiple Inputs Single Output) and using an acoustical source as excitation. The mode shapes and modal parameters of the structure are given by the identification of the Frequency Response Functions (FRF) obtained by acoustic pressures measurements of the excitation in the near field of the structure at a large number of locations (according to the considered number of degrees of freedom) and by only one acceleration measurement of the structure response. A new MISO model is being developed for treating fully correlated excitation sources. As a first development step, the results from a computer simulation are compared with theoretical results to show the validity of the model.

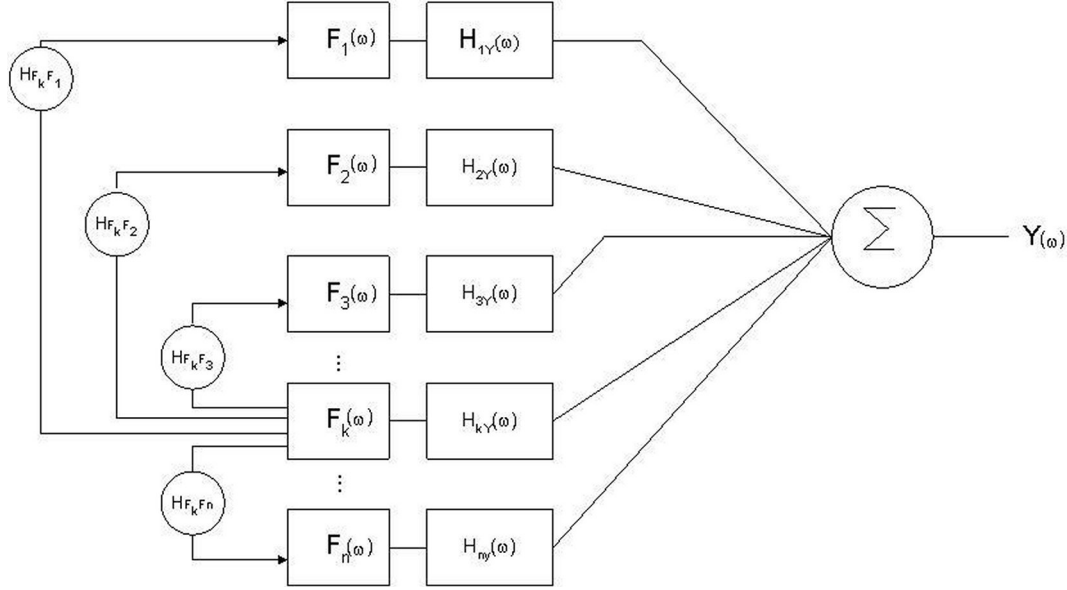
### 2 - MISO MODEL WITH COHERENT EXCITATIONS

A MISO system is defined by the application of several input forces to a structure and by the measurement of only one response as illustrated on figure 1.

When a set of perfectly coherent external forces  $F_i(\omega)$  is applied to the structure we can define the transfer function between the force  $i$  and the force  $j$  as being  $H_{F_i F_j}(\omega)$ . This relation is expressed as follows by choosing the force  $k$  as a reference:

$$S_{F_k F_i} = H_{F_k F_i} \times S_{F_k F_k} \quad (1)$$

where  $S_{F_k F_i}$  is the cross-spectrum between the force  $k$  and the force  $i$  and  $S_{F_k F_k}$  is the autospectrum of the force  $k$  (considered as the force reference).



**Figure 1:** MISO system.

These transfer functions  $H_{F_k F_i}$  depend on the characteristics of the excitation and varies for each load case  $\alpha$  according to the amplitude and phase relations between the forces. The dynamic mechanical system is characterised by a series of transfer function  $H_{iY}(\omega)$  which are specific to the structure and depend on the modal parameters. The cross-spectrum between the force  $F_k$  and the response  $Y(\omega)$  is expressed as follows:

$$S_{F_k Y} = \sum_{i=1}^n H_{iY} S_{F_k F_i} \quad (2)$$

We express the total response  $Y$  according to  $F_k$  in the following way:

$$Y = \sum_{i=1}^n H_{iY} H_{F_i F_k} F_k \quad (3)$$

The measurement of each input force  $F_i$  ( $i=1\dots n$ ) and of the vibration response  $Y$  leads to only one equation with  $n$  unknowns which are the transfer functions of the system  $H_{iY}$ . It is thus necessary to increase the number of equations available to  $m \geq n$  to be able to solve and identify the  $H_{iY}$ .

