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NEARLY PERIODICITY IN STIFFENED PLATES AND ITS INFLUENCE ON SOUND RADIATION

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ABSTRACT
A structure designed to be spatially periodic in its configuration cannot be exactly periodic due to material, geometrical, and manufacturing variabilities. The presence of small irregularities in a nearly periodic structure may influence the propagation of vibration strongly and localise the vibration modes. A number of papers has been addressed to localisation phenomena in simplified structures. This paper will instead focus on the mean vibration field and its influence on sound radiation in a plate stiffened by beams.

1 - INTRODUCTION
A structure designed to be spatially periodic in its configuration cannot be exactly periodic due to material, geometrical, and manufacturing variabilities. The presence of small irregularities in a nearly periodic structure may influence the propagation of vibration strongly and localise the vibration field.
Anderson [1] described localisation phenomena for nearly periodic systems in solid state physics, concerning the transport of electrons in an atomic lattice (leading to a Nobel Prize). Obviously, localisation also occurs in disordered periodic structural systems, but its theoretical investigation is more difficult than that of a one-dimensional atomic lattice, since governing equations for structural systems are generally more complex. However, a number of papers has been addressed to localisation phenomena in simplified structures.
Hodges and Woodhouse [2] describe the theory and some simple experiments carried out to demonstrate the phenomenon of Anderson localisation in an acoustical context. A simple chain of pendula coupled by springs and a string with nearly equally spaced point masses is studied. A perturbation method was used in the statistical treatment and a localisation factor was calculated in the form of an exponential decay constant. Pierre and Dowell [3] investigated the localisation of modes of free vibration in discrete disordered structural systems consisting of coupled subsystems. The degree of localisation is dependent upon two parameters: the coupling between and the mistuning among the component systems. Perturbation methods are used for slightly disordered systems because they are cost effective. At the same time, they lead to very accurate results for small perturbations. Cai and Lin [4] treat a generic one-dimensional nearly periodic system using transfer matrixes. The localisation factor is defined as the limit of the logarithm of the transmission part in the random matrixes. Thus, transmitted waves have an average exponential decaying rate. Other papers on the subject of localisation of free wave propagation in nearly periodic structures can be found in [5], containing a survey of periodic and nearly periodic solution techniques.
In problems concerning sound insulation in e.g. dwellings the excitation of the system can be seen as a superposition of spatially harmonic pressure fields. The system will then vibrate and radiate sound in the receiver room.
None of the papers concerning nearly periodic systems above deals with the questions of acoustical excitation or radiation. Whether the localisation phenomena influence these problems is still an open question. Moreover, the described analysis methods are not suited for this type of problem, they consider free wave propagation or excitation in one bay and propagation in the rest of the system. Another

problem with the described analyse methods is that they are only suited for one-dimensional problems and therefor not suited for i.e. systems built up by plates [5].
This paper will instead focus on the mean vibration field caused by spatial harmonic excitation and its influence on sound radiation in a plate stiffened by supports. The localisation factor found in the mentioned literature can be seen as a virtual damping. The aim of this paper is to investigate if additional damping, in an average sense, also is found for spatially harmonic driven systems. To the author this problem seems to be important in the fields of structural and building acoustics.

2 - THE PROBLEM

Consider a linear differential operator $L[\cdot]$. A inhomogen differential equation can then be written

$$L\left[w(x)\,e^{i\omega t}\right] = F(x)\,e^{i\omega t} \tag{1}$$

for a time harmonic displacement and the time harmonic term $e^{i\omega t}$ is henceforth suppressed. In equation (1) w can be a displacement and F a force or a pressure field. Consider a plate resting on equally spaced simple supports. Take the linear differential operator in (1) to be that of a thin plate in bending,

$$L[w] = D'\Delta\Delta w - \omega^2 m" w, \qquad L = D'\left(\alpha^2+\gamma^2\right)^2 - m"\omega^2 \tag{2}$$

where γ is the excitation wavenumber in the z-direction, w is the displacement, *D'* is the bending stiffness and *m"* is the mass per unit area. The influents of fluid loading is neglected In order to simplify, set γ=0. Moreover, let the boundary condition be the homogenous Dirichlet conditions, $w(nl) = 0$, i.e. simply supported. Let the driving force be a space harmonic force $F_d = e^{-ik_x x}$. Material damping can be introduced as a complex bending stiffness $D\cdot(1+i\eta)$. When the periodicity is perfect the solution can be found with methods similar to Mace [6]. Consider a small divergence from the exact periodic formulation.

$$L[w(x)] = F_d(x) - \sum_{n=-\infty}^{\infty} F_n \delta(x - nl - \varphi_n) \tag{3}$$

where φ_n is a random number in some given distribution, and the standard deviation σ_φ is $\sigma_\varphi \ll l$. In this case we have no general periodic description. Formally the solution is

$$w(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\tilde{F}_d(\alpha)}{L(\alpha)} e^{-i\alpha x} d\alpha - \frac{1}{2\pi}\int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F_n \frac{e^{-i\alpha(x-nl-\varphi_n)}}{L(\alpha)} d\alpha \tag{4}$$

The jet unknown parameters F_n can be found by boundary conditions at x=nl, leading to an infinite set of equations. In the periodic case the solution is found solving for only one boundary condition.