By exciting the structure with  $m$  load cases ( $\alpha=a\dots m$  and  $m \geq n$ ) and measuring each set of forces  $F_i(\alpha)$  and the response  $Y(\alpha)$ , it is possible to express the system of equations in matrix form as:

$$\begin{bmatrix} H_{F_k F_1}(a) & H_{F_k F_2}(a) & H_{F_k F_3}(a) & \dots & H_{F_k F_n}(a) \\ H_{F_k F_1}(b) & H_{F_k F_2}(b) & H_{F_k F_3}(b) & \dots & H_{F_k F_n}(b) \\ H_{F_k F_1}(\alpha) & H_{F_k F_2}(\alpha) & H_{F_k F_3}(\alpha) & \dots & H_{F_k F_n}(\alpha) \\ \dots & \dots & \dots & \dots & \dots \\ H_{F_k F_1}(m) & H_{F_k F_2}(m) & H_{F_k F_3}(m) & \dots & H_{F_k F_n}(m) \end{bmatrix} \begin{bmatrix} H_{1Y} \\ H_{2Y} \\ H_{3Y} \\ \dots \\ H_{nY} \end{bmatrix} = \begin{bmatrix} Y(a)/F_k(a) \\ Y(b)/F_k(b) \\ Y(\alpha)/F_k(\alpha) \\ \dots \\ Y(m)/F_k(m) \end{bmatrix} \quad (4)$$

where  $H_{F_k F_i}(\alpha)$ , is the transfer functions between  $F_k$  and  $F_i$  for the load case  $\alpha$  ( $\alpha=a\dots m$ ).

This system of equations can be easily resolved if  $m \geq n$  (by inverting or pseudo-inverting techniques):

$$\{H_{iY}\}_{(n \times 1)} = [H_{F_k F_i}(\alpha)]_{(n \times m)}^{-1} \left\{ FRF \left( \frac{Y(\alpha)}{F_k(\alpha)} \right) \right\}_{(m \times 1)} \quad (5)$$

where  $FRF$  is the vector ( $m \times 1$ ) containing the  $FRF$  between the force  $F_i$  (with  $i=1..n$ ) and the response of the system  $Y$ . Once the  $H_{iY}$  are obtained for each frequency, the  $n$  first mode shapes of the system can be deduced using the usual techniques.

A numerical simulation was performed on a plate with simply supported boundary conditions. The PCB material properties were used to calculate the first 7 mode shapes and natural frequencies. Numerical calculation of the vibration response of the structure for 7 load cases were performed by modal superposition. The MISO method with coherent excitations was used to determine the mode shapes and the  $H_{iY}$  of the plate. Figure 2 shows the complete transfer function  $H_{F_1 Y}$  of the system over the 0 to 400 Hz frequency domain and figure 3 shows the mode shape comparison for the first resonance frequency of the plate. The Mode Assurance Criteria (MAC) between the computed and theoretical mode shapes was equal to 1 for all the 7 identified mode shapes. This indicates a perfect mode shape identification using this model.

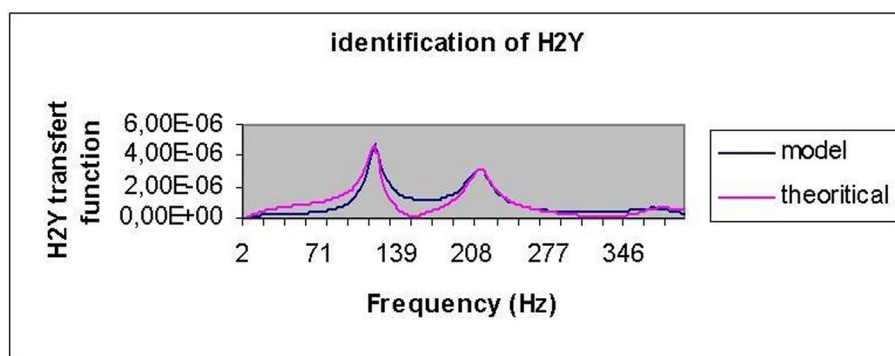


Figure 2: Identification of the transfer function  $H_{2Y}$ .

## First mode shape identification (MAC=1)

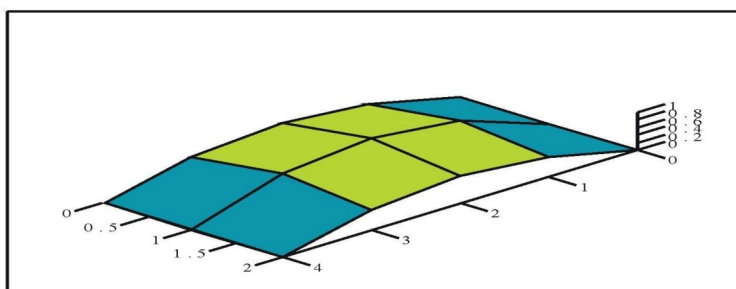


Figure 3: Identification of the first mode.

### 3 - CONCLUSION AND RECOMMENDATIONS

This experimental method requires the application on the structure of  $n$  load cases including each  $n$  forces having different amplitude and phase relations between them. The use of a loudspeaker as a source of excitation and  $n$  microphones to measure this excitation coupled with the use of an accelerometer for the measurement of the response of the structure constitutes an appropriate solution to the application of this MISO method of modal analysis. For the realisation of  $n$  load cases of the structure, the loudspeaker must be moved at  $n$  different locations and the phase and amplitude relations between the input forces

and the vibration response of the system must be measured. The advantage of this method of excitation lies in the facts that it is easy to change the load case and that it is not necessary to modify the configuration of the sensors during measurements. The method is being validated experimentally.