2.1 - Statistic considerations

We now try to take the expected value of the nearly periodic displacement.

$$\overline{w}(x) = E[u(x)] \tag{5}$$

Thus, the expected value is taken on a sum of functions of independent random variables. The rules for the expectation operator can then be used. The reaction force is a function of every stochastic variables. However, if the reaction force is independent of the harmonic term, the expectation of the reaction force can be separated from the harmonic term,

$$E\left[F_n e^{i\alpha\varphi_n}\right] \approx \overline{F_n} E\left[e^{i\alpha\varphi_n}\right] \tag{6}$$

It can now be assumed that the expected displacement and force field is to be periodic.

2.2 - Uniformly distributed

Take φ_n to be a random number, uniformly distributed in the range $\varphi_n \in \{-a, a\}$. The corresponding probability density function f_φ is given as

$$f_\varphi(x) = \begin{cases} \dfrac{1}{2a} & \text{if } x \in \{-a, a\} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Thus, the expected value taken at each term in the sum in (20) is

$$E\left[e^{i\alpha\varphi_n}\right] = \text{sinc}\left(\alpha a\right) \tag{8}$$

which yields the mean displacement. The jet unknown parameter $\overline{F_0}$ can be found by boundary conditions at x=nl=0. The boundary condition is fulfilled in a average sense.

2.3 - Numerical example

As a numerical example, consider the plate to be a chipboard plate whit Young's modulus E=4.6×10^9 N/m^2, density ρ=650 kg/m^3, damping η=0.03 and thickness 5 mm. The periodic length is L=1m. Gracing incidence, that is $\theta = \pi/2$, $k_x = k\text{sin}\theta$.

In fig. 1 the magnitude of the vibration velocity is plotted and a peak is studied closely. Five different cases are studied; the periodic case and four cases with uniform distribution with increasing spread. The frequency region is taken from 90 to 150 Hz, with a frequency resolution of 0.01 Hz. The position is x=2L/3.

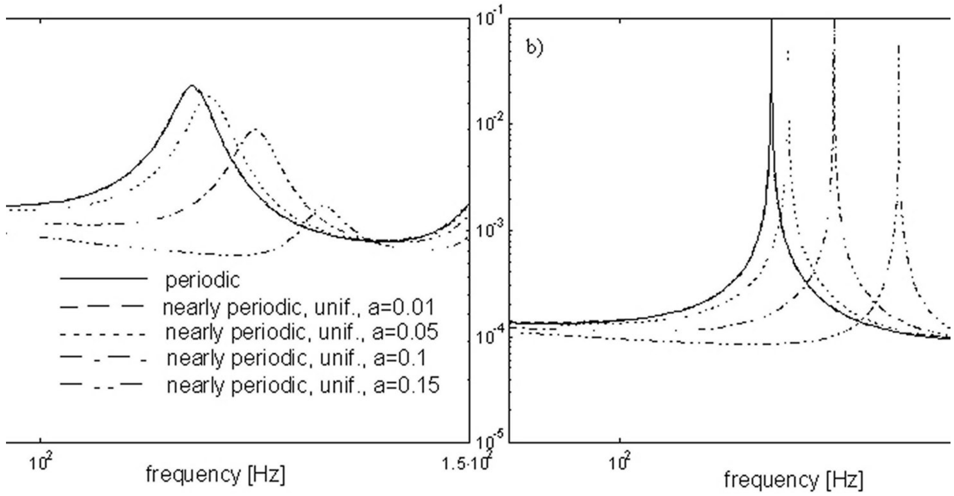

Figure 1: Magnitude of velocity due to pressure excitation, x=0.67 m, a) with damping, b) no damping.

2.4 - Conclusions from the numerical example

The tendency in the numerical example is that increasing the amount of irregularity shifts the peaks to increasing frequencies and in most cases decreases the height of the peaks. It can therefor be concluded that the irregularities increases the stiffness and the damping in the expected vibration field if material damping is present. If material damping is not present the irregularities increases only the stiffness.

3 - THE ACOUSTIC FIELD AND THE RADIATION

The vibration of the plate will cause acoustic radiation on the side opposite of the excitation, a transmitted pressure field. The Fourier transformed transmitted pressure fields are

$$\tilde{p}_t\left(\alpha, y\right) = -\frac{\omega^2\rho\cdot\bar{w}\left(\alpha\right)}{\mu\left(\alpha\right)}\cdot e^{-\mu(\alpha)y},\ y>0 \tag{9}$$

where

$$\mu^2 \equiv \alpha^2 + \gamma^2 - k_o^2$$

The transformed displacement field have only discrete wave components and are therefor easy to inverse transform, and then is the radiation determined.

4 - CONCLUSIONS

The effects of small irregularities in a nearly periodic spatially excited structure is studied with a new statistical approach. The method used is suited for sound insulation problems. Also the acoustic radiation is considered.

The irregularities causes extra damping and stiffness in the mean vibration field if material damping is present. If no material damping is present only a increase in the stiffness can be seen.

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